UCDAVIS

SUSTAINABLE TRANSPORTATION ENERGY PATHWAYS

An Institute of Transportation Studies Program

Year 2008 **UCD-ITS-RR-08-26**

Modeling of Energy Production Decisions:

An Alaska Oil Case Study

September 8, 2008

Author
Wayne Leighty
wwleighty@ucdavis.edu

Advisors
Cynthia Lin
Joan Ogden
James Wilen

Institute of Transportation Studies * University of California, Davis
One Shields Avenue * Davis, California 95616
PHONE: (530) 752-6548 * FAX: (530) 752-6572
WEB: http://its.ucdavis.edu/

Table of Contents

Glossary and Abbreviations

Adjustment cost: the cost of increasing or decreasing (adjusting) production rate. Generally, a company cannot instantly change its rate of production without incurring some costs of adjustment, which may include fixed costs that are the same for any change and variable costs that depend on the magnitude or rate of change. Oil producers generally face a tradeoff between project timeline and cost since fixing unforeseen events quickly costs more than slower response. This implies a variable adjustment cost since making an adjustment more rapidly (i.e., with project timeline the binding constraint) increases the project's cost. Other costs of oil production adjustment, like labor time and equipment replacement, may be fixed or variable costs. See the sidebar in section 4.4 for further discussion.

Cooperative (and non-cooperative) models: Models of non-cooperative behavior use game theory to account for strategic interactions. For example, two competing oil producers may impact each other's profits if they are large enough to influence market price or if they are producing from the same oil reservoir. Consequently, they each consider what the other may do in deciding on their own course of action. Models of cooperative behavior do not include such game theory. See the sidebar in section 2.1 for further discussion.

Economic Limit Factor (ELF): used to adjust severance taxes in Alaska from 1977 to 2006, the ELF was a fraction between zero and one. The nominal severance tax rate (12.5 or 15 percent) was multiplied by the ELF calculated for each field to determine the tax rate actually paid. For marginal fields near the "economic limit"

Abbreviations

AC	Adjustment Cost
ADR	Alaska Dept. of Revenue
ANWR	Arctic National Wildlife Refuge
AOGCC	Alaska Oil and Gas Conservation Commission
API	American Petroleum Institute
AS	Alaska Statute
BC	Base Cost
CEO	Chief Executive Officer
CCF	Composite Cost Function
CRWells	Constant Returns Wells plane
DCS	Drilling Cost Scalar
DDCS	Dampened Drilling Cost Scalar
Dmp	Dampener for the Drilling Cost Scalar
OLS	Ordinary Least Squares
DR	Discount Rate
DRTS_M	Decreasing Returns to Scale Margin, a factor that shifts the slope of the composite cost function
DRWells	Decreasing Returns Wells surface

of viable production, the ELF reduced the severance tax rate to encourage continued production. Calculation of the ELF was based on the number of wells and total production rate (see footnote 46). In 2006, when the ELF was eliminated as part of comprehensive revision to severance tax policy in Alaska, no field was paying 100 percent of the nominal severance tax (ELF factors by field in March, 2006 were 0.82415 for Prudhoe Bay, 0.00032 for Kuparuk, 0.0000 for Endicott, and 0.6856 for Northstar; personal communication, Dick Tremaine and Jenny Duval, Alaska Department of Revenue, July, 2007).

Economically recoverable oil: oil that can be produced at a profit given the production cost and market price. Only a fraction of the oil in a reservoir is technically recoverable, and only a fraction of the technically recoverable oil is economically recoverable. As technology improves, the fraction that is technically recoverable increases and the fraction that is economically recoverable also increases because production cost decreases. Higher market price also increases the fraction that is economically recoverable.

Extraction externality: the effect of one leaseholder's oil production reducing the reserves available for neighboring leaseholders to produce if the oil reservoir is common among them. See the sidebar in section 2.1 for further discussion.

ELF	Economic Limit Factor, an adjustment to severance tax eliminated from Alaska statute in 2006
FOB	Free On Board
NGL	Natural Gas Liquid
OIP	Oil In Place
OPEC	Organization of the Petroleum Exporting Countries
PC	Production Cost, dollars per barrel
Q	Oil Production Rate, millions of barrels per month
S	Technically Recoverable Reserves Remaining, millions of barrels
TAPS	Trans-Alaska Pipeline System
TCF	Trillion Cubic Feet
USGS	United States Geological Survey
WHV	Wellhead Value
WS	Wells Scalar

FOB (free on board): another way of saying the value (price) of oil at the point of production (i.e., the well where it comes out of the ground) rather than at the point of sale (e.g., an oil refinery). Free on board (FOB) price is equivalent to wellhead value since the buyer pays the transportation cost from origin to final destination.

Information externality: the improvement in knowledge about the likelihood of finding oil gleaned by one leaseholder from observing the results of a neighboring lease holder's exploration. See the sidebar in section 2.1 for further discussion.

Net Social Benefit: generally defined as the social benefits of an action minus the social costs of the action, where "social" implies a broad summation of benefits and costs that includes externalities. Net social benefit can be thought of as the size of the pie available for distribution. In the case of our research, we define the net social benefit of oil production as the sum of producer profit and tax revenue.

Oil in Place: the amount of oil originally present in reservoir rock. Only a fraction of oil in place is technically recoverable with current production methods, and only a fraction of technically recoverable oil is economically recoverable (i.e., can be produced at a profit).

Production cost: the cost of producing output from known fields. Generally, oil production cost includes the costs of exploration to find and evaluate oil reserves, development costs to build the infrastructure for producing the reserve, and the variable costs of actually producing output from existing wells. Since we model the production decisions of unit operators for known oil fields, "production cost" as we defined and estimated it does not include exploration costs because rational economic agents should make production decisions for known fields without regard to past exploration investments. This distinction requires care in interpreting our results for "producer profit," which are profits from oil production from which exploration and overhead costs should be deducted to approximate net corporate profits.

Severance tax: imposed on the extraction of a natural resource to compensate for the removal of the resource from the area in which it originated. In Alaska, the severance tax rate was 12.25 percent of the gross value of production prior to July, 1981, changed to 15 percent, and then changed to a tax on the net value of production in 2006. See the sidebar in section 2.2.2 for further discussion.

Structural economics: the use of economic theory to define the structure for statistical modeling. Economic theory is the basis for defining mathematical relationships between observable "endogenous" variables and "explanatory" variables. The use of economic theory to define the structure often enables estimation of parameter values with meaningful interpretation, like elasticities. See Reiss and Wolak (2007) for further discussion.

Technically recoverable oil: the quantity of oil that can possibly be recovered, regardless of cost or time, from a particular reservoir. The technically recoverable oil for a given reservoir changes as oil production technology improves.

Unitization: the process by which an agreement is reached among leaseholders to cooperate in the production from an oil reservoir that is common among them. Unitization is legally required in Alaska prior to production, to mitigate *extraction externalities*.

Unit operating agreement: the result of unitization, the unit operating agreement specifies the share (percentage) of oil and gas production each leaseholder receives and specifies one unit operator. The unit operator makes all operating decisions for the field, subject to approval from the other leaseholders.

Wellhead value: the value (price) of oil at the point of production (i.e., the well where it comes out of the ground) rather than at the point of sale (e.g., an oil refinery). The wellhead value is generally the market price less transportation cost from the production well to market.

Acknowledgements

The author would like to thank the Sponsors of the Sustainable Transportation Energy Pathways (STEPS) Program, the Chevron Graduate Fellowship at UC Davis, and the Graduate Automotive Technology Education (GATE) fellowship program for financial support of this work. Several Alaska state agencies were kind in providing data, especially Stephen McMains with the Alaska Oil and Gas Conservation Commission. The author also thanks Cynthia Lin for guidance and mentoring throughout this project and Joan Ogden and James Wilen for valuable comments and reviews of this report. The views and opinions expressed in this report are those of the author alone and do not necessarily represent those of any sponsoring organization or outside reviewer.

1 Introduction

Understanding the dynamics of optimal oil production has been a major application in economics of theories regarding finite resource extraction and dynamic programming for many decades. Recent high oil prices have caused oil-holding nations and states to revise their tax policies. Many of these revisions have tipped the tax slope (i.e., more share of both upside potential and downside risk via higher tax rate) and have introduced a variety of credits and deductions for oil company investments in the area.[1]

This report seeks to inform such policymaking by investigating the effect of government tax policy on dynamic firm behavior in oil production in Alaska. The main novelty of our paper is modeling the effects of a wide variety of tax structures (not just tax rates) on dynamically optimal oil production paths. We also develop a method for estimating field-specific cost functions without direct observations of production cost. Our research addresses questions like the following:

- Have oil producers approximated dynamically optimal production despite imperfect information (e.g., inability to predict future oil price) and stochastic production processes (e.g., equipment failures)?
- Can tax policy encourage more rapid or gradual energy production in the future?
- Does government policy create inefficiency in the oil industry?
- Are there tradeoffs between maximizing the net social benefit from energy production and achieving a desired allocation of producer profit and tax revenue?

We find that changing the tax rate alone does not change the oil production path except for marginal fields that cease production. Introducing credits or deductions into the tax policy, however, can change the oil production path, but at the expense of net social benefit, meaning either oil companies or the government will be made worse off (i.e., lower profits or lower tax revenue). Analyses of Alaska's oil production industry are particularly valuable now because of Alaska's potential role in the next several decades of US energy supply.

1.1 Organization of the Report

This report is organized into several distinct sections, with subsections in each. In the remainder of the introduction, we describe the objectives of our research, outline our modeling approach in general terms, and describe the research importance and relevance to future transportation energy supply. The purpose is to establish the motivation for this research with broad understanding of the context, methods, and potential application of results.

Section 2 provides background information on related literature, the Alaska oil and gas industry, and the concept of a dynamic model of oil production. The information in this section will be helpful for understanding the remainder of the paper, particularly the data described in section 3 and the modeling methods described in section 4.

[1] The author participated in one such policy revision as staff for an Alaska state senator. In that debate, the oil company response to tax credits and deductions for investment was assumed to be increased investment, which was assumed to improve the government tax revenue. Yet these assumptions were not supported by research. In fact, some research that did exist at the time (but was not cited in the debate) suggested that such policy changes would have negligible effect on oil company behavior (Kunce, 2003).

Section 3 provides description of the data used in this research, the estimation of an exogenous price function, and our novel method of building field-specific cost functions from relatively little cost data. Although the motivation for needing the data presented in section 3 may not be clear without understanding of the modeling methods presented in section 4, we decided to present the data – and particularly the estimation of cost and price functions – first to obviate confusion regarding modeling methods in section 4.

Section 4 presents our dynamic model of Alaska oil production, including a simple model formulation without taxes, the complete model with taxes included, sensitivity analysis, model calibration to historical production data, and the tax scenarios we evaluated with the calibrated model.

Finally, Section 5 presents results pertaining to the ability to do such modeling with limited data, evaluation of dynamic optimality in historical production decisions, and how taxes can affect production paths, profits, and tax revenue. We offer recommendations for interpretation of these results. Section 6 concludes with discussion of possible extensions of this research and recommendations for future work.

There is a glossary at the beginning of the report for clarification of terminology that is either technical or unique to this paper. The glossary also includes a list of abbreviations. Sidebars in the text, shaded grey, provide additional explanation of key concepts.

1.2 Three Primary Objectives

We have three primary objectives for this research. First, we evaluate the ability to do such modeling, with both analytic and empiric approaches, given data constraints. To address this topic, we test the limits to complexity in dynamic modeling while considering what features need to be added to the model to bring economic theory close to observed behavior (i.e., minimize discrepancy between modeled optimal production paths and actual production histories for each of the seven North Slope production units). We find that adjustment costs with fixed initial production rate are key components, as well as discount rate.

Second, we evaluate whether producers have been dynamically optimal in their production decisions by comparing the discount rates that best fit the model to historical production with the discount rate range that is considered “reasonable” for the oil industry.[2]

Third, we simulate the effect of alternative tax policies on production paths and present discounted values of producer profits and state tax revenue. We present results for a range of tax policies, including the actual historical policies and an approximation of the new policy enacted in 2006 (revised in 2007) by the Alaska legislature, with

[2] Interpretation is complicated by two competing explanations. On one hand, theory could be inadequate. Producers are successful dynamic optimizers and historical production data show the optimal path, with perturbation for stochastic events (accidents, etc.). The path computed based on theory deviates from the historical path because the theory is not adequate for predicting the optimal path. On the other hand, economic theory might accurately predict the dynamically optimal path, with the benefit of perfect hindsight, and the deviation from this path in historical data is the degree to which producers failed to be dynamically optimal in their production decisions.

implications for designing policy to maximize net social benefit (defined as the sum of producer profit and tax revenue).

1.3 Modeling Overview

We propose a simple dynamic model of oil production for seven production units (fields) on Alaska's North Slope, add taxes to the model structure, use adjustment cost and discount rate to calibrate the model against historical production data, and use the calibrated model to simulate the impact of tax policy on production rate. In Alaska, the efficiency of petroleum production may be influenced by tax and leasing policies and contract structures. Our research approach is to simulate the optimal production path and compare it to actual production data to evaluate differences. We present empirical estimates for wellhead price, drilling cost, an inverse production function for producing wells, and production cost functions. A variety of modeling frameworks are discussed and the potential benefits of such modeling are proposed.

1.4 Research Importance

Our research in Alaska seeks better understanding of oil production decision-making and how to model these decisions. By improving upon a simple dynamic model of oil production to incorporate more realism in producer decision-making, cost functions, and the policy context in which decisions are made, we examine whether producers have been successful dynamic optimizers. The resulting insights are useful for the design of efficient policies that will be important for future petroleum development and may be relevant for other energy industries as well.

- The degree to which actual production history deviates from the modeled optimal path may represent an unclaimed profit opportunity for producers as well as lost tax revenue for government.
- With the methodological question of what features need to be added to bring theory close to reality answered, simulation of production paths and resulting revenue streams can indicate which tax policies are likely to yield higher producer profit and/or government revenue.
- For Alaska, the state legislature made major changes to the oil and gas tax system in 2006 and 2007. The government should consider what impact those changes may have on production decisions since tax revenue is determined by the combination of tax policy and production decisions.

Alaska has 37.5 trillion cubic feet (TCF) of proven natural gas reserves and over 100 TCF of likely resources (USGS, 2005). This quantity is sufficient for supplying approximately six percent of total United States demand for 30 years, but the gas is stranded without construction of a $25 billion pipeline (Alaska Gas Pipeline website, 2007). As the state develops policy for the commercialization of this resource, lessons about pipeline sizing and the influence of policy on production paths in the oil industry may apply.

For other energy supplies, like wind power or biofuels, lessons from the oil industry may help to inform what policy levers would be effective stimulus for faster development and production. The models of dynamic behavior that emerge from our research are models of firm-level decision-making and of the effects of policies and institutions on these decisions. As such, similar models may be used in the future to examine firm behavior and decisions under many pending policies, like the low carbon fuels standard in California and, more broadly, all types of policies that employ incentives or penalties to encourage developing alternative sources of energy.

Implications of this research include the following. For oil producers, evaluation of the dynamic optimality of past production may inform future production decisions. For policy makers, evaluation of the effects of policy tools on producer behavior may provide insight into means for encouraging more rapid or gradual energy production in the future, and tradeoffs between maximizing net social benefit and achieving a desired allocation of producer profit and tax revenue.

1.5 Relevance to Future Transportation Energy

The question of how policy effects oil production is important for our transportation future for several reasons. In the short-term, our current transportation systems depend on oil for more than 95 percent of their energy supply (EIA, 2008) and Alaska accounts for 13 percent of total US oil production and supplies five percent of total US oil consumption (EIA, 2008b). Understanding Alaska oil production is important for understanding our transportation energy supply in the short term.

In the long term, understanding past energy production decisions and how policy can impact these decisions will help us understand how future energy development and production may occur and be guided. It is increasingly clear that new, low carbon transportation fuels will emerge over the next several decades for several reasons. Public policy is beginning to internalize the costs of global climate change, which will re-shuffle the relative costs of energy alternatives by adding cost for CO_2 and other GHG (California Assembly Bills 32 and 1493; Leighty et al., 2007). The world will reach peak oil production rates, at which point supply will begin to diverge from demand and alternative primary energy sources will become more competitive (Campbell and Laherrere, 1998; Rogner, 1997). New technologies and energy conversion devices will change the value proposition of energy forms (Williams, 2006).

Many researchers have focused on the systems optimization for emerging energy markets, from engineering economic optimization of hydrogen pipeline systems (Johnson et al., 2005) to biomass feedstock gathering and plant location (Parker, 2007) to impacts of new transportation fuels on the electric grid (McCarthy et al., 2007 and 2008). Economics-based research is also needed to explore the potential effect of policy on industry behavior in these emerging energy markets. Our research takes a step toward this goal by developing a flexible dynamic framework that may be adapted to other energy industries.

Our development of a model for understanding dynamic production behavior in the Alaska oil industry may provide a foundation for similar modeling of the potential Alaska natural gas industry, which may be an important component of the future domestic energy supply, and other low-carbon energy sources.

2 Background

This section provides background information on related literature, the Alaska oil and gas industry, and the concept of a dynamic model of oil production that may be useful for understanding the remainder of the paper, particularly the data described in section 3 and the modeling methods described in section 4.

2.1 Related Literature

The oil crises of 1973 and 1979/1980 motivated modeling designed to forecast future supply and demand for US crude oil resources. A dichotomy formed between models based on economic theory describing supply and demand interactions (Dasgupta and Heal, 1979; Pindyck, 1982; Horwich and Wimer, 1984; Griffin, 1985) and engineering-process models that simulate the exploration, development, and production processes (Davidsen et al., 1990). The former generally exclude physical and engineering factors that influence the supply of oil while the latter generally exclude economic forces (e.g., prices) that influence supply and demand. Neither approach accurately forecast future supply and demand (Kaufmann, 1991).

What is "optimal" and why is oil production a "path"?

Many disciplines seek to optimize a situation by maximizing or minimizing an "objective function." In economics, the objective function is often assumed to be either cost or profit. For example, the plant manager's optimization problem is to minimize cost for a particular level of output while the CEO's optimization problem is to maximize profit. Since profit is generally defined as revenue less cost, the CEO is taking the plant manager's cost minimization as given and is trying to maximize revenue.

To find the profit-maximizing production plan, a method is needed for adding up profit that is earned over time in a consistent manner. The notion of discount rate is used for this purpose. One way to understand the discount rate is to realize that a dollar in your pocket today is worth more to you than a dollar next year because you can put the dollar to work earning interest in a bank. In the case of oil production, this means the CEO is trying to maximize the "net present value" or "present discounted value" of the entire stream of future profits, where profits earned next year are worth slightly less than profits earned today. Consequently, unless specified otherwise, the "optimal" oil production path is the one that maximizes the present discounted value of profits.

However, in some cases it may also be important to recognize objectives other than profit maximization as well. For example, investing in exploration to increase the quantity of oil a company can access for production (known as "bookable reserves") can increase stock value by improving the prospects for future production. Increasing the size of a company can improve the CEO's cache. If the oil producer is a national oil company, it may have social goals like delivering short-term revenue for building infrastructure to help diversify the economy.

Consequently, the profit-maximizing objective function is an assumption, albeit a standard one, underlying most oil production modeling exercises.

Finally, the optimal oil production plan is called a path because the plan is specified for all periods into the future (tracing a path on a production vs. time graph). Oil production is an inherently dynamic optimization problem, meaning current-period decisions impact future-period opportunities, because each reservoir contains a finite quantity of oil. Thus, production today impacts future period profits by reducing the reserves that remain. Consequently, the production plan that optimizes the present discounted value of profits will specify the production in every period into the future – an entire production path. See Figure 20 for an example of this "path."

Ruth and Cleveland (1993) extended this literature by using a nonlinear dynamic model of oil exploration, development, and production coded in STELLA[3] to simulate optimal depletion paths for the 48 contiguous United States in the period 1985 to 2020. They used the theoretical model of optimal depletion developed by Pindyck (1978), which considers supply and demand, with their own econometric estimation of supply and demand parameters. The authors model demand and all three phases in oil supply – exploration, development and production – to "derive optimal time paths for drilling rates, discoveries, production, costs, and prices of crude oil." Similarly, Rao (2000, 2002) used a dynamic model to examine the "joint production-investment decision for the entire supply process from drilling through production" for petroleum resources in India.

These integrated modeling efforts for oil industries produced interest in more detailed consideration of producer-level decision making. A series of papers on the Gulf of Mexico oil industry is perhaps the best example of structural econometric modeling of decision-making in an oil industry. Papers by Hendricks and Kovenock (1989) and Hendricks and Porter (1993, 1996) analyzed the learning and strategic delay caused by information externality associated with exploratory drilling and found that plausible non-cooperative models generated reasonably accurate and more descriptive equilibrium predictions than cooperative models. However, these papers were based on theoretical models and reduced-form empirical analyses. The subsequent work by Lin (2007) improved upon these models by using a structural model to estimate the effects of a neighbor's actions on firm profits and by adding real options theory in the structural econometric context to model the multi-stage investment timing game of exploration and development.

[3] STELLA is systems modeling software that uses an icon-based graphical user interface rather than the command-line coding common in other software packages. STELLA is well suited for modeling system evolution over time with stocks and flows for discrete or continuous processes.

What are cooperative versus non-cooperative models?

Modeling behavior in an industry like oil production requires some knowledge or assumption about the industry structure (sometimes testing for elements of industry structure is a research objective). Intuitively, we would expect behavior to be different for monopolies (one company), oligopolies (a few companies), and situations of perfect competition (many homogenous companies). These structural differences require differences in model construction as well. For example, the ability of monopolies (and to some extend oligopolies) to influence both supply quantity and price means price is an endogenous part of the model (via the market-clearing equilibrium of supply and demand). Conversely, perfect competition is modeled with exogenous price.

In an oligopoly situation, which is often the case for oil production where there are relatively few large producing companies, the dichotomy of cooperative vs. non-cooperative modeling becomes important. If companies are coordinating their production plans (i.e., cooperating, like OPEC has done in the past), then one should model optimization of oil production for the whole region. It's as if there is a single CEO of a single company making production decisions, with production then allocated among the several actual companies (in fact, this is essentially how OPEC has operated in the past).

If companies are not cooperating, but rather are making production decisions independently and in competition with one another, then one should include strategic interactions in modeling each company's optimization. Modeling with strategic interactions implies game theory and the fundamental concept of best responses – each company considers the likely response of other companies to its actions and formulates a best response given the others' likely responses.

Thus, the distinction between cooperative and non-cooperative modeling is essentially a distinction between including game theory in the model or not, which is determined by whether the industry structure implies strategic interactions are important. Non-cooperative models incorporate game theory to account for strategic interactions.

The work by Lin (2007) documented the potential impact of government leasing policy on multi-stage investment timing decisions in oil exploration and development in the Gulf of Mexico. The focus in Lin's work was on the potential for lease tract size, set by government leasing policy, to induce wasteful non-cooperative strategic behavior due to competing information and extraction externalities. The modeling we develop for the Alaska oil industry is different in methodology and specific research questions, but addresses the same fundamental question of whether government policy creates inefficiency in the oil industry. Our model of Alaska unit production decisions[4] incorporates the impact of government policy on the dynamic optimal control problem inherent in these decisions, but focuses on the production phase rather than exploration and development investments.

[4] The concepts of unitization and unit production are described in section 2.2.

Externalities in Oil Production

One way to define the term "externality" is an impact on one party that is not considered by the actor causing the impact. For example, you may benefit from seeing and smelling my flower garden, but I may plant it for my own reasons without considering its impact on you. The flower garden provides a positive externality. Untaxed pollution from industry is often cited as a negative externality because the industry does not consider the costs associated with health effects and environmental damage in its financial decision to pollute.

For oil production, two forms of externality have been well documented (see Lin, 2007, for further discussion of both). Both stem from the fact that oil occurs in particular locations around the world in geologic formations that trap the oil (i.e., reservoirs) and the general practice of leasing tracts of land to confer the right to explore for and produce oil vertically beneath these leases.

Information externality relates to the exploration stage of oil production. If oil is found one leased tract, there may also be oil beneath neighboring leases if the leases are close in proximity because they may share the same geologic structure and same oil reservoir or may share similar geologic features that contain multiple oil reservoirs. Thus, if one lease holder can observe her neighbor's success in finding oil, an information externality is conferred since the neighbor's action in finding oil impacts our hypothetical lease holder's assessment of her own likelihood of finding oil. Technically, this is referred to as Bayesian Updating wherein the prior assessment of the probability of finding oil is updated with new information. In fact, observing a neighbor's discovery of oil is quite easy if she acts on it because a production rig is visually distinguishable from an exploration drilling rig. The result is a positive externality – like the flower garden – where one leaseholder's exploration investments confer a benefit to neighbors who can observe the results and get a better idea of whether they should invest in exploration. The incentive in this case is to delay one's own exploration in hopes of benefiting from this positive externality.

Extraction externality relates to the extraction stage of oil production. If neighboring leases share a common oil reservoir, several leaseholders will be producing from the same resource and the production decisions of one will impact the others' ability to produce. Actor Daniel Day-Lewis explained this notion of "drainage" in the 2007 film "There Will Be Blood" as follows,

> *If you have a milkshake, and I have a milkshake, and I have a straw… my straw reaches across, and starts to drink your milkshake. I drink your milkshake! {slurp} I drink it up!*

The result is a negative externality – like pollution – where one leaseholder's oil production can effectively reduce the reserves available for neighboring leaseholders to produce. The incentive in this case is to produce faster than one's neighbors to mitigate the negative externality by getting more of the common oil resource.

The study by Kunce (2003) is directly related to our consideration of the impact of severance tax policy on oil production industries. It extended previous research by Deacon et al. (1990) and Moroney (1997) by generalizing the analysis to all U.S. states with a dynamic profit-maximizing framework.[5] Kunce continued in the vein of integrated modeling of exploration, development and production by embedding tax policy into Pindyck's (1978) theoretical model of exhaustible resource supply.[6]

In this paper, we model producer behavior at the field level, taking known fields as given (i.e., the exploration stage is complete) and modeling production decisions only. That is, we do not model the exploration stage of oil production but rather model the profit maximizing extraction of a known reserve. Consequently, our model is not intended to forecast future production but rather to simulate the effect of tax policy on field-level production decisions. Despite the difference in methods, we produce results similar to those found by Helmi-Oskoui et al. (1992) and Kunce (2003), namely that tax policy has relatively little affect on the optimal time path of production but does change the allocation of surplus between producer profit and government revenue.[7] However, we offer the additional insight that tax policy <u>can</u> affect the production path if distortionary components like credits and deductions are introduced. Future work may expand our Alaska case study with one or more of the integrated dynamic empirical frameworks discussed above, to examine the impact of tax policy on exploration and development activities and the impact of modeling method on results. Case studies of the Alaska oil industry are particularly relevant now since the state revised its severance tax system in 2007, is considering a pipeline contract and policy structure under which 37.5 TCF of natural gas will be commercialized, and likely holds oil reserves that may be produced in the future (see section 2.2).

---

[5] Deacon et al. focused on California; Moroney focused on Texas. Another example of previous work considering the impact of tax policy on oil production industries is Pesaran's (1990) econometric model of offshore oil production in the UK which was extended to include taxes by Favero (1992). However, the shadow value of oil in these analyses is not always positive, suggesting overestimation of the impact of taxation on profit (Kunce, 2003).

[6] Other "Pindyck-based simulation studies" that consider the effects of taxation on exploration and production include Yucel (1989) and Deacon (1993). These studies were focused on "assessing the generality of theoretical results obtained in more limited settings" rather than empirical case study of a particular oil industry or change in state tax policy (Kunce, 2003).

[7] Helmi-Oskoui et al. (1992) added the interesting twist of using reservoir pressure (based on actual well data and reservoir characteristics) as a control variable in their dynamic model of joint oil and natural gas production. They argue that, "controlling the reservoir pressure and bottom well-hole flowing pressure of the producing well are key elements in petroleum production from a given reservoir" (Helmi-Oskoui et al., 1992). We do not include reservoir pressure explicitly in our modeling, but proxy for it with the diminishing returns to production rate built into our cost function. Helmi-Oskoui et al. also included the effect of tax policy on production in their modeling, but found that "production and severance taxes, federal corporation income taxes, and depletion allowances do not affect the optimal time path of oil and gas production… because the tax deductions and depletion allowances only affect the net revenue but not the production and energy requirement," which is also consistent with Uhler (1979). However, Helmi-Oskoui et al. did find that, "the imposition of taxes increases the present value of the revenues of the state and federal governments and decreases the revenues of the firms for all discount rates" and the "discount rate is an important factor in the determination of joint production rates and the length of production periods." Our findings are quite similar, with the added insight that tax policy <u>can</u> impact the production path if distortionary components like credits and deductions are introduced.

2.2 The Alaska Oil and Gas Industry

The oil and gas industry in the state of Alaska presents a unique "laboratory" for the study of primary energy production for several reasons. The state is isolated, with only one export point for oil at the port of Valdez. Oil from Alaska's North Slope is delivered to market via 800 miles of Trans-Alaska Pipeline System (TAPS) and approximately 3,000 miles of tanker travel (Alyeska Pipeline Service Company, 2007; Kumins, 2005). As such, the physical boundaries of the market are well defined.

2.2.1 History of Oil Production

The history of oil production in Alaska runs from the late 1950s to the present. The first oil leases were sold in the Cook Inlet area near Anchorage in 1959. But the discovery of the Prudhoe Bay oil field on Alaska's North Slope in 1968 signaled the start of what we now consider Alaska's oil and gas industry. The Prudhoe Bay field contained nearly 20 billion barrels when discovered, making it more than double the size of the second largest oil field in the United States, the East Texas oil field (personal communication, Vincent Monico, BP-Alaska, 2 July 2007). Completion of the Trans-Alaska pipeline in 1977 created a means for delivering this oil to market. The pipe carried peak flow of 2 million barrels per day in 1988 and currently carries just under 1 million barrels per day (Alyeska Pipeline Service Company, 2007). Structural breaks define the following three distinct periods.[8, 9]

- 1957 to 1977 was a period of oil discovery, exploration, and limited development that occurred before completion of the 800-mile Trans-Alaska Pipeline connecting the North Slope oil fields to the port of Valdez. These events occurred under Alaska's initial tax laws, including the corporate income tax, property tax, royalty, and severance tax.
- 1977 to 2006 was a period of oil production after revision of Alaska's severance tax system to include the Economic Limit Factor (ELF), that was intended to spur new exploration and development investment.[10]
- 1987 to 2006 was a period of oil production after significant revision of Alaska's corporate income tax.

The composition of firms active in oil exploration and development in Alaska has changed over time, leading to the present situation of only three primary oil producers active in the state: BP, ConocoPhillips, and ExxonMobil. The small number of players presents a situation where economic theory would suggest the possibility of strong strategic considerations and the potential for collusive actions. The questions of whether

---

[8] These structural breaks afford the opportunity to evaluate the impact of changing conditions – especially the construction of infrastructure for delivering oil to market and several changes to the tax landscape – on producer investment decisions, and provide valuable reference points for modeling strategic behavior. However, much of the potential for use of these structural breaks in modeling strategic behavior is left to future work.

[9] Note, the requirement for unitization was passed in 1955 (amended in 1978 and 1980; AS 31.05.110), which was before production on the North Slope began, so it is not a structural break that is relevant for our modeling.

[10] The petroleum production tax (PPT), passed in 2006, replaced the gross-profits-based ELF system with a net profits tax, thereby creating another structural break and defining the start of a new period in Alaska's oil industry (Petroleum Production Tax website, 2007; Alaska Department of Revenue website, 2007; Alaska State Legislature website, 2007).

strategic considerations and collusion are substantial in Alaska are important to policy makers in the state.

What are structural breaks?

Modeling behavior in an industry like oil production requires some knowledge or assumption about the industry "structure" – how many companies are in business and under what rules do they interact. To help identify the appropriate structure to use in modeling, economists ask whether companies in the industry are similar in their production methods and products, whether there are barriers to entry of new companies, whether the companies have good information about the marketplace, and whether any one company is large relative to the market size. Sometimes testing for elements of industry structure is a research objective in itself.

The results from a model built on one particular industry structure (e.g., perfect competition with exogenous price) are only valid for that particular industry structure. If something in the underlying structure changes, the model forecasts may no longer hold. For example, a nice paper by Moschini and Meilke (1989) identified a structural break in the demand for red meat and poultry when the health effects of cholesterol were documented and publicized. There was a shift in demand that market models predicated on no-cholesterol-knowledge demand structure could not have predicted.

In oil production, tax and regulatory policy changes are common sources of structural change. An oil producer makes production plans based on the current tax regime but likely cannot predict what future policymakers will enact. A change in the tax policy, however, may change the rules of the game in a way that would change the producers' optimal production plan. This kind of structural break is one of the main topics of our research.

However, the legal requirement for unitization prior to production in Alaska likely mitigates the potential for strategic interactions in oil production. This requirement (described below) is somewhat unique to Alaska and impacts our modeling of production decisions.

Oil leases are two-dimensional polygons on the earth's surface, many of which may be located vertically above the same oil resource. If multiple different lease-holders are producing from the same common resource, strategic considerations may lead to inefficient results (e.g., a race to pump faster than optimal since some oil is lost to the other lease holders as a consequence of waiting to pump) (Lin, 2007). The policy of mandatory unitization is intended to mitigate this extraction externality. When a new oil field is found in Alaska, its extent is carefully mapped and all lease-holders with a claim on the reserve must agree on a unit operating agreement prior to any production (AS 31.05.110). The primary components of this agreement are the production shares (percentages of total production) for oil and gas and designation of a unit operator (the company that will make all operating decisions, subject to approval from all the other companies involved). Production shares are based on geologic assessment of the percentage of the reserve beneath each lease and are extremely contentious and valuable.[11] Some companies want to be unit operators to gain experience with technology and operations while others do not (personal communication, Vincent Monico, BP-Alaska, July, 2007). For this research, the salient point is that these required unit agreements eliminate the strategic interactions present in other places during the production phase since the unit operator makes production decisions for the entire field. Thus, we can consider the decisions of the unit operator as the single owner of the resource, optimizing production without strategic consideration with regard to the other owners of the common resource. Hence, we model oil production for the seven individual units on Alaska's North Slope: Prudhoe Bay, Kuparuk River, Milne Point, Endicott, Badami, Colville River, and Northstar (Appendix A).[12]

In practice, however, it is not as simple as we have described and some strategic interactions persist. For example, the production shares for oil and gas are usually quite different since some leases are located above the oil reserve while others are above the gas cap. For example, the production shares for Prudhoe Bay are the following: 51% oil and 14% gas for BP; 22% oil and 42% gas for Exxon; 22% oil and 42% gas for ConocoPhillips (Libecap and Smith, 1999). Since natural gas on the North Slope is stranded without a pipeline to deliver it to market, the unit operator may wish to process associated gas into natural gas liquids (NGL) for shipment down TAPS or for re-injection to boost oil recovery (flaring is not permitted), depending on its relative oil and gas shares of production (ibid). For example, when BP took over as unit operator of Prudhoe Bay in 2000, it was clear that BP would benefit from re-injection while the other companies would benefit from NGL processing, and litigation over unit management decisions ensued (ibid).[13] Although such strategic interactions are largely resolved in negotiation and court rooms rather than by non-cooperative strategic behavior in the marketplace, future work may include consideration of the impact of unit agreement contract structures on production decisions.

2.2.2 Future Oil and Natural Gas Production

High oil prices are prompting major new policy development and infrastructure investment in Alaska. The Alaska Legislature adopted an entirely new severance tax system in August 2006 and then again in November 2007.[10] The state is also currently negotiating the contractual context for construction of a $25 billion, 3,000-mile natural gas pipeline to bring 37.5 trillion cubic feet (TCF) of known natural gas reserves on the North Slope to market.[14] Analyses of Alaska's oil production industry are particularly

---

[11] Production shares are carried out to the tenth decimal and revision of the fifth decimal for the Prudhoe Bay field equates to tens of millions of dollars (personal communication, Vincent Monico, BP-Alaska, 2 July 2007).

[12] We will use the terms "unit" and "field" interchangeably hence forth.

[13] Prudhoe Bay was comprised of the East Operating Area (operated by ARCO) and the West Operating Area (operated by BP) prior to 2000 (BP, 2006). We abstract from this complexity by treating Prudhoe Bay as a single field in our modeling.

[14] USGS, 2005; Petroleum Production Tax website, 2007; Alaska Gasline Inducement Act website, 2007. In fact, the former governor of Alaska, Frank Murkowski, negotiated a contract for the construction of this natural gas pipeline, but the legislature did not approve the contract before the end of his term of office (Alaska Gas Pipeline website, 2007).

valuable now because of Alaska's potential role in the next several decades of US energy supply.

A Primer on Oil Taxes

Oil production in the United States is taxed in four ways - royalty, severance, property, and income taxes. The relative magnitudes of these four types of taxation differ greatly among oil producing states (see Deacon et al., 1990 for comparison of Alaska, California, Louisiana, Oklahoma, Texas, and Wyoming).

Royalty refers to payments made to a landowner for the rights to produce oil. If the landowner is the federal government, these royalty payments are 12.5 to 16.7 percent of the value of the oil and gas actually produced (12.5% for onshore, 16.7% for offshore). In Alaska, most oil production occurs on state-owned land. Lease terms for these state lands have varied over time for different lease sales and areas, but the most common royalty rates are 12.5% and 16.7% as well. Finally, royalties can often be paid in value or in kind, with the former payment made in dollars based on market price (less downstream costs incurred) and the latter payment made in barrels of oil, which the recipient must then market and sell. The option for royalty in kind is often used infrequently only as a check on producer-reported market sales revenue because establishing their own oil sales capability is difficult for royalty recipients.

Severance tax is imposed on the extraction of a natural resource, for its severance from the state in which it originated. This tax is generally levied by the state regardless of the landowner as recompense for the general population for the removal of a natural resource from their state. In Alaska, the severance tax rate was 12.25 percent of the gross value of production prior to July, 1981, when it was changed to 15 percent, and then was changed to a tax on the net value of production in 2006. Since at least 25 percent of severance tax receipts are deposited in the Alaska Permanent Fund, which now has a balance of more than $35 billion and pays annual dividends to all Alaska residents based on a rolling average of earnings on the principal, the conversion of natural resource wealth into financial wealth implied by the concept of severance tax is literal and for all Alaska residents.

Property tax for oil production is generally very similar to other types of property tax. The tax is based on a small percentage of the value of all capital assets owned in a particular area. In the case of oil production, these assets are often pipelines, drilling rigs, production platforms, and the like. The property tax is often collected for use by local government.

Similarly, corporate income tax for oil production is similar to other corporate income taxes, levied as a percentage of net profit from operations in a particular jurisdiction.

Natural gas is often cited as a clean fossil energy source for future energy systems.[15] If climate change becomes a more significant motivation in energy decisions, demand for low-carbon natural gas will grow. Thus, understanding future natural gas supply in the United States is relevant to a wide range of future scenarios, from business as usual to hydrogen-fueled vehicles. Alaska's proven reserves of 37.5 TCF of natural gas is projected to provide 6.5 percent of United States supply for the period 2016 to 2030 (Alaska Gas Pipeline, 2007). But infrastructure for delivering this gas to market has not been built for a variety of reasons, including strategic considerations (Leighty, 2007).

Similarly, the potential for additional oil exploration and development in Alaska (e.g. in the Arctic National Wildlife Refuge, ANWR) will likely be a perennial topic of interest as oil becomes more scarce, and will require development of new institutional and regulatory frameworks.

Consequently, studying the effects of institutions and policies on production decisions in Alaska to find policy parameters that lead to socially desirable outcomes is especially important. By analyzing dynamic behavior under existing policies and institutions, we can improve national energy planning and policy for the future.

2.3 A dynamic model of unit production

We focus on production decisions rather than exploration and development investment decisions because the conditions in Alaska are not conducive to econometric analysis of the first two stages. In the Gulf of Mexico, Lin modeled exploration and development investment timing decisions in a situation where many producers compete and make these decisions independently (Lin 2007). In Alaska, there are few oil producers and cooperation is required by law (in the form of unit operating agreements) prior to oil production. The mandate for eventual cooperation would likely complicate modeling of the exploration and development stages leading up to production. We avoid such complication by starting our modeling after unitization and by not including exploration or development investment timing decisions. This separation of the production phase from preceding exploration and development phases is justified by the notion of forward-looking rational economic agents who make production decisions based on future revenue without regard to past activities.

[15] For example, ongoing research suggests on-site reformation of natural gas will be the low cost hydrogen production method for vehicle fuel until significant market penetration (perhaps 10 percent) of hydrogen vehicles is achieved (Personal Communication, Nils Johnson, presentation in STEPS seminar at UC Davis, 2007). Understanding future natural gas supply in the United States is relevant to scenarios for hydrogen-fueled vehicles.

Three Stages of Oil Production

The production of crude oil can generally be divided into three stages – exploration, development, and extraction (or production). The exploration stage involves seismic geologic and geophysical mapping of the reservoir rock to identify likely reservoirs and "wildcat" drilling to confirm the presence of oil.

The development stage involves drilling the production and injection wells necessary to recover oil in large quantities and building the surface infrastructure to process the oil and send it to market. Surface facilities generally include roads, well pads, equipment and maintenance facilities, employee housing and facilities, and collector pipelines to bring oil together from several wells. In Alaska, surface facilities also included the $8 billion Trans-Alaska Pipeline system to bring oil 800 miles to the tanker terminal in Valdez and, since flaring of associated gas is not permitted, a $2 billion central gas processing facility to separate natural gas liquids for shipment down TAPS and natural gas for re-injection into the oil reservoirs.

The extraction stage is where actual oil production occurs. In addition to the variable costs of extraction like labor, energy for equipment and pumping, and equipment depreciation and replacement, extraction may also require some well drilling. This is because initial producing wells are often drilled "downdip" of the reservoir "crest" (i.e., below the highest point) and injection wells are often drilled below the oil/water contact in a reservoir. As oil is produced and water injected, the oil/water contact rises, causing initial wells to "water out" and requiring new wells to be drilled "updip."

We use economic theory and empirical data to model both the physical component and behavioral component of oil production. The physical component is the extraction of a finite resource (i.e., reserves remaining equal original reserves less cumulative extraction) and the behavioral component is the maximization of an objective function (we assume profit maximization). A dynamic model is appropriate for oil production modeling since production today impacts reserves quantity tomorrow, meaning current period decisions will impact future period profits.

The theory for dynamic modeling of non-renewable resource extraction dates back to the work of Harold Hotelling (Hotelling, 1931). As Lin has carefully documented (Lin 2008), many researchers have subsequently used and built on this basic theory. We continue this approach, combining the Hotelling model with optimal control theory to compare simulated optimal oil production with historic actual oil production in Alaska. The general approach is to develop an understanding of the physical processes and economic conditions that characterize an industry, define these processes and conditions in the equations of a dynamic optimization model, and then estimate parameters in the equations via matching the model to real-world data. The motivation for comparing model results to historical production is to better understand how well producers have optimized production, how economic theory differs from reality, and how policy may affect production decisions.

The result of this research is a model for estimating the optimal oil production path and how that path may change under different government tax policies and unit contract structures, and for evaluating how closely Alaskan oil producers have approximated the optimal rate of production. The model will also enable evaluation of whether tax and leasing policies and contract structures have introduced inefficiencies in Alaska petroleum production, thereby informing the design of policies and institutions that lead to more socially efficient and desirable outcomes.

2.3.1 The Multi-Stage Investment Timing Game

Firms producing petroleum in Alaska make the following decisions: 1) whether to bid on a lease; 2) whether to invest in a seismic study of a particular area; 3) whether to apply for exploratory (or any) well drilling; 4) whether to proceed with exploratory well drilling; 5) whether to initiate, participate in and/or complete a unit agreement; 6) whether to invest in infill drilling in a producing unit to maintain or boost production; 7) whether to invest in production infrastructure; 8) whether to invest in major infrastructure such as TAPS, a gas treatment facility, or collector pipes; and 9) production rates.

Unit operators' production decisions are dynamic because current period decisions will impact next period profits.[16] Current-period production impacts next-period reserves quantity, exploration investment decisions impact future reserves quantity through new finds, and sequential investment decisions necessary prior to production impact the future ability to produce. Taken together, the sequential nature of decisions and investments causes the situation to be dynamic, making it a multi-stage game. That is, for example, unitization must come before production in Alaska, so the decisions leading up to unitization comprise one stage and production decisions after unitization comprise a second stage.

There are several sources of strategic behavior in Alaska. In the leasing process, the game is a closed-bid auction, where each company uses its private information (and public information) to assess the value of lease tracts and determine their bids. Each company's optimal bid will be the lowest possible such that it is larger than all other bids, but still lower than their valuation of the tract. Thus, the bidding is a game with each player's strategy contingent on the play of the others.

In the exploration phase after leasing, each company proceeds with the knowledge that a unit agreement must be negotiated before production. Thus, the goal of exploration is both to find oil and to document that a large share of the oil exists under the leases a particular company owns. Since exploration is costly, there is an optimal amount of exploration, which is related to the amount done by other companies. On the one hand, a company would save money by letting other companies explore to find the oil and then getting a share during the unit negotiations. However, the unit negotiation will require enough information to credibly argue for a large share of the production. This could be accomplished by having skilled geologists to review the information provided from the other companies' exploratory activities and/or independent exploration by the particular company in question. In addition, there is the issue of whether other companies will do exploration quickly enough and in the locations most advantageous to the company in

[16] Kunce (2003) also makes the argument that since “firms extracting nonrenewable resources are tied to an immobile reserve base that represents the key component of their capital stock, [they] view time, rather than space, as the most important dimension over which to substitute in response to changes in tax policy.”

question. Thus, it would seem that the companies most intent on finding new resources due to their firm-specific business model would do more exploration rather than wait for others, whereas the companies least intent on finding new resources would do less exploration. Similarly, it would seem that companies with large tracts of leases and/or no nearby leases would be more prone to invest in exploration (since no one else is going to find the oil under their leases) than in cases with mixed lease ownership all in close proximity.

As mentioned previously, our research focuses on the production stage only, in which strategic considerations are mitigated by the requirement for unitization. Consequently, we develop a dynamic model without strategic components that is an isolated model of the unit operator's production decision (i.e., not integrated with exploration and development activities that would increase reserves). Thus, we model each field with an initial stock that does not increase over time.[17] Each unit operator is treated as an independent decision-maker, not influenced by other unit operating decisions.

[17] See section 6 for several ways to relax this assumption, by adding satellite fields incrementally as they were discovered or by using an integrated modeling framework like those used by Kunce (2003) and others.

3 Data, Cost Estimation, Price Estimation

3.1 Data

In developing a dynamic model for oil production, we needed data for a number of variables. These are listed in Table 1 below. Data for this research came from a variety of federal and Alaska state government agencies, industry reports, research documents, and personal communication with personnel active in the Alaska oil industry (Table 1). More detailed explanation of these data follow; summary statistics are shown in Table 4. For all monetary data, we used the urban consumer price index to adjust to 1982-84 constant US dollars.[18]

[18] We chose to use 1982-1984 constant dollars for monetary units rather than a different reference year (e.g., 2006 or 2008) for two reasons. First, the period 1982-84 is used by the US Department of Labor as the reference for calculating the consumer price index. This makes the reader's own scaling of our results to alternative reference years relatively easy via simple multiplication by the consumer price index for her preferred reference year. Second, we are both hindcasting historical production and forecasting future production in our modeling, which raises the potential for misinterpretation of our results. For the hindcasting, using a reference in the historical period mitigates the risk of interpreting current-dollar profits as actual profits earned in past years. For the forecasting, using a future-year reference would avoid similar misinterpretation, but would require some prediction of future inflation, which would be unwise. Consequently, we chose to use a reference year during the historical period of production. We acknowledge, however, that some readers may find interpretation of current dollars more intuitive than constant 1982-84 dollars.

Variable	Units	Definition of Original Data	Source	Sample Mean
OIP_i	Billion barrel	Original Oil in Place for unit *i* (billions barrels)	AOGCC[1]	5.5
S_{it}	Billion barrel	Reserves remaining for unit *i* in month *t*, where S(0) = 50% of OIP (billions barrels)	calculated	2.4
Q_{it}	Million bbl/mo.	Quantity oil produced from unit *i* in month *t* (millions barrels per month)	AOGCC[2]	10.6
$AKWHV_t$	$/barrel	Alaska wellhead value, weighted average for all destinations, annual 1978–2006 ($/bbl, 1982-84 US dollars)	ADR[3]	$12.19
$USWHV_t$	$/barrel	USA spot price, FOB, average weighted by volume, weekly, 1997–2004 ($/bbl, 1982-84 US dollars)	EIA[4]	$13.46
$FWHV_t$	$/barrel	Forecast USA wellhead value, 2004–2030, reference, low- and high-price cases (annual, $/bbl, 1982-84 US dollars).	EIA[4]	$24.42 $17.91 $36.57
C_s	$/barrel	Total facilities investment cost of production (capital cost) in 2003 by field size, (13 categories, $/bbl, 1982-84 US dollars)	USGS[5]	$1.64[i] $1.35[ii]
$WELLS_{it}$	Count	Number of active wells by field for each month of production	AOGCC[2]	270
DC_t	$ mil./well $/ft.	Well drilling cost data for Alaska ($ millions per well and $ per foot, 1982-84 US dollars)	API[6]	$3.6 $341

Table 1: Variable definitions, data sources, and sample means. Free on board (FOB) price is equivalent to wellhead value since the buyer pays the transportation cost from origin to the final destination. Data sources are: 1) Alaska Oil and Gas Conservation Commission (AOGCC), 2008; 2) Personal communication, Stephen McMains, Alaska Oil and Gas Conservation Commission, June, 2007; 3) Alaska Department of Revenue (ADR), 2007; 4) Energy Information Administration (EIA), 2007; 5) Attanasi and Freeman, 2005; 6) American Petroleum Institute (API), 1969-2004. [i] average of the 13 categories defined by Attanasi and Freeman (2005). [ii] average of facilities investment cost of production for all monthly production observations for all seven fields on the Alaska North Slope.

3.1.1 Resource Data

To understand how producers make decisions about production, we need to know how much oil was originally in place in each unit area in Alaska. Data on original oil in place (OIP) are estimates from a variety of published sources compiled by the Alaska Oil and Gas Conservation Commission in "pool statistics" documents for each field. The OIP data were aggregated into units as follows (see appendix A). These seven units account for more than 90 percent of the OIP in Alaska.

- **Prudhoe Unit** = Prudhoe + Aurora + Borealis + Midnight Sun + Orion + Polaris + Lisburne + Niakuk + North Prudhoe + Point McIntyre + West Beach + Raven
- **Kuparuk Unit** = Kuparuk + Meltwater + Tabasco + Tarn[19]
- **Milne Unit** = Milne + Sag River + Schrader Bluff
- **Badami Unit** = Badami (no associated fields)
- **Colville Unit** = Alpine + Fiord + Nanuq + Nankup + Qannik
- **Endicott Unit** = Endicott + Eider + Ivishak
- **Northstar Unit** = Northstar (no associated fields)

Published estimates for original OIP were not available for the Lisburne (est. 400 million bbl), Raven (est. 10 million bbl), Nankup (est. 20 million bbl), and Qannik (est. 20 million bbl) fields, which account for 1.5% of the Prudhoe unit and 4.3% of the Colville unit. The estimate for original OIP for Kuparuk was revised to exclude the heavy/viscous oil in West Sak (approx. 15 billion barrels) which is not yet technically recoverable, making the estimate for Kuparuk 5 billion barrels. See Table 4 for OIP data by unit.

It is evident from this list that most of the seven production units on the North Slope have many associated satellite fields. We decided to include these fields in the initial estimate of OIP for each unit since this total is the best representation of the quantity of oil actually present initially in each unit. However, many of the satellite fields were discovered some time after the original discovery in each unit. Thus, we have inherently assumed perfect information regarding total resources that the producers did not have when developing each unit. The dilemma for how to include imperfect information in modeling producer behavior will appear elsewhere in this paper and is left to future work. For example, future model revisions could add the reserves of satellite fields incrementally as each one came online.

Only a fraction of OIP is technologically recoverable, and only a fraction of technologically recoverable oil is economically recoverable. The technologically recoverable fraction has been between 20% and 50% of original OIP (personal communication, Emil Attanasi, USGS, August, 2007), but this fraction has been increasing over time as technology improves. For this research, the original OIP data were scaled by 50% to estimate initial technologically recoverable reserves (see Table 4).[20]

---

[19] West Sak was not included because its heavy oil is not currently technically recoverable.

[20] Note, scaling by 20% and 35% result in historical production greater than initial reserves, a nonsensical result. Thus, it appears that estimates of original OIP were conservative or a higher fraction of original OIP has been technologically recoverable in Alaska.

3.1.2 Production Data

To validate our model, we need to compare actual production data to model predictions. For the modeling of production decisions described in this paper, the unit is taken as the level of production decision-making and thus production data are aggregated at the unit level. Thus, we use the quantity of production from each unit by month and year. Production data were obtained from the Alaska Oil and Gas Conservation Commission.[21] These data are summarized by year in Table 2.

Year	Prudhoe	Kuparuk	Milne	Badami	Colville	Endicott	Northstar	N.Slope Total
1978	34.41							34.41
1979	39.82							39.82
1980	46.25							46.25
1981	46.35	1.80						48.15
1982	46.62	2.76						49.37
1983	46.75	3.39						50.14
1984	46.50	3.97						50.46
1985	47.86	6.76	0.41					55.03
1986	47.35	8.02	0.35			0.00		55.72
1987	48.72	8.51	0.00			1.03		58.26
1988	47.74	9.32	0.00			3.07		60.13
1989	43.09	9.09	0.36			3.03		55.57
1990	40.33	8.91	0.55			3.16		52.95
1991	39.61	9.59	0.62			3.44		53.26
1992	36.78	9.84	0.57			3.48		50.67
1993	33.66	9.57	0.57			3.23		47.02
1994	33.20	9.29	0.56			2.84		45.89
1995	31.15	8.87	0.74			2.75		43.50
1996	29.53	8.26	1.24			2.17		41.21
1997	26.69	8.00	1.59			1.74		38.02
1998	23.40	8.03	1.70	0.14		1.43		34.69
1999	19.92	7.86	1.63	0.09		1.16		30.67
2000	18.47	7.14	1.59	0.08	1.44	1.00		29.70
2001	16.60	6.63	1.62	0.05	2.75	0.86	0.59	29.10
2002	15.66	6.44	1.55	0.05	2.92	0.75	1.53	28.88
2003	15.04	6.40	1.56	0.02	2.98	0.79	1.99	28.78
2004	13.64	5.96	1.56	0.00	3.05	0.62	2.06	26.88
2005	12.63	5.49	1.31	0.02	3.67	0.53	1.82	25.47
2006	9.87	5.20	1.08	0.04	3.69	0.43	1.57	21.87

Table 2: Average annual production for each unit in millions of barrels per month. Note, maximum TAPS throughput is approximately 2.033 million barrels per day, or 60.99 million barrels per month. Source: personal communication, Stephen McMains, Alaska Oil and Gas Conservation Commission, June, 2007.

[21] The AOGCC is an independent quasi-judicial state agency charged with preventing the "physical waste of hydrocarbon resources, promot[ing] greater ultimate recovery, protect[ing] underground supplies of drinking water, and afford[ing] all owners of oil and gas rights an equal opportunity to recover their fair share of the resource."

3.1.3 Price Data

The price of oil is a key factor in production decisions. A combination of three sources of price data were used to estimate a price function for Alaska oil. These data are for the wellhead value of oil, or the market price less shipping costs. Historical data for Alaska North Slope wellhead value were calculated annually by the Alaska Department of Revenue for the Alaska fiscal year spanning from July 1 to June 30 (ADR, 2007). There is also a one-month lag between production data and tax data because taxes are filed monthly and revenue from production in one month is taxed in the next month. These details become important when estimating the price function. Historical data for average United States wellhead value (reported as price, FOB and weighted by volume) were compiled by the Energy Information Administration weekly for the period 1997 to 2006 (EIA, 2007). Finally, the Energy Information Administration has also developed price forecasts (also reported as price, FOB) for reference-, low-, and high-price cases through the year 2030 (EIA, 2007).

3.1.4 Production Cost Data

Data for estimating production cost are often the crux of econometric modeling since most production cost data are proprietary and not available. Our research is no exception and future refinement of our models will benefit from improved cost data.

The total "facilities investment cost" of oil production on the Alaska North Slope was estimated by the United States Geological Survey (Attanasi and Freeman, 2005). These costs, expressed in dollars per barrel of oil produced, include the cost of drill pads, flow lines from drilling sites, central processing units, and infrastructure required for housing workers (including amenities). In other words, these are the capital costs of oil production. The costs were estimated for a generic oil field on the Alaska North Slope, specifically in ANWR, in the year 2003. Attanasi and Freeman developed a "cost relationship that specified investment cost per barrel as a function of peak fluid flow rates…" and expressed their cost estimates by discreet accumulation size class, where field size is technically recoverable resource (Table 3). The facilities investment cost estimates provide a reasonable approximation of total production costs since the Alaska oil industry is capital dominated, meaning labor and other costs of production are small relative to the facilities investment cost (personal communication, Neal Fried, Alaska Department of Labor, July, 2007).

Field Size (MMBO)	Cost ($/bbl)
32	4.51
48	3.39
64	2.77
96	2.09
128	1.73
192	1.41
256	1.22
384	1.00
512	0.86
768	0.71
1,024	0.61
1,536	0.50
2,048	0.43

Table 3: Facilities investment (capital) cost for the Alaska North Slope, in 1982-84 dollars, by initial field size in millions of barrels of technically recoverable oil (Attanasi and Freeman, 2005)

3.1.5 Well Data

Data on the number of producing wells and the well-days of production by field for each month of operation were provided by the Alaska Oil and Gas Conservation Commission (personal communication, Stephen McMains, AOGCC, 2007).

3.1.6 Drilling Cost Data

Data on the drilling cost per well and per foot were compiled from the American Petroleum Institute's Joint Association Survey of the U.S. Oil and Gas Industry from the years 1969 through 2004 (API, 1969-2004). These costs are Alaska-specific, based on industry responses to the annual API survey. The survey has been used extensively for cost data in previous studies of oil production (e.g., Kunce, 2003 and Lin, 2007). For our modeling of oil production, we used the cost of onshore oil wells and dry holes (i.e., we did not use cost data for offshore or gas wells).

	Prudhoe	Kuparuk	Milne	Endicott	Badami	Colville	Northstar
Start Date	Jan.1978*	Nov.1981	Oct.1985	Jun.1986	Jul.1998	Oct.2000	Sept.2001
Initial OIP	28,764	5,351	1,747	1,127	240	920	247
Initial Technically Recoverable Reserves							
	14,382	2,675	874	564	120	460	124
Technically Recoverable Reserves Remaining in 2006							
	2,902	478	624	114	115	231	15
Historical Production							
Mean	33.02	7.29	0.98	1.82	0.05	3.11	1.72
Max.	51.85	10.52	1.83	3.70	0.22	4.18	2.44
Min.	6.00	1.09	0.00	0.00	0.00	0.53	0.00
Std. Dev.	13.03	2.04	0.59	1.19	0.04	0.65	0.45
Wellhead Value ($/bbl, 1982-84 dollars)							
Mean	12.19	11.95	10.69	10.59	13.66	15.86	16.47
Max.	27.90	27.90	27.90	27.90	27.90	27.90	27.90
Min.	5.05	5.05	5.05	5.05	5.05	9.43	9.43
Std. Dev.	5.61	5.52	4.99	5.04	6.45	5.97	6.26
Wells							
Mean	701	378	86	50	5	37	13
Max.	961	552	142	64	7	59	19
Min.	113	1	1	1	2	13	1
Std. Dev.	264	138	45	14	1	13	5

Table 4: Summary statistics for historical data by unit. All quantities are in millions of barrels (production is millions barrels per month). *The first well at Prudhoe Bay produced oil on March 12, 1968, but the first oil flowed down TAPS in January, 1978.

3.2 *Cost Estimation*

We assume maximization of the discounted stream of future profits as the producers' objective function. Consequently, a function to define the cost of oil production is necessary. Information on the cost of oil production, however, is guarded as proprietary and there is a paucity of publicly available data. Chakravorty et al. (1997) used cost data compiled by the East-West Center Energy Program to estimate extraction cost functions econometrically. Dismukes et al. (2003) compiled information on per-unit costs for oil and gas activities by water depth in the Gulf of Mexico to develop an industry-specific expenditure profile. But the distinct environment (arctic) and location (remote on-shore) of Alaska's North Slope suggest production costs very different from other oil production operations. Consequently, we needed to develop an estimation of Alaska-specific costs. Furthermore, to model seven unique fields, we needed field-specific cost functions. To accomplish this task, we developed a novel method for estimating cost functions from available data that may be applicable to other modeling exercises as well.

We estimate the cost function from available data by scaling an average Alaska North Slope cost function (Attanasi and Freeman, 2005) by a constructed Alaska-specific drilling cost scalar and field-specific wells scalar. The result is a production cost surface with marginal cost increasing as reserves are depleted and as production rate exceeds limits to reservoir flow rates. Lack of original cost data (i.e., observations of production cost and other variables like production rate and reserves quantity) necessitated our development of this novel approach rather than a more standard econometric approach of estimating the parameters of the cost function from data using an econometric model of the cost function.

Economic theory and reservoir geology suggest a production cost function should incorporate the following three effects:

1) Economies of scale for increasing field size as captured in the USGS facilities investment cost estimates (Attanasi and Freeman, 2005; Figure 1). The assumption that production cost is a decreasing function of stock size is common in the economic literature (e.g., Farrow, 1985; Hartwick, 1982; Pindyck, 1978; Ruth and Cleveland, 1993).

2) A time trend as the North Slope industry developed, technology improved and adapted to the arctic environment, rigs and labor became less limiting, and learning occurred for arctic operations, as indicated by well drilling costs from the American Petroleum Institute (API, 1969-2004; Figure 2).

3) Diseconomies of scale for very high production due to physical constraints on oil flow rate, as indicated by State of Alaska data on the number of wells producing on each field across time and production rate (personal communication, Stephen McMains, AOGCC, 2007; Figure 3).

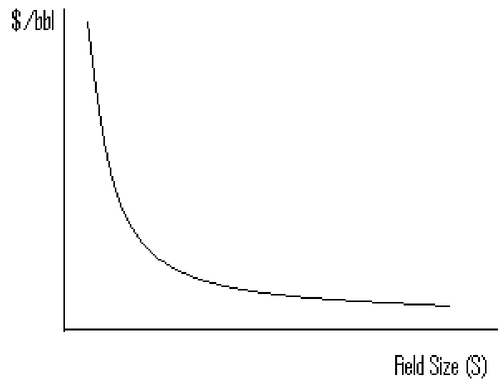


Figure 1: A generic (not from data) average production cost curve showing economies of scale for increasing field size.

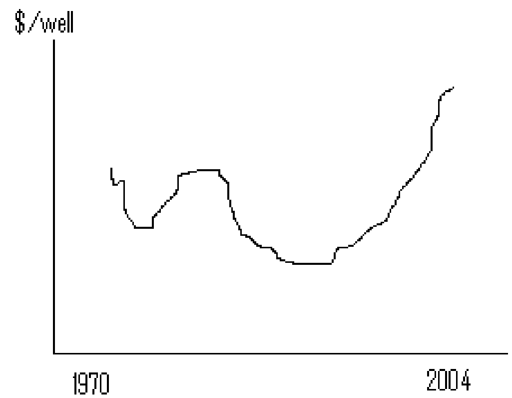


Figure 2: A generic (not from data) time trend in production cost indicated by Alaska well drilling costs ($/well).

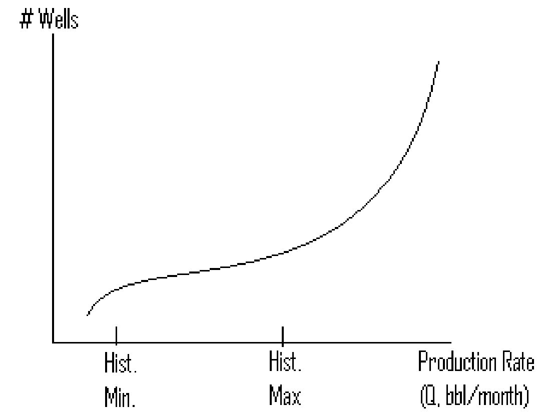


Figure 3: A generic (not from data) wells function showing diseconomy of scale for high production rate, indicated by the number of wells required. Historical maximum production rates tend to be below the range of significant diseconomies of scale.

There are three variables in a cost function that combine these three effects: production rate (Q), reserves remaining (S), and time (T). Allowing only one to vary at a time, the desired result in a composite cost function is as follows:

1) For a given field size and year, there are economies of scale as production increases up to some point where geology becomes limiting and excessive pumping causes diseconomies of scale (Figure 4).

2) For a given quantity of production and year, marginal, average, and total costs are lower for larger fields (Figure 5).

3) For a given quantity of production and field size, costs generally peaked around completion of TAPS, declined for a decade, and then began a steep climb in the late 1990s (Figure 6).

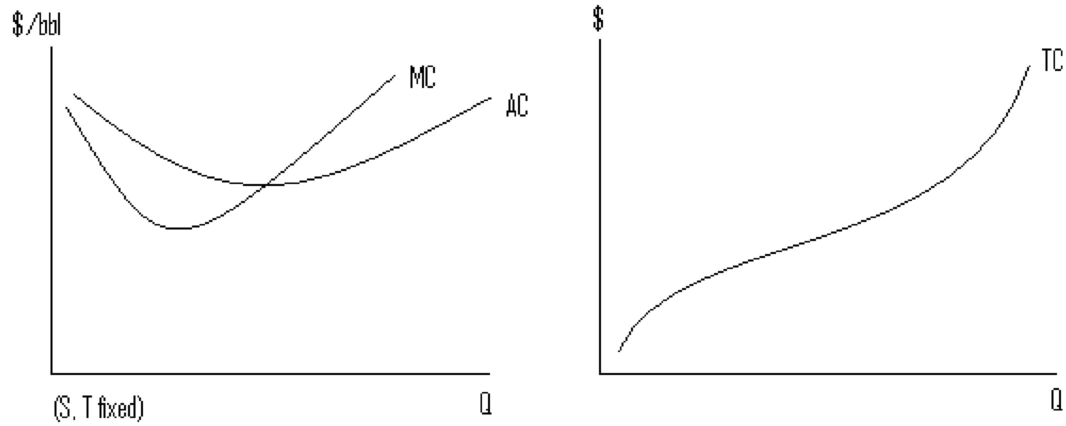


Figure 4: Behavior of a theoretical production cost function. For a given field size and year, marginal production cost initially decreases as production rate increases, but then begins to increase when production rate exceeds the reservoir's natural flow rate.

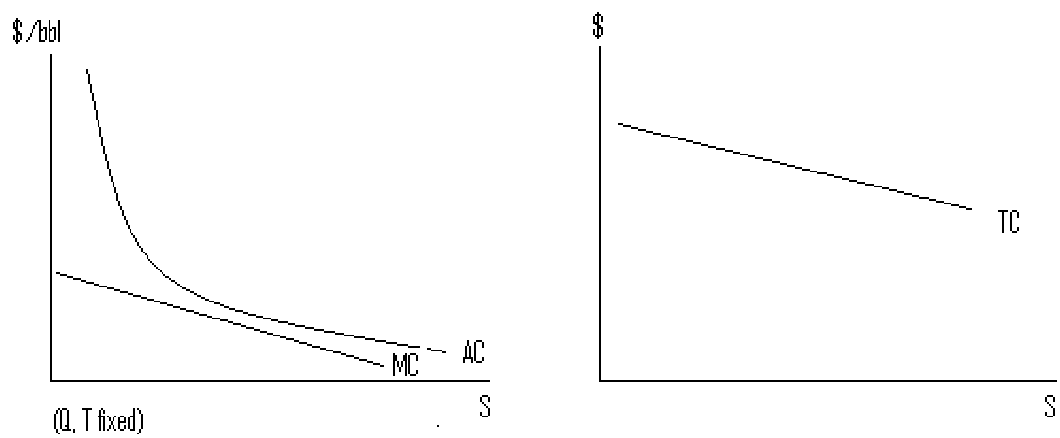


Figure 5: Behavior of a theoretical production cost function. For a given production rate and year, production cost is lower for larger quantity of reserves remaining.

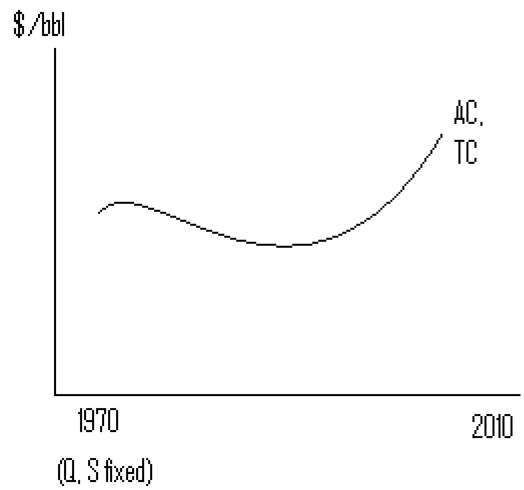


Figure 6: Behavior of a theoretical cost function. For a given production rate and quantity of reserves, production cost peaked around the completion of TAPS (1977), declined for a decade, and then climbed in the late 1990s.

Finally, it is important to note that each field is unique in its geology, oil properties, and context of development. Consequently, it is logical to estimate field-specific cost functions, as we do in this paper.

Our general approach for estimating a "composite" cost function with the attributes just described was as follows. The USGS data (Attanasi and Freeman, 2005) were used to estimate a "base" cost function that describes the fundamental facilities investment cost of production (capital cost, which approximates total cost) for a particular field size on the Alaska North Slope in 2003. Next, the field-specific wells data were used to construct a scalar for production rate, multiplying the base cost function. Then, the Alaska-specific API well-drilling-cost data (API, 1969-2004) were used as a proxy for the time trend in production cost to construct a second scalar for the base cost function. Finally, the composite cost function was defined as the product of the base cost function and one or more of the scalars, depending on conditions in the modeling. We now describe the estimation of the composite cost function in detail, taking each of the three effects described above in turn.

3.2.1 Base (Average) Cost

We began by estimating a continuous function for average cost ($/bbl) for oil production by fitting a function to the total facilities investment cost (capital cost) of oil production estimated by the USGS (Attanasi and Freeman, 2005). The results are shown in Figure 7.

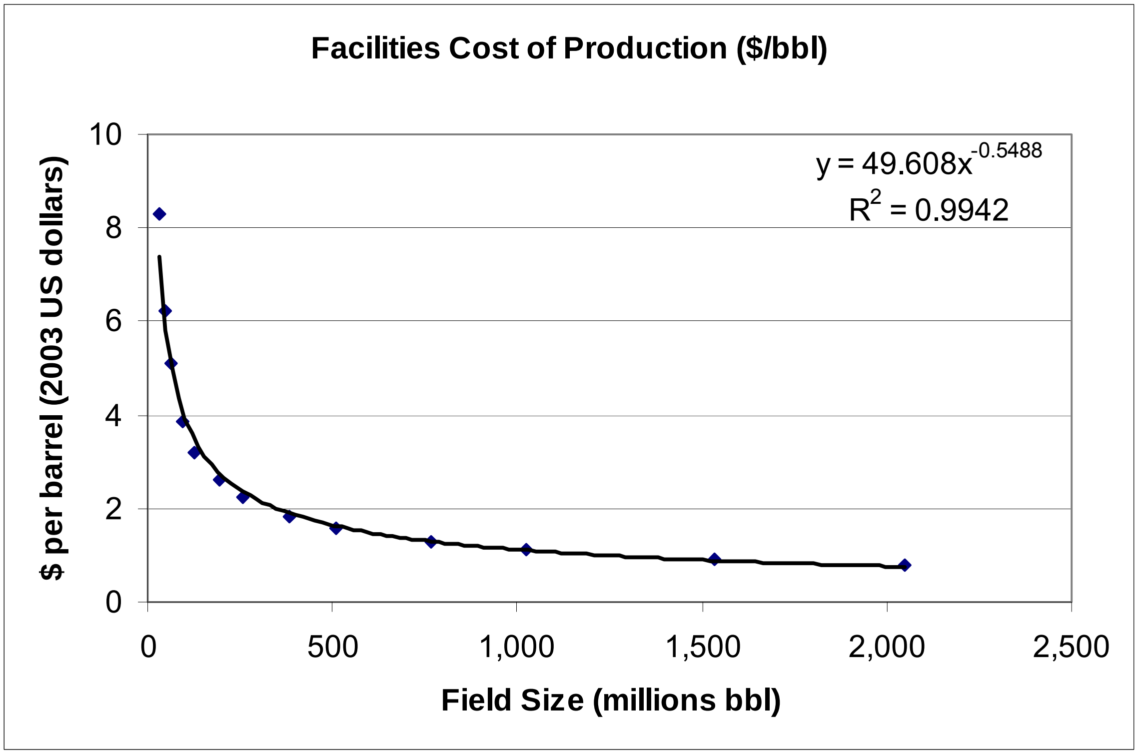


Figure 7: Average facilities investment cost (capital cost) of production ($/bbl) function fit to data from Attanasi and Freeman (2005).

For dynamic modeling of oil production decisions, however, marginal cost is necessary. In other words, we needed the cost per barrel for any particular combination of production rate (Q) and reserves remaining under the ground (S) at any moment in time since this is the relevant cost for production decisions. The field size categories in facilities investment cost estimates from Attanasi and Freeman (2005) were based on the original field size, so their cost estimates were for average cost rather than marginal cost (i.e., an estimated single average cost for a field's entire life based on the initial field size). This made estimation of a stock effect in the marginal cost of production from these data impossible.[22, 23]

Consequently, the next step for estimating the "base" cost function required the following assumption. Consider an oil field. When first discovered, the situation matches what Attanasi and Freeman quantified—namely, a field of that particular size may be expected to have an average cost per barrel for production over its lifetime equal to what

---

[22] Estimation of the facilities cost of production ($/bbl) was motivated by the question of what the cost of production would be for the field sizes that might be found in ANWR. The facilities cost is a reasonable approximation of total production cost since labor cost is a relatively small portion (personal communication, Neal Fried, Alaska Department of Labor, July, 2007).

[23] The term "stock effect" refers to the increase in production cost that generally occurs as reserves are depleted. Average cost data for the entire production life of a field do not contain information on such changes in production cost.

Attanasi and Freeman estimated. Now, imagine the same field 10 years later from the dual perspective of a potential buyer. There is less oil in the ground because some of the initial reserve has been produced. The average facilities investment cost of production, however, could be estimated for the future of that field and, in fact, would be the same as a newly-found field of the same size since the cost of facilities are amortized over their useful life and the remaining life is included in the purchase price. Thus, the average production cost by field size estimated by Attanasi and Freeman should apply equally to newly-discovered fields and producing fields, at any particular moment in time.

With an estimate of the initial reserves for each field, and monthly data on the production rate (bbl/mo), we calculated the reserves remaining in each field for each month and used the facilities investment cost function shown in Figure 7 to associate this with an average cost of production ($/bbl) for that month. Multiplying by the quantity of production in that particular month yields the total cost of production. Thus, we constructed data on production rate (Q), reserves remaining (S), and total cost of production (C) for each field in each month. These calculations were made for the 12 months of 2003 for each field since the facilities investment cost of production data were estimated for 2003. Costs are deflated to 1982-84 dollars for consistent constant-dollars units used throughout our modeling.

These data enabled estimation of a total cost function of the form $cost_i = c_1 Q_i^{c_2} S_i^{c_3}$, which is similar in form to previous studies of oil production and incorporates both production and stock effects (Lin & Wagner 2007; Lin 2007).[24] A log-linear form was used to estimate parameters by ordinary lease squares (OLS), where S is reserves remaining measured in millions of barrels, Q is production rate measured in millions of barrels per month, and cost is measured in constant 1982-84 US dollars (eq. 1, Figure 8).

1 Base total cost of production: $TC = c_1 Q^{c2} S^{c3} = 91495468(Q^{1.00065})(S^{-0.549262})$
Standard error:[25] (0.0037146) (0.000474736) (0.000651287)
Adjusted R^2: 0.999985
All coefficients are statistically significant at the 0.1% level.[26]

---

[24] Production and stock effects relate to the conceptual figures at the beginning of this section in the following ways. Production rate affects production cost if economies of scale exist (see Figure 4). Decreasing stock of reserves remaining as production occurs generally causes increased production cost as reserves are depleted (see Figure 5).

[25] The standard error reported for c_1 is for the estimate of $\ln(c_1)$ calculated by linear regression rather than for c_1 itself.

[26] The estimated magnitude of c_2 is interesting because it indicates the elasticity of total cost with respect to production rate. The estimated magnitude suggests slightly more than unitary elasticity, meaning total cost increases more than one percent for a one percent increase in production rate.

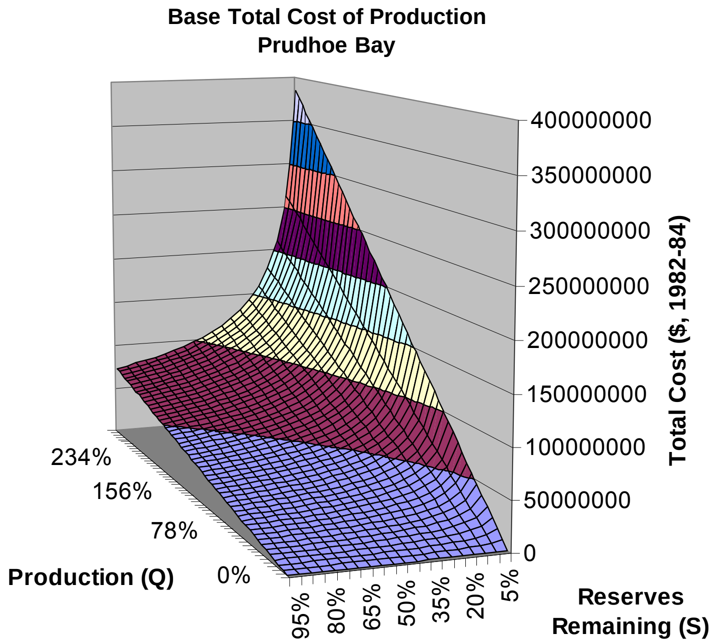


Figure 8: The base total cost of production for any combination of reserves remaining and production rate is plotted in three dimensions. The axes for production and reserves are in percentage terms, in this case for Prudhoe Bay, from zero to 100 percent of original reserves in the field and from 0 to 300 percent of historical maximum production rate. The vertical axis is in dollars, normalized to 1982-84 dollars.

3.2.2 Drilling Cost Scalar

With the base cost function defined, our next task was to incorporate the evolution of capital costs over time into the cost function. The majority of oil production costs in Alaska are facilities and equipment costs (i.e., labor is relatively small). Furthermore, changes in drilling cost may be a reasonable indicator for changes in total facilities and equipment costs due to use of similar inputs. Consequently, since drilling costs have fluctuated over time (Figure 9), it may be logical to scale the cost function in any particular year based on the drilling cost in that year (or a prior year for lagged effect on production cost) by multiplying by the ratio of drilling cost in that year relative to the reference cost in 2003. We made this assumption, but included a dampening parameter for use in sensitivity analysis.

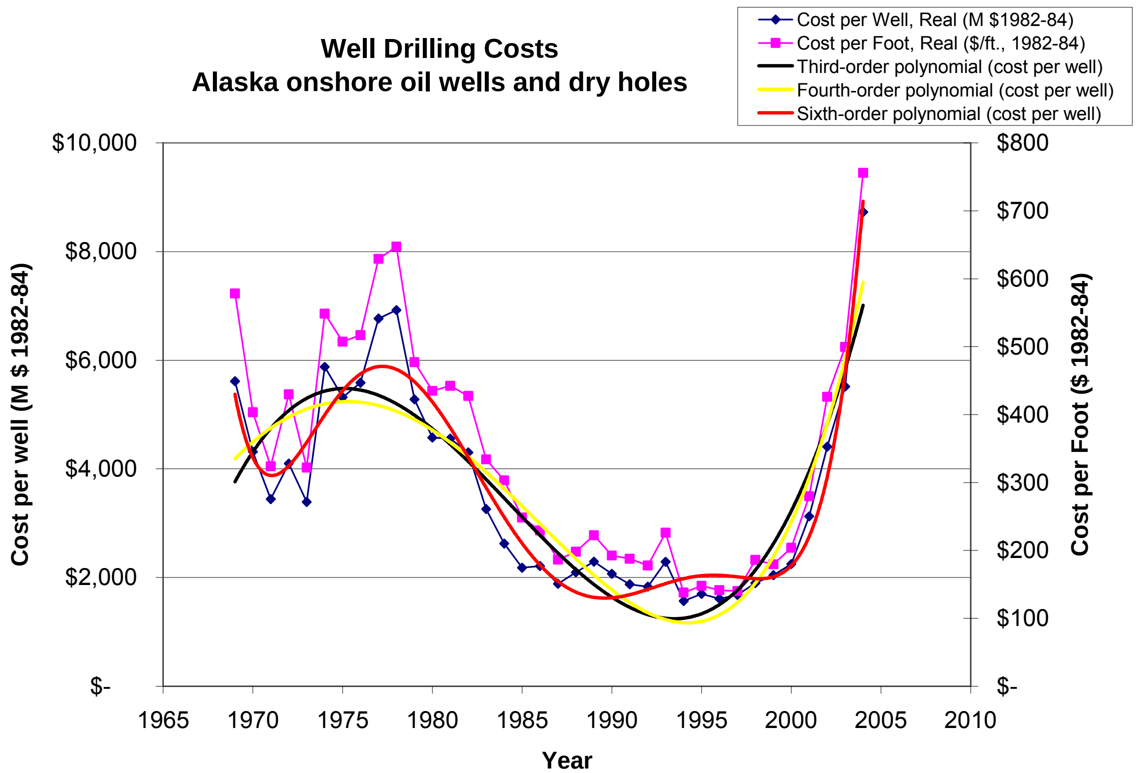


Figure 9: Well drilling costs in Alaska over time, per well and per foot, with third-, fourth-, and six-order polynomial regressions shown (API, 1969-2004).

Drilling costs in Alaska have fluctuated over time (Figure 9). One explanation is quasi-rents from drilling equipment scarcity, materials costs, technological change, and improvement in operational knowledge. A boom in exploration and development followed the discovery of Prudhoe Bay in 1968, which included the construction of TAPS (completed in 1977). The shortage of skilled labor, materials, and equipment associated with this boom coincides with the first peak in drilling costs from 1970 to 1980. With TAPS and the initial rush of exploration and development completed, labor and equipment became readily available. Since Alaska's North Slope was one of the first arctic oil developments, the technological and operational learning curves for arctic oil production were steep. These events coincide with the decline and trough in drilling costs from 1980 to the late 1990s. In recent years, global demand for materials and skilled labor may have pushed drilling costs upward again. In this light, it is reasonable to think of a scalar for oil production cost based on drilling cost that is an approximation of similar fluctuations in the cost of oil production factors.[27]

[27] An alternative explanation, however, is changes in the quality of drilling sites in response to oil price. If more marginal sites are given the green light for drilling when oil price is high, then the first peak in drilling cost may correspond to the high prices caused by the oil crises of 1973 and 1979, the decline and trough in drilling cost from 1980 to the late 1990s may correspond to the relatively low oil prices of this

The API data (API, 1969-2004) were deflated to 1982-84 dollars and scaled so the value is approximately equal to one in 2003, thereby creating a multiplier that will scale the cost function in other years appropriately for changes in oil production costs (as proxied by drilling costs).

Third, fourth, and sixth order polynomial functions were evaluated for regressing the cost of well drilling on time using a time index (1969 = 1) rather than the actual year to avoid overflow errors (e.g. when 1970 is raised to the sixth power). A user-defined lag parameter (Lag) was added to account for the delay between an increase in drilling costs translating into an increase in oil production cost. The sixth order polynomial regression best fit historical data on well drilling costs by accurately mapping five inflection points (Figure 10). Consequently, we defined the Drilling Cost Scalar (DCS) as a sixth-order polynomial function of the indexed and lagged year (YrIL).[28]

2 Drilling Cost Scalar:

$$DCS = c_4 + c_5 YrIL + c_6 YrIL^2 + c_7 YrIL^3 + c_8 YrIL^4 + c_9 YrIL^5 + c_{10} YrIL^6$$

	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
Coeff.	1.413501	- 0.5839932	0.161024	-0.0175783	0.0008877	-0.0000211	1.92E-07
Std.Error	.156303	.1086034	.0242834	.0023998	.0001165	2.72e-06	2.44e-08

Adjusted $R^2 = 0.9233$; all coefficients are statistically significant at the 0.1% level.

period, and the recent increase in drilling cost may correspond to recent increases in oil prices. In this case, a scalar based on drilling cost may have less relationship with oil production cost.

[28] For example, with year equal to 1985 and a lag of 2 years between drilling costs and production costs, the variable YrIL equals 1985 - 1968 - 2 = 15.

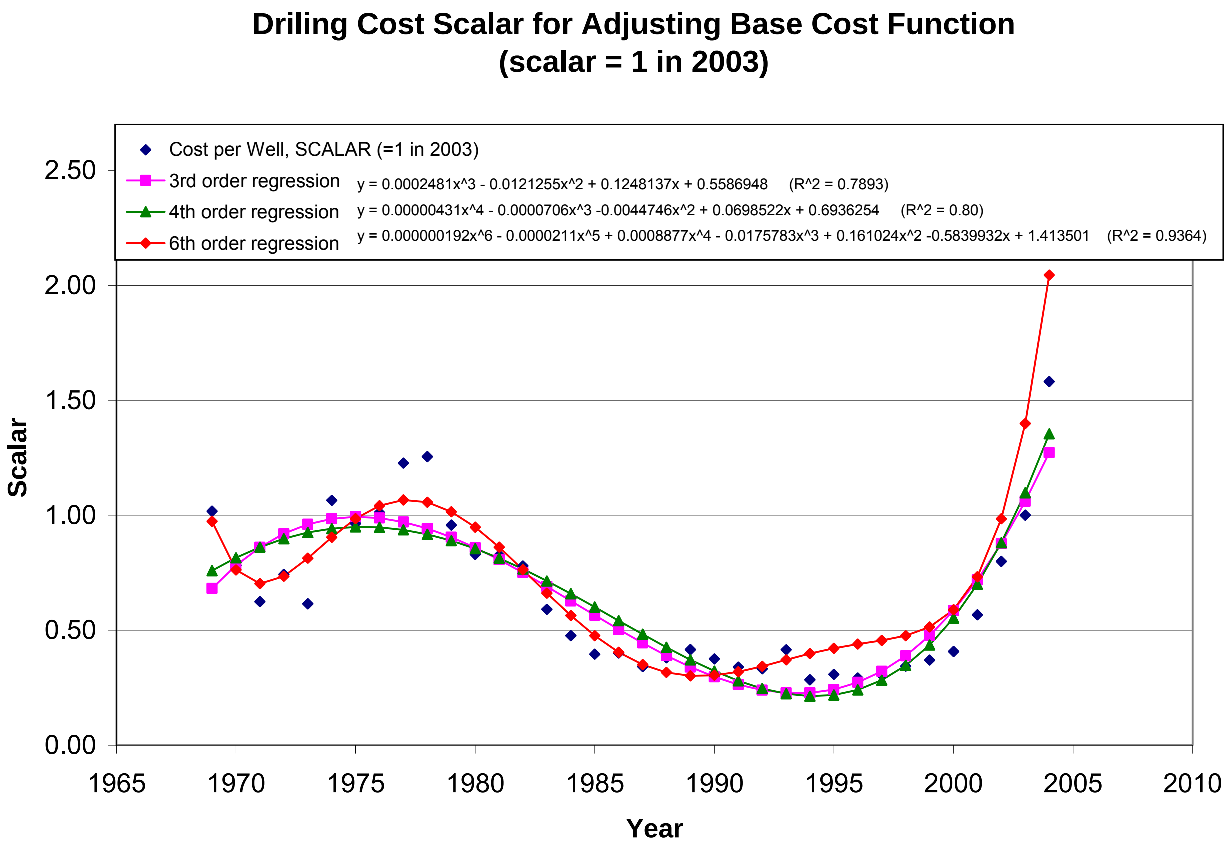


Figure 10: Drilling cost scalar (DCS) for multiplication of the base cost function to account for the evolution of drilling costs – which proxies for changes in oil production costs – due to quasi-rents from scarcity in drilling equipment, material and labor supplies and to improving knowledge of oil production in the arctic environment. A dampener was included to allow sensitivity analysis since an alternative explanation for drilling cost may be changes in the quality of drilling sites in response to oil price, in which case the DCS may have less correlation with oil production cost.

We used the DCS to scale the base cost function to adjust for changes in production cost over time. However, the validity of drilling cost as a proxy for production cost is weakened if the evolution of drilling cost was due to changes in the quality of drilling sites in response to oil price rather than to changes in quasi-rents and the cost of inputs like materials, equipment, and labor. Consequently, the scalar range from 0.28 to 1.6 may cause overly large changes in the cost of production. To account for this possibility and to examine the impact on results with sensitivity analysis, we added a "dampener" (Dmp) to the drilling cost scalar that can be used to restrict its range. The dampened drilling cost scalar (DDCS) is defined as follows,

3 Dampened Drilling Cost Scalar: DDCS = 1+(DCS-1)/Dmp

where *DCS* is the drilling cost scalar defined by equation 2 and *Dmp* is a user-defined dampening factor.

Finally, it is evident from Figure 10 that well drilling costs were increasing rapidly in the period 2000 to 2004 and that this trend is incorporated into the DCS and DDCS. Consequently, we applied the DDCS in the composite cost function for the historical period for which we have data only (i.e., 1969 to 2004), implicitly assuming drilling costs remain constant (other than inflationary change) at 2004 levels into the future.

3.2.3 Decreasing Returns to Scale

The last piece of reality to incorporate in the composite cost function is the notion of decreasing returns to scale as production rate exceeds the geologic limit to flow rate for each particular field (Bedrikovetsky, 1993; Allain, 1979). In other words, more wells are needed to produce at a faster rate and at some point the number of additional wells needed per additional increment of production rate increases rapidly as producers try to draw oil out of the ground faster than the rock is willing to yield it.

Data from the North Slope fields show this pattern (Figure 11). In this graph, the number of wells increases in order to maintain a certain production rate while reserves remaining declines. In fact, the increased number of wells is often insufficient to maintain a production rate, causing the typical tailing-off of production for the field. The tailing-off of production is typically not due to a decrease in the number of operating wells until very near the end of the field's life. Thus, it appears producers have made the rational decision to produce below the point of diminishing returns to additional wells. That is, they do not devote resources to using many wells to pump oil faster than the geology is willing to yield it.

When a field is discovered, it is generally characterized by how much oil there is (OIP), how much is technically recoverable (typically 30% - 50% of OIP), and the anticipated maximum production rate (and thus lifetime of the field). This information comes largely from geologists. Thus, the geology sets maximum production rate, not economics, and we are faced with the task of reflecting this physical reality in our cost function for economic modeling. We used estimation of functional relationships between wells and the rate of oil production to tackle this challenge.[29] The resulting inverse production functions give the number of wells needed in each field for any particular combination of production rate and level of reserves remaining.

[29] We anticipated finding a non-linear increasing trend for the number of wells needed for production as the production rate became exceedingly high, since such extreme production would require extra inducement for oil to flow faster than the predominant geology would dictate. However, changing well technology could also influence the number of wells needed to produce oil at a certain rate, ceteris paribus, so our regressions may suffer from omitted variable bias. Lacking data on well technology in use on the North Slope, we considered adding a time regressor to account for evolutionary change. But development of well technology may have been lumpy (personal communication, Frank Kareeny, BP-Alaska, July, 2007) and including time in our wells scalar made the derivations used in solving the boundary value problem for optimization prohibitively complex. Thus, including well technology is left to future work.

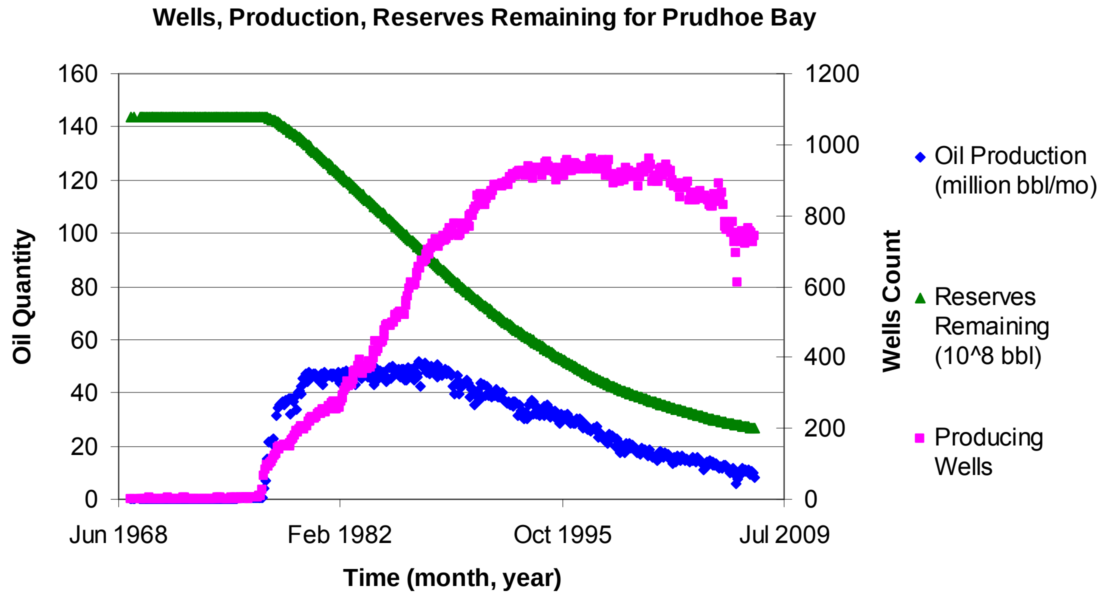


Figure 11: The number of producing wells, production rate, and reserves remaining for Prudhoe Bay. Similar plots for other units are shown in appendix B.

To establish the relationship between wells and oil production, we regressed the number of producing wells on oil production rate and reserves remaining (to control for the influence of field size on the number of wells necessary for a given rate of oil production).[30] We estimated two well functions, one presuming constant returns to scale (i.e., a plane, eq. 4) and a second presuming decreasing returns to scale (i.e., a convex surface, eq.

5). Since the number of wells required for oil production is highly reservoir specific, we allowed the functional specification for the latter estimation to vary among fields.[31]

4 Constant returns wells plane: $CRWells = c_{11} + c_{12}Q + c_{13}S$

5 Decreasing returns wells surfaces:

Prudhoe Bay: $DRWells = c_{14P}+c_{15P}Q+c_{16P}Q^2+c_{17P}Q^3+c_{18P}S+c_{19P}S^2+c_{20P}S^3$

Kuparuk River: $DRWells = c_{14K} + c_{15K}Q + c_{16K}Q^2 + c_{17K}Q^3 + c_{18K}S$

Milne Point: $DRWells = c_{14M} + c_{15M}QS + c_{16M}Q^2S + c_{17M}Q^3$

Endicott: $DRWells = c_{14E}+c_{15E}QS+c_{16E}QS^2+c_{17E}Q+c_{18E}Q^2+c_{19E}Q^3+c_{20E}S$

Colville: $DRWells = c_{14C} + c_{15C}Q + c_{16C}Q^2 + c_{17C}Q^3 + c_{18C}S + c_{19C}S^2$

Northstar: $DRWells = c_{14N} + c_{15N}Q + c_{16N}Q^2 + c_{17N}Q^3 + c_{18N}QS$

---

[30] One would expect a smaller field to require more wells to achieve a given rate of production since production at a given rate from a small field will require encouragement of faster flow rates via more wells.

[31] We recognize that well technology may differ between fields due to reservoir differences as well as across time. We do not include this complexity in the current work.

where *CRWells* and *DRWells* are the number of wells, for the constant returns and decreasing returns cases respectively, *S* is reserves remaining measured in millions of barrels, and *Q* is production rate measured in millions of barrels per month.

We used a stepwise variable selection technique for model specification based on significance at the 5 percent level. The stepwise technique combines forward and backward variable selection by starting with the zero model, using the forward selection technique to add variables, and the backward selection technique to evaluate the result.[32] However, this technique failed to produce acceptable forms (i.e., erratic forms and/or non-decreasing returns to scale) for the Kuparuk and Prudhoe Bay fields. Consequently, we used iterative model specification to define the decreasing returns model specification for these fields. Due to this heavy-handed approach, we withheld 10% of observations (selected randomly) for model validation. The results of these regressions are presented in Table 5 and Table 6, and example plots for the Colville River field are shown in Figure 12 (see appendix C and E for other fields). The Durbin-Watson statistics presented include a correction for first order serial autocorrelation using a Cochrane-Orcutt procedure (Ramanathan, 2002).[33]

	Constant	Q_index	S_index	Adj. R^2	DW stat.
Colville	88.7405***	0.402406	-0.155815***	0.97882	2.06323
Std. error	3.33065	0.540647	0.00645563		
Endicott	63.341***	3.68028***	-0.0677256***	0.963985	2.55515
Std. error	4.19891	0.665917	0.0148555		
Kuparuk	567.158***	2.75862**	-0.118193***	0.996603	2.00689
Std. error	32.3279	0.937735	0.0214271		
Milne	341.272***	18.7964***	-0.354895***	0.990938	2.64668
Std. error	42.4184	2.01307	0.0556701		
Northstar	16.87012***	2.481977***	-0.1251358***	0.9468	1.356
Std. error	0.5410759	0.2916317	0.003565918		
Prudhoe	1066.89***	1.09976**	-0.0616073**	0.998266	2.61257
Std. error	147.569	0.337912	0.0202845		

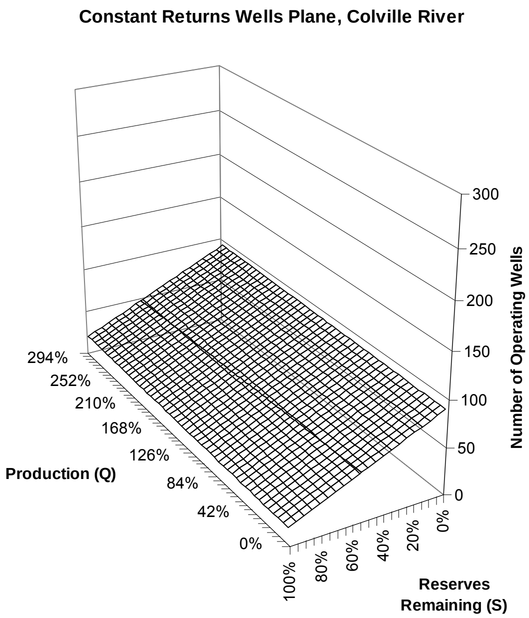
[32] The forward selection technique adds variables to the regression model one at a time with the sequence based on choosing the variable that minimizes the residual sum of squares provided the variable is significant at our chosen 5 percent level. The backward selection technique eliminates statistically insignificant variables (F-statistic below the critical value for our chosen 5 percent level) from the regression model one at a time with the sequence based on choosing the least significant. The stepwise procedure offers an improvement over the forward selection and backward elimination procedures on their own because it guards against any variables becoming statistically insignificant with the addition of the next variable to the model.

[33] The Durbin-Watson is a statistical test for the presence of first-order serial correlation (i.e., first-order autoregressive or AR(1)) that is centered around the value two. Failure to correct for serial correlation in OLS regression produces unbiased, consistent, but inefficient estimates because the standard assumption of independence of errors across observations is violated. Although not important for our research since we do not perform formal hypothesis tests, the inefficiency of OLS estimates in the presence of serial correlation will cause bias and inconsistency in test statistics because standard errors are biased and inconsistent. Consequently and to conform with best practices, we used the Cochrane-Orcutt method for correcting for serial correlation, which is an iterative procedure that begins with OLS to obtain residuals, calculation of an estimated serial correlation coefficient from these residuals, transformation of the data with the estimated serial correlation coefficient, and generalized least squares (GLS) on the transformed data.

Table 5: Parameter estimates for the constant returns wells plane, for Q, S in millions barrels. Statistical significance for coefficient estimates is indicated at the 5% level (*), 1% level (**), and 0.1% level (***).

Colville	Constant	Q	Q^2	Q^3	S	S^2		Adj. R^2	DW Stat
Coeff. Est.	68.2945***	8.07675	-2.95748	0.35801	-0.0631439	-0.00014196		0.978864	2.0309
std. error	12.4577	7.73784	2.82791	0.335916	0.0552247	8.3671E-05			
Endicott	Constant	QS	QS^2	Q	Q^2	Q^3	S	Adj. R^2	DW Stat
Coeff. Est.	65.1632***	0.171746***	-0.000207604***	-7.46746	-8.06115***	0.854042**	-0.111207***	0.971796	2.2
std. error	2.31291	0.0175386	0.0000213501	4.03617	1.96804	0.322885	0.00575513		
Kuparuk	Constant	Q	Q^2	Q^3	S			Adj. R^2	DW Stat
Coeff. Est.	513.389***	18.2461**	-1.62634	0.0503533	-0.106936***			0.996686	2.10729
std. error	42.9937	6.2585	1.0215	0.0538752	0.0298598				
Milne	Constant	QS	Q^2S	Q^3				Adj. R^2	DW Stat
Coeff. Est.	144.02**	0.0729054***	-0.0396082**	5.55127				0.992833	2.35932
std. error	48.005	0.00990767	0.0132308	3.33221					
Northstar	Constant	Q	Q^2	Q^3	QS			Adj. R^2	DW Stat
Coeff. Est.	0.844783	21.46647***	-7.329782*	1.265286	-0.08179707***			0.8904	0.771358
std. error	1.568916	4.062971	3.455657	0.8765398	0.003815506				
Prudhoe	Constant	Q	Q^2	Q^3	S	S^2	S^3	Adj. R^2	DW Stat
Coeff. Est.	68.0352	10.7607***	-0.304704***	0.00288675***	0.343354***	-4.5943E-05***	1.51203E-09***	0.998519	2.38227
std. error	40.9523	1.49721	0.0647278	0.000738853	0.01654	2.1625E-06	8.71092E-11		

Table 6: Parameter estimates for the decreasing returns wells surface. Statistical significance for coefficient estimates is indicated at the 5% level (*), 1% level (**), and 0.1% level (***).



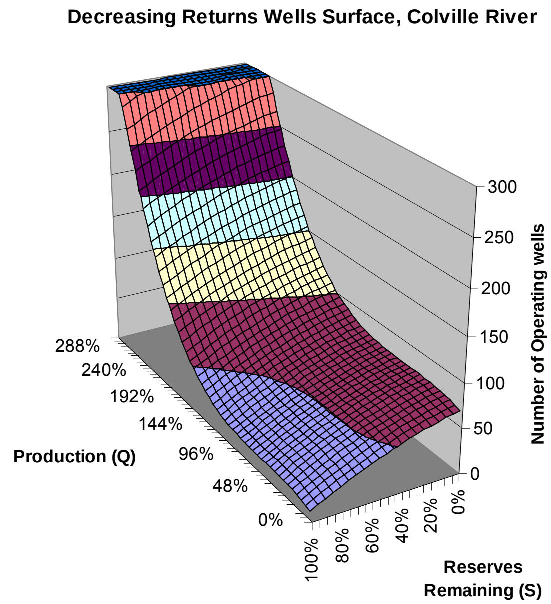


Figure 12: The constant returns wells plane (left panel) and decreasing returns wells surface (right panel) for Colville River. The axis for reserves remaining extends from the original quantity of technically recoverable oil to zero. The production axis ranges from zero to three times the maximum historical rate of production. The vertical axis is the number of operating wells.

In both the constant returns and diminishing returns plots, more wells are required to maintain a given rate of production as the reserves remaining declines and more wells are required to produce faster, given a level of reserves remaining. However, the rates of change for these well requirements are greater for the decreasing returns graph.

The type or size of well and/or well capacity influences the number of wells needed to produce oil at a given rate. If such specifications for wells on the North Slope changed over time, adding a time regressor would pick up the impact of this change. But if the change occurred in one brief period of time, it would confound our regression attempts. Coil tube drilling was developed in Alaska in the early 1990s and has enabled development of some smaller fields and drilling multiple wells from the same pad (personal communication, Frank Kareeny, BP-Alaska, July, 2007).[34] This technology may have changed the capacity of a well for production. There are also two basic categories of prospects. Infrastructure led exploration (ILX) is for satellite fields where the field size is small but it is close to existing infrastructure. Industry generally pursues these only if there is better than a one in three chance of finding oil. The other type is

[34] Water injection began in 1984 at Prudhoe Bay and miscible gas injection (ethane, propane, butanes) began in 1987 with construction of the Central Gas Facility (CGF) and Central Compressor Project (CCP) (ibid).

wildcat or corex, where the field is far from existing infrastructure but the size is large enough to cover the cost of new infrastructure and large enough to justify further investigation even if the chance of oil is as small as 1 in 10 (personal communication, Vincent Monico, BP-Alaska, July, 2007). There may be systematic differences in the production capacity for wells drilled at satellite fields versus wildcat tracts due to differences in the equipment that can be brought in to each location. Finally, there is a possibility for larger-capacity wells to be drilled in larger, easy-to-extract pools. We have abstracted away from these complexities in the current modeling by assuming all the wells in a particular field are about the same capacity and estimating unique wells functions for each field. Future work may examine this assumption more carefully.

Having defined well functions for each unit, the remaining task is to incorporate this information regarding the increasing number of wells needed as pumping rate increases and/or reserves remaining decrease (i.e., decreasing returns to scale) into the composite cost function. Our general approach was to define a “wells scalar” that will multiply the cost function and increase the cost of producing oil if the model chooses production rates that are high enough to be in the range of decreasing returns to scale. We defined this scalar as the ratio of the decreasing returns wells surface to the constant returns wells plane and invoke it only when the ratio is greater than one (i.e., the cost function is left unmodified so long as production is in the range of constant returns, but is scaled upward if production exceeds this range). We also added a user-defined parameter (the decreasing returns to scale margin, DRTS_M) to shift the constant returns plane up or down, thereby enabling sensitivity analysis of the point in production rate at which the wells scalar begins to increase cost. Thus, we define the complete wells scalar (WS) as

6 WS = DRWells/(CRWells*DRTS_M)

where *CRWells* and *DRWells* are the number of wells, for the constant returns and decreasing returns cases respectively, and *DRTS_M* is a user-defined parameter used to shift the point at which decreasing returns to scale begin.

Since the base cost function includes the impact of production decisions on production cost over a reasonable range, the intent of this wells cost scalar is to scale the cost function up only when modeled production rates exceed geologic limits to flow rate. Thus, the base cost function is a minimum cost, which can be scaled up by the wells scalar if the scalar is greater than one. Figure 13 illustrates this concept graphically.

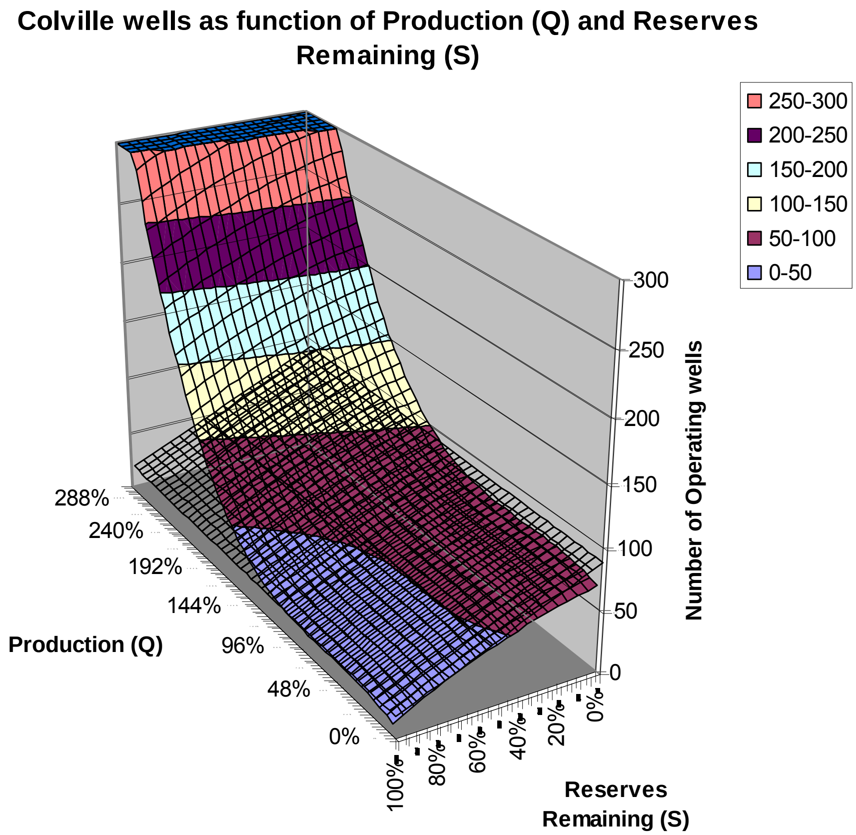


Figure 13: the constant returns plane and the decreasing returns surface for Colville River. The wells scalar is invoked when it is greater than one, which is true for Q, S combinations that cause the decreasing returns surface to climb higher than the constant returns plane. Note this occurs when production rate is approximately 150% of historical maximum, implying rational producer behavior in choosing production rate less than the level at which diminishing returns occur. Conversely, assuming rational producer behavior implies validity of the results from our approach of using wells data to estimate diminishing returns due to physical reservoir properties.

3.2.4 The Composite Cost Function

The base cost function was estimated assuming the rational behavior of producing in the constant-returns region (i.e., not pushing the production rate past geologic limits to flow rate) in the year 2003. Since our application for the cost function is in a dynamic optimization model that does not constrain production rate and covers the period 1978 to 2170, we added decreasing returns in production rate and time trends in production cost to the base cost function to create the "composite" cost function (CCF). We defined the CCF as follows. Figure 14 shows an example of the CCF for Colville River.

$$
7 \qquad CCF = \begin{cases} BC & \text{if } (CRWells*DRTS\_M) > DRWells \text{ and } Year > 2004 \\ BC*DDCS & \text{if } (CRWells*DRTS\_M) > DRWells \text{ and } Year \leq 2004 \\ BC*WS & \text{if } (CRWells*DRTS\_M) \leq DRWells \text{ and } Year > 2004 \\ BC*WS*DDCS & \text{if } (CRWells*DRTS\_M) \leq DRWells \text{ and } Year \leq 2004 \end{cases}
$$

where *BC* is the base cost defined by equation 1, *DDCS* is the dampened drilling cost scalar defined in equation 3, *WS* is the wells scalar defined in equation 6, *CRWells* and *DRWells* are the number of wells (for the constant returns and decreasing returns cases, respectively), and *DRTS_M* is a user-defined parameter used to shift the point at which decreasing returns to scale begin.

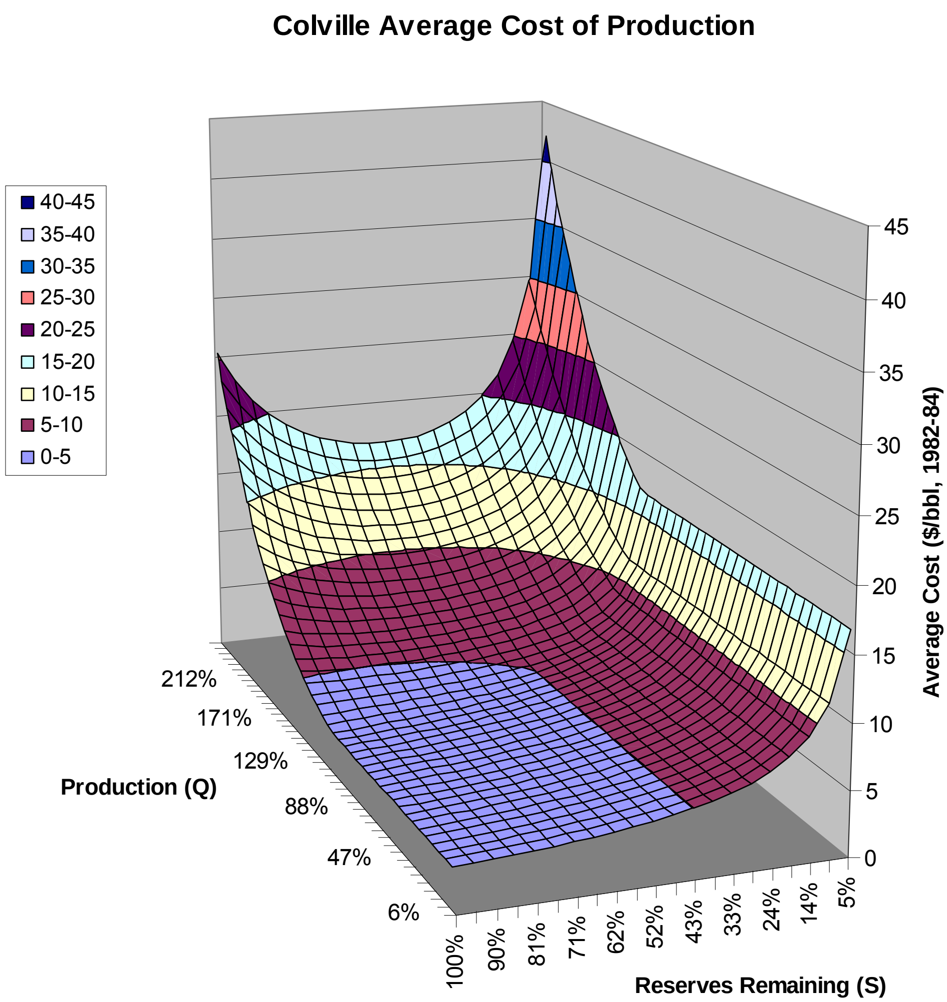


Figure 14: The composite cost function for Colville River in 2003 with DRTS_M = 1, Lag = 1 and DCS Dampener = 2. In the range of historical production rates, production cost is approximately $3 to $6 per barrel in 1982-84 dollars. But getting the last few barrels of technically recoverable oil might cost $15 or more, and producing at twice the historical maximum rate might cost $12 per barrel.

3.2.5 Discussion of Methods for Cost Estimation, part I

Our novel method for construction of the CCF was born from limitations in available cost and reservoir data and from the need for integration of engineering and economic approaches to improve modeling of oil production. Using a scalar, estimated from data on the number of operating wells, to reflect reservoir characteristics via decreasing returns in the CCF proved to be an effective approach for incorporating more "technical knowledge about the operation of the fields" and "realism in the description of reserves" without using engineering computer models that require additional data (Salehi-Isfahani, 1995).

The engineering literature on oil production modeling is based on dynamic fluid flow models, where the objective is usually assumed to be maximization of total ultimate recovery from the reservoir (e.g., Feraille et al., 2003). This approach, however, generally omits the economic consideration of discounting, which implies that future production is worth less to the profit-maximizing firm than production today.

The economic literature on oil production modeling often relies on production functions with "generic propert[ies] of scale or substitution (e.g. the constant elasticity of substitution production function), or on approximating forms (such as the translog and Leontief" (Gao et al., 2008). Parameter values for these functions are estimated from data with econometrics. But the choice of functional form can be restrictive and may not adequately represent the physical realities of a particular reservoir. A strict econometric approach also requires adequate cost data from oil producers that include observations for production rates extending into the range of decreasing returns to scale. Such data are rare and difficult to obtain.

Consequently, integration of engineering and economic approaches to modeling oil production is warranted. In a review of oil market models, Salehi-Isfahani (1995) wrote, "… Depending on the type of geological structure, oil may be lost due to pressure and seepage. Unfortunately, the economic literature has so far not incorporated much technical knowledge about the operation of the fields. Mining engineers often predict a production path from a given field as an inverted U, with a unique peak. Economists on the other hand emphasize the role of price in extraction. Adelman (1993) correctly criticizes the exhaustible resource models for their lack of realism in the description of reserves…"

Motivated by these observations, Gao, Hartley and Sickles (2008) used an "engineering computer model of dynamic fluid flow" to simulate reservoir data for use in economic modeling of dynamically optimal oil production in Saudi Arabia. Their paper presents one approach to developing an integrated "economic and engineering-based methodology to model the dynamic production decisions from an idealized oil field." As Gao et al. wrote, "Specifically, we use an engineering computer model of dynamic flow (*Workbench* Black Oil Simulator, 1995) to simulate the effects of water injection rates, the cumulative production of the field, and the number of oil wells on the cost of production and short-run production capacity" (Gao et al., 2008). Rather than estimate field-specific inverse production functions from available wells and production data as we did, Gao et al. used field-specific reservoir characteristics to model the production function directly with an engineering model. The engineering model simulation generated the data used to estimate a "short-term dynamic production function" that was then used in the dynamic optimization model.[35, 36] They then modeled the dynamic optimization problem with the number of wells as the control variable.

Our novel method for construction of the CCF is an alternative for integrating economic and engineering-based methodology that delivers a cost function that extends beyond the range of historical data, shows the onset of decreasing returns to scale predicted by economic and reservoir theory, and does so without needing data on reservoir characteristics or simulated data from engineering models. As Gao et al. noted, "in practice, oil wells typically are abandoned well before the reservoirs are depleted. Depletion raises the costs of extraction until they make continued recovery unprofitable." They incorporated this phenomenon into their modeling with an engineering computer model that predicts an increasing number of wells and quantity of water injection to maintain oil production as a reservoir is depleted. We incorporate the same phenomenon econometrically by using data on the number of operating wells to estimate a scalar that introduces decreasing returns to production rate into the composite cost function. Our method is an alternative for addressing a fundamental aspect of reservoir geology without requiring explicit engineering data or modeling.[37] The resulting cost function approximation is well behaved over the entire range of production rates and reserve quantities that may be encountered during the empirical dynamic optimization process.[38]

3.2.6 Discussion of Methods for Cost Estimation, part II

The astute reader may suggest there should be dynamics in the wells function model to account for friction since it appears that wells in the current period are affected by wells in the previous period and because control is not instantaneous (i.e., there is an adjustment cost to turning wells on and off). In fact, the lagged dependent variable(s) in a dynamic regression specification often explain a large portion of the variation in time series data (Ramanathan, 2002). For a dynamic specification in this case, we looked at autocorrelation and partial autocorrelation of the dependent variable (wells) and observed the geometric decline in statistical significance in lag-1, -2, and -3 terms that is typical for

[35] Gao et al. assumed that "current oil production affects reservoir conditions and hence future production costs and ultimately the total resources that will be extracted from the reservoir," and modeled the reservoir conditions with an engineering computer model. In contrast, we estimate production cost econometrically as a function of the rate of oil production and the reserves remaining.

[36] The estimated dynamic production function that resulted from the Black Oil simulation was "consistent with the hypothesis that short-term overproduction will jeopardize the producing environment of a particular well. In particular, high levels of current water injection exacerbate the negative effect of cumulative past production on future maximum producing capacity" (Gao et al., 2008).

[37] Gao et al.'s use of an engineering computer model to simulate production cost implies modeling a "stylized oilfield" while our approach seeks to estimate field-specific cost functions econometrically from available data so we can model each of the seven actual production units on the North Slope. (They do argue that their modeling is a representation of the Ghawar field in particular since the reservoir characteristics used in the Black Oil simulator "mimic those of Saudi Arabia's largest light oil field, Ghawar.")

[38] Iterative model specification was necessary to find acceptable forms given the fundamental problem of trying to estimate a function showing decreasing returns to scale in production rate from well data that do not enter such a realm (i.e., producers have not produced beyond geologic limits to flow rate).

an AR(1) process.[39] However, the implication for our modeling is not what the reader originally supposed. It is not clear whether the importance of the lagged variable stems from adjustment costs or dynamic production management. Moreover, our intended use of the wells function is in scaling the cost function, which is an exogenous function in the dynamic model of oil production. Furthermore, the dynamic model includes an adjustment cost factor determined by the absolute difference between current-period and previous-period production. Logically, it makes sense that the cost function we develop for use in the dynamic modeling is a static cost function (i.e., it gives the cost of production for all possible combinations of Q and S that may be encountered in the dynamic production path that is solved for in the overall dynamic model). The data from Attanasi and Freeman (2005) are for average cost over the lifetime of a field and we are adding decreasing returns to the marginal cost by scaling by the observed need for producing wells to achieve all possible combinations of Q and S in each field. Thus, the wells function is not intended to be dynamic, and not intended to be used for hypothesis testing. Rather, it is meant to forecast the number of wells that would be needed for any particular combination of Q and S and, consequently, how much production cost should be inflated if that combination of Q and S are chosen. Thus, while dynamic specification of the wells function would likely be appropriate if the estimation was an end product intended for use in hypothesis testing or policy simulation, a static specification is appropriate for our use as an intermediary input to a larger dynamic model. The appropriate place to introduce the dynamics of adjustment cost in bringing wells into or out of production in our modeling is in the dynamic model via a cost associated with the change in production from one period to the next rather than in the wells function regression specification. The magnitude of this adjustment cost could be informed by the coefficient estimated in a dynamic wells function specification, but would not be identical since wells are not equivalent to cost. Thus, without a better way to estimate the adjustment cost, it remains a parameter in the dynamic model that is used for model calibration and is subjected to sensitivity analysis.

The astute reader may also note that estimating the wells function amounts to estimating a function for discrete data (since wells is a count of the number of operating wells). For large numbers of wells (i.e., more than 50), estimating the function under the assumption of normal distributions is acceptable, especially if the variance is low (i.e., relatively few wells are coming on and off line). However, for the fields with very low numbers of wells (Badami and Northstar, each with 10-30 wells), the dependent variable is a count and we should use a logit model rather than OLS. Since Badami was omitted from our analysis for other reasons, we leave modeling of Northstar with a logit model of the wells function to future work.

3.3 Price Estimation

As with the cost function estimation, a price function was estimated exogenously via linear regression analysis of the historic wellhead value data and EIA forecast data described in the data section.

---

[39] An AR(1) process is first-order autoregressive, meaning there is correlation from one time period to the next. A few fields also have significant AR(12) terms (e.g., Kuparuk, Milne), indicating the need for a monthly lag as well due to the seasonality of work on the North Slope.

We appended future oil price projections from the Energy Information Administration (FOB, through 2030; EIA, 2007) to historical data on Alaska wellhead value to incorporate the modeling done by the EIA in forecasting future prices into our function of future price behavior. The EIA data are for wellhead value (a.k.a. FOB price) and thus measure a consistent commodity with the Alaska data. However, the EIA data are for "average lower-48" oil rather than Alaska North Slope oil, which presents two discrepancies for which we corrected.

To concatenate the data, we adjusted for the discrepancy in shipping cost for lower-48 versus Alaska oil. Assuming a homogenous commodity (approximately true for oil) sold into the world market at a single price, the lower-48 wellhead value should differ from the Alaska wellhead value by the difference in cost of delivering the oil to market. We used a seven-year period of overlap in the data to estimate this difference. However, the Alaska wellhead values were calculated by the Alaska Department of Revenue for the Alaska fiscal year, which runs from July 1 to June 30 (personal communication, Michael Williams, Alaska Department of Revenue, July 2007),[40] while EIA forecasts were made for each calendar year. Fortunately, the EIA data for the seven years of overlap were reported monthly, so we calculated the average wellhead value from EIA data for Alaska's fiscal year (July–June) rather than the calendar year (January–December).

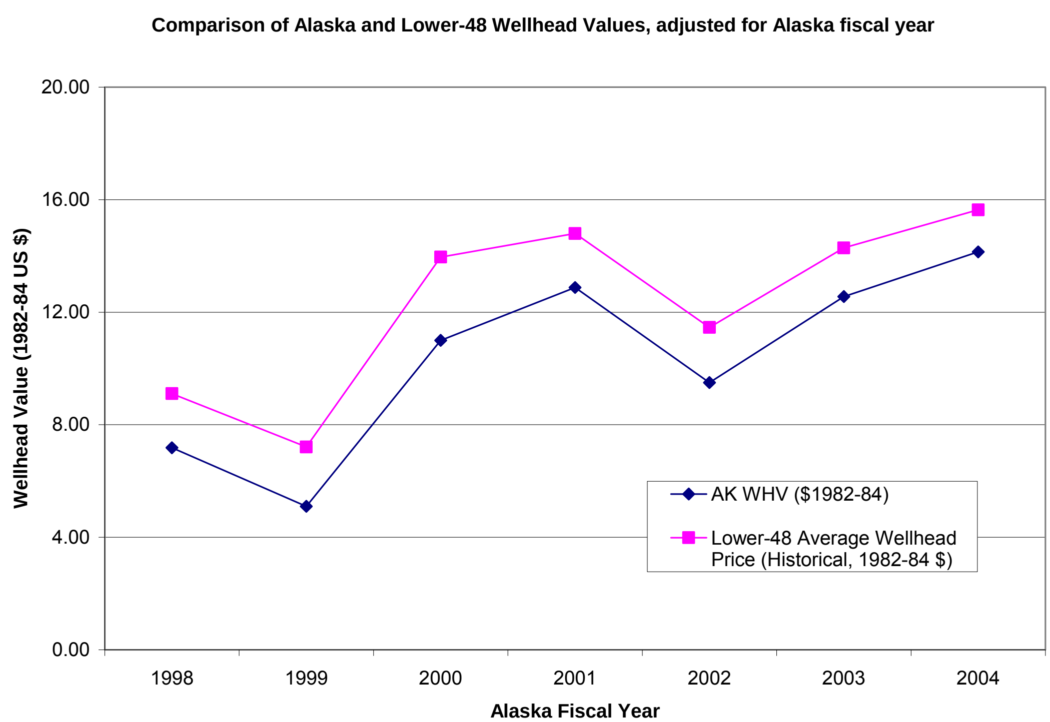


Figure 15: Comparison of Alaska wellhead values reported by the Alaska Department of Revenue with "lower-48" wellhead values reported by the EIA and averaged for the Alaska fiscal year. The consistent discrepancy is explained as the difference in shipping costs.

[40] Note, there is also a one-month lag between production data and tax data because taxes are filed monthly, so revenue from production in June is taxed in July. Thus, the state's July 1 to June 30 fiscal year corresponds to taxes on oil production from June 1 to May 30. It also takes several weeks for oil produced on the North Slope to reach market.

The discrepancy between Alaska and lower-48 wellhead values appears relatively consistent (Figure 15), which accords with the assumption stated previously that the discrepancy should be the difference in cost for delivering the oil to market (a relatively constant parameter). Thus, we scaled EIA data to match the Alaska data by simply subtracting a constant difference in shipping cost and shifting the Alaska data by six months to accurately reflect its calculation for the Alaska fiscal year rather than the calendar year.[41] The resulting concatenation is shown in Figure 16.

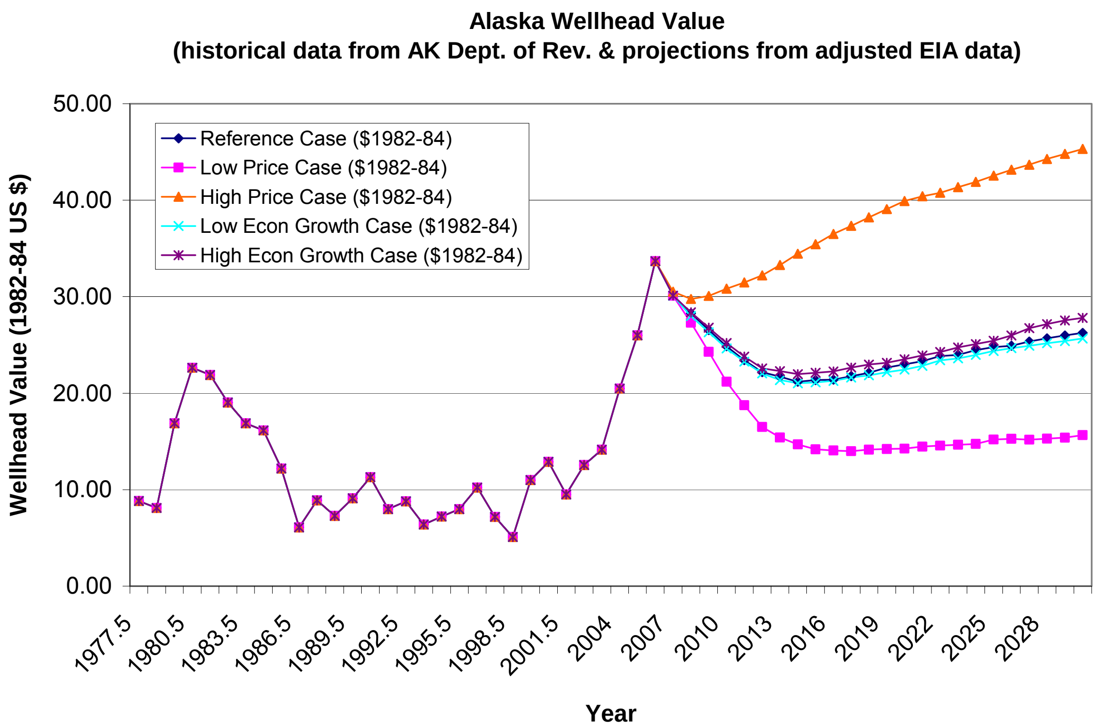


Figure 16: EIA wellhead value forecast data, adjusted for Alaska's fiscal year and the discrepancy in shipping cost for Alaska and "lower-48" oil, were appended to historical Alaska wellhead value data to create the entire wellhead value data set.

From these compiled data, it appears the EIA reference case and two economic growth variants are quite similar. Consequently, the five EIA scenarios were collapsed into three: reference, high price, and low price. Finally, the price function for these three scenarios was estimated with simple linear regression for the following second degree polynomial functional form (results are shown in Figure 17 and Table 7):

8 $$P(t) = c_{21} + c_{22}\text{Month} + c_{23}\text{Month}^2$$

[41] For example, the Alaska data reported for the year 1997 is actually the average value for July 1, 1996 to June 30, 1997 and thus should be reported as "1996.5" data)

where P(t) is wellhead value in 1982-84 dollars per barrel and Month is indexed to January, 1978 (i.e., equals zero in January, 1978).

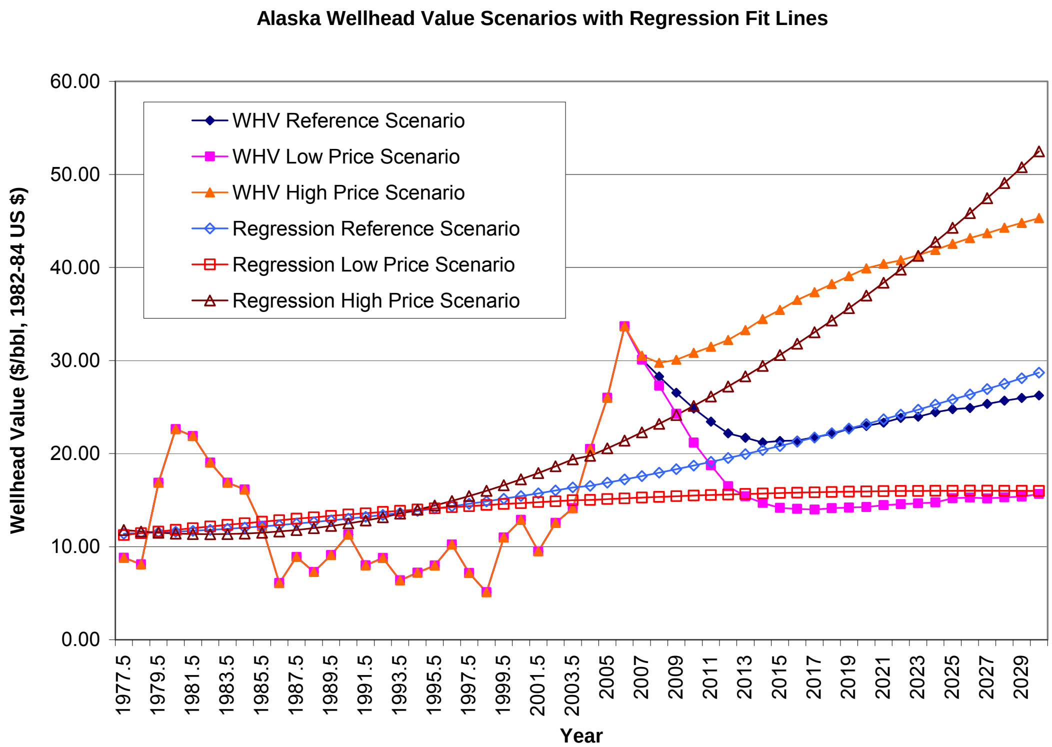


Figure 17: Reference, low-price, and high-price price functions are shown with the historical Alaska wellhead value data and adjusted EIA forecast data from which the price functions were estimated.

	constant	Month	Month^2	Adj. R^2
Low Price	11.9031***	0.0132529	-0.0000101	0.0207
(std. error)	2.506673	0.0183633	0.0000283	
Reference	12.10955***	0.0020134	0.0000404	0.4427
(std. error)	2.377859	0.0174196	0.0000268	
High Price	12.31032***	-0.0169067	0.0001313***	0.8221
(std. error)	2.425985	0.0177722	0.0000274	

Table 7: Wellhead price function parameter estimates (1982-84 dollars per barrel; Month indexed to January, 1978). Statistical significance for coefficient estimates is indicated at the 5% level (*), 1% level (**), and 0.1% level (***).

The regressions produce statistically significant results only for the high-price scenario (adjusted R^2 = 0.8221; two coefficients significant at the 0.001 level). The low-price case has a dismal fit (adjusted R^2 = 0.0207) and the fit for the reference case is still poor (adjusted R^2 = 0.4427). Neither case shows statistical significance at even the 0.1 level for any coefficients. As with the user-defined parameters in the CCF, we built the dynamic optimization model with the option for selecting any of these three price scenarios to enable sensitivity analysis.

A fourth option included in the dynamic optimization model is a fixed price or fixed price with stepwise increase over time. The rationale is based on conversations with producers which suggested that long-range planning in the oil industry is generally performed with a single price estimate rather than a functional form of price projection (personal communication, Simon Harrison, BP Exploration Alaska, July 2, 2007). The thinking is that future prices are so uncertain that any forecast is bound to be incorrect, so it is more informative to perform the analysis with several scenarios for a single price rather than more complicated price specifications.[42] Consequently, a fixed-price option in the dynamic optimization model may most accurately reflect the decision-making process within the oil industry.

---

[42] The results of our sensitivity analyses for price scenarios, however, demonstrate the impact of a price trend on the optimal production path. Increasing price spreads production into future periods while a constant price, regardless of level, pushes production into earlier periods, ceteris paribus.

4 Modeling Alaska Oil Production

We use the standard assumption that perfectly competitive producers choose a production path to maximize the sum of the discounted stream of future operating profits accruing each period from the sale of the resource. Modeling with perfect foresight, the unit operator's problem is to choose the optimal time path for production given known future output prices and taxes.[43] The simplest model of the dynamic production decision faced by Alaska unit operators is one in which profits are given by revenues less cost, where revenue is price (wellhead value) times quantity and cost is the total production cost. In other words, this is a model of profit at the wellhead, ignoring downstream activities and prior exploration and development activities.[44] Since Alaska law requires unitization prior to production from all oil fields, it is each unit operator that faces the production decision of how to maximize profit from extraction of the non-renewable resource in the particular field. Of course, there are many other factors involved in production decisions. We added some complexity to our model, as discussed below, while striving to meet the equally important goals of capturing the salient features while enabling straightforward solution and interpretation.

4.1 Objective Function and Optimal Control Problem

Assuming Alaskan unit operators are price takers (i.e., sell into the world oil market at market price), their objective function is to maximize profits from oil production, which are given by total revenue minus cost.[45] Thus, the first part of their optimal control problem is to choose the production profile {Q(t)} to maximize the present discounted value of the entire stream of future profits.

9 $$\text{Max}_{\{S(t)\}} \int_0^{\infty} \{P(t)Q_{it} - CCF(Q_i(t), S_i(t))\} e^{-\rho t} dt$$

where the subscript *i* indexes units and the subscript *t* indexes time, P(t) is the wellhead value (market price less shipping cost) for Alaska North Slope crude, S_{it} is reserves remaining measured in millions of barrels, Q_{it} is oil production per period measured in millions barrels per month, $CCF(Q_i(t),S_i(t))$ is the total cost of production given by the

---

[43] See Wilson (1999) for a review of this type of basic tax competition model in which governments commit to a tax policy which is reacted to as exogenous by affected firms.

[44] As described previously, production cost as we defined and estimated it includes what are generally called "development costs" (e.g., surface infrastructure, well drilling, and maintenance) and "production cost" (e.g., variable costs of operation and engineering), but not "exploration costs" (e.g., geophysical surveys, drilling exploration wells). The omission of exploration costs is appropriate for our modeling of unit operators' production decisions, which should be made without regard to past investments, but requires care in interpreting our results for "producer profit." Specifically, we report profits from oil production from known fields, from which exploration and overhead costs should be deducted to approximate net corporate profits. Ignoring downstream activities is appropriate for modeling the profit-maximizing unit operator's production decisions, which may or may not be influenced by objectives of the parent company.

[45] We are aware that objectives other than profit maximization might also be plausible especially since BP-Alaska, ExxonMobil-Alaska and ConoccoPhillips-Alaska are divisions of global corporations who, consequently, do not make production decisions in isolation. Increasing share value, corporate net worth, or CEO cache may all be short-term objectives for individual operations in addition to profit maximization.

composite cost function, and ρ is the discount rate. As described previously, both the price and cost functions are exogenous to the model, so the control variable is quantity, which is chosen to maximize profits.

Unit operators are constrained, however, in this maximization by four physical realities of non-renewable resource extraction: the stock available in the first period equals the initial reserve of the resource, the change in reserve is equal to the rate of production, the rate of production is nonnegative (i.e., producers do not re-inject oil), and the stock is nonnegative (i.e., no more oil can be produced when the stock is depleted). Consequently, equation 9 is maximized subject to the following:

10 $S(0) = S_o$

11 $Q(t) \geq 0$

12 $S(t) \geq 0$

13 $\frac{dS(t)}{dt} = -Q(t)$

Thus, the complete optimal control problem is to choose the production profile {Q(t)} to maximize the present discounted value of the entire stream of future profits, given the initial stock S(0) and the relationship between production Q(t) and the remaining stock S(t), and subject to the constraints that both production and stock are nonnegative.

This model structure is simple for many reasons. First, and most importantly, it omits components of production cost which include a variety of state and federal taxes that can sum to more than 60% of gross revenue. We now consider how incorporating taxes changes the model specification.

The state of Alaska collects four types of tax related to oil production: royalty, severance tax, corporate income tax, and property tax. Royalty and severance tax are the largest. Until recently, the severance tax was adjusted by the economic limit factor (ELF), which was a fraction between one and zero.[46] Focusing on just the largest two components of state taxation, net revenue in the unit operator's objective function becomes a fraction of total revenue, with lease royalties (LR) and severance tax (ST), adjusted by the ELF, subtracted. That is,

14 $\pi(Q_{it}) = P(t)Q_{it}(1 - LR_{it} - ST_{it}ELF_i(Q_i(t)) - CCF(Q_i(t), S_i(t))$

[46] The ELF adjustment factor was determined for each unit by a formula set out in Alaska statute based on the volume of production from the unit. Essentially, the ELF was designed to reduce the tax burden on "marginal" fields near their "economic limit," so the ELF factor is lower for fields with low production volume (i.e., "marginal" fields). Alaska statute 43.55.011 specified the ELF formula as follows:

$$ELF(Q_i(t)) = \left(1 - \frac{300 * WELLS_{it} * DAYS_t}{Q_{it}}\right)^{\left(\frac{150,000}{Q_{it}/DAYS_t}\right)^{\left(\frac{460*WELLS_{it}*DAYS_t}{300*WELLS_{it}*DAYS_t}\right)}}$$

Rather than estimate functions from data for the element "well-days" in the ELF formula, we used the average number of days producing for wells on the North Slope (25.7) to multiply by the number of wells predicted from the wells function to get well-days for use in the ELF formula.

The unit operator's optimal control problem can now be written as follows:

15 $$\text{Max}_{\{S(t)\}} \int_0^\infty (P(t)Q_{it}(1 - LR_{it} - ST_{it}ELF_i(Q_i(t))) - CCF(Q_i(t),S_i(t)))\, e^{-\rho t}\, dt$$

$$\text{s.t.} \quad dS(t)/dt = -Q(t)$$
$$Q(t) \geq 0$$
$$S(t) \geq 0$$
$$S(0) = S_o$$

where the added elements are the lease royalty percentages (LR_{it}) and severance tax percentages (ST_{it}) as modified by the economic limit factor ($ELF_i(Q_i(t))$).

4.2 Solving the Optimal Control Problem

We tried two numerical approaches for solving the optimal control problem of dynamic oil production: one with an ordinary differential equation boundary value problem and the other with a Bellman equation value function (Dorfman, 1969; Pontryagin, Boltyanskii, Gamkrelidze and Mishchenko, 1962). The boundary value problem approach was burdened by rapidly expanding complexity in differential equations as taxes were added to the modeling (see appendices G and H) and was ultimately blocked by our inability to impose the nonnegativity constraints on production and reserves remaining in Matlab's "bvp4c" solving routine.[47] The complete derivation of the boundary value problem approach is presented in appendix G, and will not be discussed further.[48, 49]

We developed the numerical approach based on the Bellman equation in Microsoft Excel. The task for each field is to maximize the net present value of profits from the entire production path ($\max_{Q_t} \sum_{t=0}^{\infty} \beta^t \pi(Q_t)$), where β is the discount factor 1/(1+discount rate), subject to the initial stock S(0) and equation of motion that relates

[47] The bvp4c solving routine in Matlab is designed for solving boundary value problems for ordinary differential equations by integrating a system of ordinary differential equations subject to two-point or multipoint boundary value conditions. The bvp4c routine uses a finite difference code implementing the three-stage Lobatto IIIa collocation formula. See Shampine et al. (2008) for more information.

[48] The reader will notice in appendix G that adding taxation into the objective function complicates both the first order condition (FOC) for static optimality in the current period and the FOC for the evolution of the shadow price over time to ensure inter-period optimality because price is net of taxation. The transversality condition is unchanged. In particular, derivations in the first step of the boundary value problem become very complex because the term $\frac{\partial ELF}{\partial Q}$ in the FOC for static optimality becomes $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$, which contains the term $\frac{\partial Q}{\partial t}$ and consequently requires solving all equations in order to isolate all $\frac{\partial Q}{\partial t}$ terms.

[49] Including the drilling cost scalar may make the problem non-autonomous, meaning the optimization is no longer a function of control and state variables alone but also has a time-dependent drilling cost variable, which makes solving differential equations difficult and could cause problems in reaching a numerical solution. The ELF factor could also cause problems in reaching a solution to the differential equations boundary value problem.

current stock and production to future stock. By the principle of optimality, we can break equation
15 into a finite problem (current value function) and discounted future stream (future value function). In this case, the value function and equation of motion are as follows.

16 Value function:
$$V(S_{it}) = (P(t)Q_{it}(1–LR_{it}– ST_{it}ELF_i(Q_i(t))) - CCF(Q_i(t),S_i(t))) + \beta V(S_{t+1})$$

17 Equation of Motion: $S_{t+1} = S_t\text{-}Q_{it}$

Thus, we can write the Bellman Equation as follows.

18 Bellman Equation:
$$V(S_{it}) = \max_{Q_{it}}[(P(t)Q_{it}(1–LR_{it}– ST_{it}ELF_i(Q_i(t))) - CCF(Q_i(t),S_i(t)))] + \beta V(S_{t+1})$$

where the subscript *i* indexes units and the subscript *t* indexes time, P(t) is the wellhead value (market price less shipping cost) for Alaska North Slope crude, Q_{it} is the oil production per period measured in millions of barrels per month, LR_{it} is the lease royalty percentage, ST_{it} is the severance tax percentage, modified by the economic limit factor ELFi(Qi(t)), $CCF(Q_i(t),S_i(t))$ is the total cost of production given by the composite cost function, and β is the discount factor defined as 1/(1+discount rate).

We used the Solver function in Microsoft Excel to solve for the entire path of production that maximizes the first period value function, subject to the equation of motion, initial reserves, and the restrictions that production and reserves remaining are nonnegative. Although oil production is theoretically an infinite horizon proposition, we approximated the optimization with a long finite horizon (i.e., year 2176 or more) and zero scrap value. Given limits in the number of periods the Solver can handle, we modeled in annual increments and used linear interpolation to calculate monthly increments in the production path. Imaginary numbers caused by negative numbers raised to fractional exponents forced us to impose the nonnegativity constraint for reserves remaining crudely by setting production cost absurdly high ($1 million per barrel) if $S<0$. However, this approach reduces the number of restrictions defined in Solver, increasing its capacity for periods from (approximately) 100 to 200. The initial guess for production was set to one barrel per month in all periods prior to running Solver to ensure consistent results. Future work may involve developing an infinite horizon model in GAMS although we do not expect results to differ significantly from the long-finite-horizon results presented in this paper.

4.3 Sensitivity Analyses

Prior to looking for results with our model calibrated to historical production (see section 4.4), we ran the uncalibrated model for a variety of scenarios to test its sensitivity to key parameter values. This set of scenarios, which we call the "uncalibrated model" scenarios, are summarized in Table 8.

Uncalibrated Model Scenarios:
dynamic optimization unconstrained by initial production rate or adjustment cost

	DR	WHV	Taxes	AC	DRTS_M
Reference Parameters	5%	Fixed, $20/bbl	none	none	1.1
Sensitivity Analyses					
Impact of High Price	5%	High P Scen.	none	none	1.1
Impact of Reference Price	5%	Ref P Scen.	none	none	1.1
Impact of Low Discount	2%	Fixed, $20/bbl	none	none	1.1
Impact of High Discount	10%	Fixed, $20/bbl	none	none	1.1
Impact of Steeper Cost Func.	5%	Fixed, $20/bbl	none	none	0.7
Impact of Shallower Cost Func.	5%	Fixed, $20/bbl	none	none	1.5
Impact of Taxes, No ELF	5%	Fixed, $20/bbl	1	none	1.1
Impact of Taxes with ELF	5%	Fixed, $20/bbl	2	none	1.1

Parameters common across all units and all scenarios include Lag = 1 and Dmp = 2

Table 8: The uncalibrated model scenarios were unconstrained for initial production rate and did not include adjustment costs. This table summarizes the parameter values used for sensitivity analysis of the uncalibrated models. Abbreviations include discount rate (DR), wellhead value (WHV), adjustment cost (AC), and a factor that changes the slope of the production cost function (the “decreasing returns to scale margin,” DRTS_M).

4.4 Model Calibration

To calibrate the model to historical production paths for each field, we constrained production in the first period to be equal to historical production in the first period and introduced the notion of an adjustment cost. Estimation of the facilities investment cost of production upon which our base cost function was estimated assumed efficient field development. Conversely, increasing or decreasing production rapidly may introduce additional adjustment costs (see sidebar below). Consequently, we defined an adjustment cost function as follows.

19 Adjustment cost: $AdjC_t = c_{25}*(Q_t-Q_{t-1})^2$

where Q_t is the production rate in the current period, measured in millions of barrels per month, and Q_{t-1} is the production rate in the previous period. Subtracting the adjustment cost from revenue in equation 16 yields the value function used in the calibrated model.

20 Value function for Calibrated Model:

$$V(S_{it}) = (P(t)Q_{it}(1 - LR_{it} - ST_{it}ELF_i(Q_i(t))) - AdjC_{it} - C(Q_i(t),S_i(t))) + \beta V(S_{t+1})$$

What are adjustment costs, really?

The notion of adjustment costs is more than an ad hoc tool to calibrate our model to data. In fact, adjustment costs capture important aspects of reality for oil production. It is difficult, however, to distinguish between several potential underlying drivers.

First, there are physical limitations to oilfield development that constrain the ability to ramp up production. In Alaska, these limitations come from a finite number of available drilling rigs, which was especially limiting during the initial boom of North Slope exploration and development, and a working season limited to the winter months due to hauling equipment over frozen tundra. Accounting for these limitations could be modeled with a simple maximum rate of change in production from one period to the next rather than an adjustment cost, although the adjustment cost captures the effect as well and allows for pushing these limits by spending more money.

Second, increasing or decreasing production rapidly may cause inefficiency as the project timeline becomes a constraining factor, causing insufficient labor, materials, or equipment supply to cost more and causing management decisions in favor of expediency rather than cost minimization. In this case adjustment costs reflect the higher project costs due to inefficiency in implementation.

Third, the gradual ramp-up in oil production observed in data may be hedging behavior against the risk and uncertainty in reservoir characteristics. Since any particular pattern of wells limits the range of possible future production adjustments, it is costly to miss-judge the reservoir characteristics and implement a development plan that is not optimized for the reservoir. Consequently, producers may intentionally ramp up production slowly, drilling in a dispersed pattern to gather more information about the reservoir with which to revise their development plan along the way. Our current modeling does not account for risk aversion, meaning we are not able to examine this explanation in detail. Future work may investigate whether the spatial pattern of well drilling in Alaska has been more consistent with implementation of an established development plan or with gathering of reservoir information.

Fourth, reservoir engineering considerations not captured in our current modeling may dictate gradual increase and lower peak-production rate than our "uncalibrated model" suggests. If rapid initial production causes too much loss in reservoir pressure, which compromises ultimate recovery, the adjustment cost parameter serves as a proxy for the foregone future production.

The parameter c_{25} and the discount rate were set for each field to calibrate the model to historical actual production. We used an iterative procedure over a coarse and fine mesh of adjustment cost and discount rate parameters to identify the best-fit combination based on minimizing the sum of squared errors between the simulated optimum production path and historical actual production path. The coarse mesh incremented the discount rate in one percentage-point increments starting from one percent and incremented the adjustment cost parameter by orders of magnitude starting from one. The fine mesh was tailored to each field to close in on more refined best-fit estimates for these parameters by incrementing the discount rate in 0.1 percentage point increments and incrementing the adjustment cost parameter in even increments dictated by the magnitude of the coarse mesh approximation. Results from this procedure are shown in Table 9. We repeated the same sensitivity analyses with the calibrated models as we did with the uncalibrated model, albeit with different values for low and high discount rates and cost function slopes (Table 9).

The calibrated models: dynamic optimization with initial production and adjustment cost used to match model results with actual production history

	Best Fit		Sensitivity Analyses*	
	Discount Rate	c_{25} (Adj. Cost)	Low Discount	High Discount
Prudhoe Bay	7.4%	8*10^7	2%	15%
Kuparuk	8.6%	4*10^8	2%	15%
Milne Point	9.5%	6*10^9	2%	15%
Endicott	12%	7.5*10^7	5%	20%
Colville	20.5%	9*10^7	10%	30%
Northstar	46%	1.8*10^7	30%	60%

* Additional sensitivity analyses involved consistent parameter changes for all units:

Impact of High Price:	High Price Scenario
Impact of Fixed Price:	Fixed Price, $20/bbl **
Impact of Steeper Cost Function:	DRTS_M = 0.5
Impact of Shallower Cost Function:	DRTS_M = 2

Table 9: Parameter values used in calibrated models. Initial production was set equal to historical initial production; adjustment cost and discount rate parameters were used to calibrate the model to the actual historical production path. The adjustment cost parameter c_{25} gives the cost in 1982-84 constant dollars per one million barrels-per-month change in production rate (e.g., c_{25} = 80,000,000 implies $80 million adjustment cost to increase production capacity by 1 million barrels-per-month in one month, or $80 to increase production capacity by one barrel-per-month in one month). The abbreviation DRTS_M refers to a factor that changes the slope of the production cost function (the "decreasing returns to scale margin").

The tax policy for the calibrated models was set to the actual historical policy for Alaska state royalty and severance taxes, with the ELF factor and 12.25 percent severance tax through June, 1981 and 15 percent thereafter (Alaska state corporate income and property taxes and federal taxes were omitted; see Table 10). We assumed a one-year lag for drilling cost to impact production cost since wells are often drilled and brought online in one work season (i.e. Lag = 1). We reduced the effect of the drilling cost scalar (Dmp = 2) due to uncertainty for whether drilling costs are indicative of scarcity that would impact production cost (i.e., an exogenous result of world conditions and independent operating decisions at other fields) or are a response to higher oil price (i.e., endogenous higher cost of drilling in marginal areas motivated by higher oil price) (see footnote 27). We made no adjustment to our best estimate of the cost function (i.e., DRTS_M = 1) and used a symmetric adjustment cost (i.e., same for increases and decreases in production rate).[50] Finally, we used the reference price scenario for the calibrated best-fit models. Although producers may have developed production plans based on different price forecasts, the reference price scenario uses the benefit of hindsight and is therefore appropriate for calibrating the model to evaluate how closely producers were able to approximate dynamic optimality despite imperfect price foresight. In other words, we evaluate whether the best-fit discount rate and adjustment cost parameters that best calibrate the model to historical production given the reference price scenario are reasonable.[51]

4.5 Tax Scenarios

One strength of the modeling approach we developed is its flexibility for incorporating almost any tax structure imaginable. We used the model to investigate the impact of several different tax structures on the optimal production path and net present value of profits and state tax revenue. These scenarios vary severance tax only, leaving royalty unchanged and leaving corporate income, property, and federal taxes omitted. The scenarios are summarized in Table 10.

The baseline tax policy used in developing our “best fit” calibrated model was the policy that existed through 2006 under which historical production decisions were made (although without perfect foresight as assumed in our modeling).[52] The severance tax was assessed on the gross wellhead value at a rate of 12.25% through June, 1981 and 15% thereafter, adjusted by the ELF factor for small and/or low-producing fields (AS 43.55.150).

Our first hypothetical tax policy simulates a simple tax increase under the historical tax system by increasing the severance tax rate to 25% of gross wellhead value. Under such a policy, one would expect to see no change in the production path and the same net social benefit, but with tax revenue increased and profit decreased. The reason

---

[50] Although higher adjustment cost for increases in production than for decreases in production may be more realistic, further investigation of the simplification of symmetric adjustment cost is left to future work. With the current model structure, the adjustment cost parameter has negligible impact for declining production since the decline follows a path of gradual change rather than the rapid increase to bring production up to the unconstrained optimal production path (see Figure 18 and Figure 19).

[51] One argument to justify this model calibration was posited by Ruth and Cleveland (1993). The difference between model results and historical production behavior may be caused by model misspecification rather than by non-optimal industry behavior. Production costs are influenced by many factors other than production rate and reserves remaining, meaning our cost function suffers from bias due to omission of relevant variables. Data undoubtedly have measurement errors as well. Consequently, it is “appropriate to adjust parameter estimates which we know are biased to an unknown degree” (Ruth and Cleveland, 1993). In our case, however, we are adjusting the hypothetical discount rate and adjustment cost parameters rather than adjusting econometrically-estimated parameter values within confidence bounds as done by Ruth and Cleveland (1993). Thus, our manipulation is truly a simulation calibration rather than adjusting to achieve model solution (ibid).

[52] The degree to which foresight is perfect or imperfect is likely to affect the production path since production plans are based on expectations of future conditions. The implication is that establishing expectations of future policy may be more important than changing current policy for influencing production decisions. Examination of perfect versus imperfect foresight in our modeling, however, is left to future work.

is this "first-best" policy does not distort the dynamic optimization of production. Note that the baseline tax policy was actually not a first-best policy since the tax rate change in 1981 encouraged a (hypothetical) producer blessed with perfect foresight to shift production into earlier periods when the tax rate was lower. The ELF factor also adds a slight distortionary influence. Consequently, the first hypothetical tax policy serves as the first-best benchmark against which other hypotheticals will be compared.

Our remaining hypothetical tax policies simulate combinations of severance tax rates and tax credits designed to encourage more rapid production by offsetting development expenses. The second hypothetical tax policy uses a severance tax rate of 25% of gross value with 20% credit for adjustment cost (up to but not exceeding the total tax burden). The tax rate remains relatively high to offset the cost of credits refunded. Our third hypothetical tax policy repeats the simulation of the impact of 20% tax credits but with a lower tax rate on gross value (15%). Our fourth hypothetical tax policy repeats the simulation with 25% tax rate and a higher credit rate (40%). As with the first hypothetical tax policy, we expect to see no change in the optimal production path from changes in the tax rate, but expect to see the production path shifted to earlier periods by the higher credit rate since some of the adjustment cost associated with more rapid ramp-up in production is "free" for the producer. Thus, we expect to see the implementation of tax credits working to motivate more rapid production, although at the cost of net social benefit due to distortion in the dynamic optimization (i.e., increased adjustment costs are borne by the government while production decisions are made by the unit operators). Thus, the government is unable to increase the tax rate to exactly compensate for the credit payments without reducing producer profits.

Finally, our fifth hypothetical tax policy is an approximation of the new tax policy passed by the Alaska legislature in 2006 (amended in 2007). The Alaska Legislature completely revised the code for calculating severance tax (Alaska statutes AS 43.55). The revision was an immensely complex change from the former tax on the gross value of oil production (wellhead value) to a tax on net revenue, modified by a set of deductions and credits, and involving both a series of reductions to the tax rate if West Coast oil price falls below $25/bbl and increases to the tax rate if West Coast oil price rises above $92.50/bbl (AS 43.55.011).[53] Abstracting from many layers of complexity, the new law set a base tax rate of 25% of "tax value" (defined as gross value less allowable lease expense) that is modified by 20% to 40% credits for allowable expenditures (generally associated with exploration and development).

We model this tax policy with a 25% tax rate on the net "tax value" defined as wellhead value ($/bbl) less production cost ($/bbl), with 20% credit for adjustment cost. This is our best approximation of the new tax policy as it would have applied to these

[53] House Bill 2001, passed in special session in 2007 was called Alaska's Clear and Equitable Share (ACES). It made the following modifications to Alaska statutes. AS 43.55.011 specifies a 25% tax on the "tax value" under AS 43.55.160, with "floors" if west-coast price is less than $25 per barrel and "progressivity" if price increases above $92.50 per barrel. AS 43.55.023 specifies 20% tax credits for allowable expenditures (generally exploration and development activities) and 25% tax credits for carried-forward annual loss. AS 43.55.160 defines the production tax value as the gross value at the point of production (WHV) less lease expenditures (under AS 43.55.165) – i.e., the tax is on net rather than gross revenue.

existing fields after discovery (i.e., no additional credit for exploration expenses).[54] It also facilitates direct comparison with our second hypothetical tax policy to investigate the impact of taxing the "net value" rather than gross value.

The Policy Scenarios: dynamic optimization with the calibrated model under a variety of hypothetical tax policies

		Taxes	Sev.Tax
Best Fit	(i.e., actual historic tax policies)	2	3
Hyp. 1:	High Tax within Old Gross-Value system with ELF	2	25/0
Hyp. 2:	Tax on Gross with Low Credits to Offset Adjustment Cost and High Tax	3	25/20
Hyp. 3:	Tax on Gross with Low Credits to Offset Adjustment Cost and Low Tax	3	15/20
Hyp. 4:	Tax on Gross with High Credits to Offset Adjustment Cost and High Tax	3	25/40
Hyp. 5:	Tax on Net with credits (Best Approximation of 2007 Policy)	4	25/20

Note, other parameters are as specified for each unit for the calibrated models

Key to taxes	
0	No state taxes, no federal taxes
1	State royalty and severance tax, no ELF factor, no federal taxes
2	State royalty and severance tax, with the ELF factor, no federal taxes
3	State royalty and hypothetical severance tax on gross WHV, no federal taxes
4	State royalty and hypothetical severance tax on net (WHV-cost), no federal taxes

Key to Severance Taxes	
1	Pre-July 1981 rate of 12.25 percent used for all periods
2	Post-July 1981 rate of 15 percent used for all periods
3	pre-July 1981 rate of 12.25 percent used through June, 1981; post-July 1981 rate of 15 percent used after June, 1981
a/b	hypothetical tax rate / hypothetical credit rate

Table 10: Tax policy scenarios simulated with the calibrated dynamic optimization production model.

[54] In the 2006/2007 change to Alaska tax policy, credits and deductions were implemented to encourage more rapid exploration and development and were defined to apply narrowly to these two phases rather than the production phase we model. To the extent that such legal partitioning of costs is successful, the impacts of these credits and deductions on production decisions are likely to be minimal. However, there was some question as to the separation of costs that would occur in practice.

5 Results and Discussion

Summary statistics for historical production and model results for Prudhoe Bay are presented in Table 11, Table 12, Figure 18, Figure 19, and Figure 20. Results are unique for each field, but the interpretation of these results applies equally to all fields. Consequently, we present results for Prudhoe Bay in the text and complete results for all other fields and the entire North Slope in the appendices.

5.1 Our original three research tasks

We are now in a position to evaluate the three research tasks described in the introduction. These were to evaluate the ability to use economic theory and dynamic programming to model real-world oil production behavior, to evaluate whether producers have been dynamically optimal in their production decisions, and to simulate the effect of alternative tax policies on production paths and present discounted values of producer profits and state tax revenue.

First, complexity in derivatives limits the flexibility for tax policy specifications when modeling production as a differential equations boundary value problem. The inability to specify non-negativity constraints on production and reserves remaining ultimately precluded the use of this differential equations approach. Modeling production as a value function maximization in Microsoft Excel allowed virtually any tax specification but was limited to modeling a finite horizon with fewer than 200 periods. Future work may include modeling the value function maximization with an infinite horizon in GAMS, although we do not expect to find significant differences.

We were able to construct a reasonable cost function from available data, suggesting similar modeling may be possible for a wide range of energy production industries with non-proprietary data. Furthermore, model results were relatively insensitive to modifications to the cost function (Table 11, Figure 18, Figure 19). Thus, although the cost function is one of the more difficult model components to determine, it does not seem to be among the most important factors for our model results.

To calibrate the model to historical production (i.e., produce a good fit), we found that a constraint on initial production and an adjustment cost for rapid increases or decreases in production are necessary. Calibrated model results closely match historic production with reasonable discount rates and adjustment costs for four of the six fields modeled, although these parameters are field-specific (Table 11).

Second, calibrating our model to historical actual production provides some basis for evaluating whether producers have been dynamically optimal in their production decisions. A paper by VanRensburg (2000) suggests that a real discount rate of 9 percent to 12 percent is reasonable for the petroleum industry; the average nominal discount rate used in the petroleum industry was 16% in 1985 and 14% in 2000 (ibid).[55] A discount rate that best fits the model to historical production data but that is outside this range of

[55] A previous paper by Adelman (1993) suggested that a 10 percent discount rate is suitable for oil producing countries with diversified income sources (e.g., United States) while 20 percent or more is suitable for countries that are heavily reliant on oil-generated income. Although approximately 80% of the Alaska state budget is from oil revenue (ADR, 2007), oil companies rather than state government make oil production decisions. This does, however, suggest possible discrepancy in discount rates that could cause discord between oil companies and state government over oil production decisions.

"reasonable" may be interpreted as suggestive of sub-optimal historical production, perhaps due to imperfect information rather than mismanagement.[56] So much cannot be said for the particular dollar figures for producer profit and state tax revenue that emerge for each tax scenario, meaning these should be interpreted relative to one another rather than in absolute magnitude.

Four of the six fields modeled are within the range of "reasonable" for the best-fit discount rate, which suggests successful dynamic optimization by producers despite imperfect foresight (Table 9). The relatively low best-fit discount rates for Prudhoe Bay and Kuparuk River may be commensurate with the relatively low risk involved with developing these large, well-defined "elephant" fields. By the same logic, a somewhat higher discount rate is appropriate for smaller, marginal, and hence riskier fields like Milne Point and Endicott. The best-fit discount rates for two fields, Colville and Northstar, appear far outside the realm of reasonable, which suggests historical management that was not dynamically optimal. The higher discount rate needed to calibrate the model to these historical production paths indicates historical production that was too fast, perhaps due to overly-optimistic resource evaluations, overly-pessimistic price forecasts, or a combination of these and other factors.

However, the production path is sensitive to the price scenario in a similar manner as the discount rate since both affect the present discounted value of future-period revenues. We modeled with perfect foresight of the reference price scenario, which forecasts wellhead value rising to more than $100 per barrel by 2100. This trend of increasing wellhead value tends to shift production to later periods, meaning a higher discount rate is needed to push production back into early periods than if we had used a constant price forecast. Consequently, a relatively high best-fit discount rate could be interpreted as evidence of producers using a fixed price projection in their production planning rather than as evidence of sub-optimal production. For example, the best-fit discount rate for Northstar with a constant price forecast of $20 per barrel is 20 percent.

Furthermore, our sensitivity analyses for price scenarios demonstrate the impact of a price trend on the optimal production path. A trend of increasing price tends to shift production to later periods while a constant price, regardless of level, pushes production into earlier periods, ceteris paribus. The implication is that producers should include trends as well as levels in the price scenarios they use to develop production plans.

Finally, we are not aware of available estimates for "reasonable" adjustment costs in oil production. However, the magnitudes of adjustment costs incurred for the modeled production paths are small relative to producer profits and state tax revenue, which may suggest the best-fit adjustment cost parameters we defined are reasonable.[57] Adjustment costs represent the inability to turn oil production on and off. As such, it is also reasonable for the adjustment cost parameter to be smaller for larger fields, as we found, if the percentage change in production rate is what drives the adjustment cost. In other words, small changes in output across many wells at a large field can produce a large

[56] Such interpretation remains valid despite the uncertainty in our cost function specification because sensitivity analysis reveals the production path is insensitive to changes in slope of the cost function. As noted previously, however, such interpretation may suffer from simultaneous testing of the hypothesis, model specification, and data. Of course, any "unreasonable" discount rates required to calibrate the model to historical production data could also reflect mistaken assumptions and model errors.

[57] Generally the present discounted value of adjustment costs is on the order of one percent of the present discounted value of producer profit plus state tax revenue.

overall change in production rate while a similar change in production from a small field would require larger and more costly changes in output across a smaller number of wells. This would explain why Prudhoe Bay has a relatively small adjustment cost parameter while smaller fields have relatively high adjustment cost parameters (Table 9).

Third, we were able to simulate the effect of alternative tax policies on production paths and present discounted values of producer profits and state tax revenue (Table 12, Figure 20). The "best fit" production path is for the historical severance tax policy, which was a tax on gross value; an increase in the tax rate in 1981 shifts the optimum production path to earlier periods because the model presumes perfect information of future tax (and price) conditions. Future modeling will consider the impact of imperfect information on the production path and net social benefit (i.e., unforeseen tax increases). In 2007, the Alaska Legislature changed severance tax policy to levy the tax on net profits rather than gross revenue and to include a credit to offset investment costs. Modeling of these policies suggests they are effective in shifting production to earlier periods, but at the cost of lower total surplus.

5.2 How taxes can affect production paths, profits, and tax revenue

A fixed tax rate on gross revenue is the first-best policy because it does not distort the optimal production path (a fixed percentage is skimmed off the top, so the optimum production plan is unchanged). Tax policies that introduce components to influence the production path (e.g., credits) result in lower net social benefit. Thus, government can increase revenue without altering the production path or net social benefit by increasing the tax rate. Government can also shift the production path with, for example, a system of credits, but at the expense of lower net social benefit.[58] Since our model uses exogenously specified price scenarios and production cost functions, model results in conjunction with sensitivity analyses for the exogenously-specified parameters provide a "choice-set" to policy makers to help inform the development of tax policy that may seek multiple goals.

Our results are consistent with Kunce (2003), Helmi-Oskoui et al. (1992) and Uhler (1979), all of whom found that severance tax policies in the form of changes to the tax rate substantially change state tax revenue and producer profits but yield little or no change in the optimal time path of production. This result follows from the fact that a fixed percentage reduction in net profit does not change the producer's dynamic optimization problem regardless of the percentage, unless the percentage is high enough to make net profit negative (i.e., production ceases). To this, however, we add the equally logical insight that severance tax policy can affect the optimal time path of production if distortionary components such as credits and deductions that modify the dynamic optimization problem are included. Consequently, the conclusion proposed by Kunce (2003) that, "states should be wary of arguments asserting that large swings in oil field activity can be obtained from changes in severance tax rates" should be qualified by the notion that the structure of a tax policy may make more impact on producer behavior than the tax rates or magnitude of revenue collection involved.

[58] The distribution of surplus between profits and tax revenue can also be adjusted in a system of taxes and credits.

One result specific to Alaska's oil tax policy revealed by our sensitivity analyses is that the ELF, enacted with the intent to encourage continued production from marginal fields, has very little impact on the dynamically optimal production path (Figure 18), present discounted value of producer profits and state tax revenue (Table 11), or date when modeled production ceases.

Finally, the degree to which foresight is perfect or imperfect is likely to affect the production path since production plans are based on expectations of future conditions. The implication is that establishing expectations of future policy may be more important than changing current policy for influencing production decisions. Examination of perfect versus imperfect foresight in our modeling, however, is left to future work.

5.3 Observations and notes about interpretation of results

The correct interpretation of the alternate production paths produced by our modeling is a retrospective of how things might have been under alternate conditions rather than a prospective of how production will occur in the future because the model optimization is over the entire production path from start to finish, presuming perfect information about future conditions. Furthermore, we lack information (and the model lacks constraints other than adjustment costs) on producers' ability to change production path once a field development plan has been implemented. Thus, the research is most applicable to informing the creation of policy for the development of new fields. However, the model structure does permit modeling the optimal path for a particular field from any point in its history forward.

The present discounted value of profits and state taxes should be taken as representative relative to each other and/or as approximations rather than predictions of actual dollar amounts since the balancing of discount rate and adjustment cost in calibrating the model to historical data was informed judgment at best, and the absolute magnitude of profit and tax discounted values will be influenced by these factors as well as the price scenario used.

We omitted Badami from our analysis for several reasons. Anecdotally, the field's history is one of optimistic but unmet expectations for production. In our model, Badami cannot produce positive profit under any conditions when production is fixed in the first period equal to the historical rate. All optimizations without this constraint show production far below historical rates, with production delayed until prices rise in all scenarios other than fixed price and/or extremely high discount rate. There is greater uncertainty and potential for misspecification in estimating the cost function for Badami due to its small size and scarcity of wells, but our modeling efforts also lend support to the historical anecdotes that describe Badami as a marginal field with mostly heavy oil that maybe should not have been developed.

We observe more variation in production path during sensitivity analysis and model calibration for marginal fields (e.g., Milne Point and Northstar) than for Prudhoe Bay and Kuparuk River because economic production limits are more binding for the marginal fields. The effect of the ELF factor on optimal production paths and NPV of profits and tax revenue is most apparent for Endicott and Northstar because the reduction in tax rate is relatively large for these marginal fields and makes a difference in profitability due to the fields' proximity to break-even operation. In contrast, the ELF factor has nearly zero impact on production paths or profits for large fields like Prudhoe

Bay and Kuparuk River. These observations are reasonable since the ELF factor reduces the tax rate only when production rate is low, which occurs in earlier periods for small and/or marginal fields. Similarly, the steeper cost function makes the most difference in the optimal production path for Colville and Northstar since these are the only fields for which the production path takes production into the region of diminishing returns in the cost function.[59]

In general, higher discount rate pushes production into earlier periods, higher adjustment cost delays production by slowing the rate of initial ramp-up, and higher price scenarios delay production by increasing the value in future periods.

The counter-intuitive result of lower PDV of profits with the "high-price" scenario is due to the lower initial prices in this scenario. Although price rises more quickly in later periods with the high-price scenario, it starts at $12/bbl and rises more slowly than the reference scenario in the early periods. Thus, with discounting, the PDV of profits for the high-price scenario is less since profits in the early periods are less.

The concept of adjustment cost causes negative profits for actual historical production in fields whose production has fluctuated (i.e., Milne and Badami). How are such fluctuations in historical production explained in the context of adjustment costs? One hypothesis could be that producers made mistakes in the management of these fields due to imperfect information (e.g., higher viscosity oil than expected) and lost money as a result of the fluctuating production. Another hypothesis could be that there are two kinds of adjustments: one is drilling new wells and shutting down old wells, which entails significant costs; the other is turning on and off valves for short-term adjustments in production, which entails near-zero cost. If the latter hypothesis is true, and Badami and Milne were managed by turning valves on and off, then our modeling with adjustment costs based on drilling and closing wells will not fit historical production accurately for these fields.

---

[59] Note, the shallower cost function has little impact on the optimal production path for these fields, as with the others, because production remains in the constant returns region of the cost function.

Table 11: Present discounted values and correlation coefficients for Prudhoe Bay model results and historical production. P-values are given for two-sample t-statistics testing the null hypothesis of equal means for historical and simulated production paths; correlation coefficients are given for comparison of these paths as well. Results indicate good fit of the "best fit" model to historical production, with little deterioration in fit for changes in the cost function or price scenario. The discount rate, however, does alter the modeled production path significantly, thereby decreasing the fit to historical production. Hypothetical tax policies alter the production path as well, although to a lesser degree. Correlation with historical production is less for the uncalibrated models without constrained initial production or adjustment costs, and is characterized by negative correlation prior to peak production and positive correlation after peak production (data not shown in this table). Tax policies that distort the dynamic optimization deliver lower sum of profits and taxes (i.e., hypothetical tax scenario 1 is highest), but do change the distribution between producer profit and government tax revenue. Profits are reported for oil production from known fields, omitting exploration and overhead costs. A fixed 7 percent discount rate was used for calculation of PDV for all best fit and tax scenarios; a 5 percent discount rate was used for calculation of PDV for reference cases. All profits, taxes, and credits are in millions of 1982-84 US dollars. [i]including credits. [ii]net of credits.

Prudhoe Bay			**Modeled Production, through 2175**				**Hist. Actual Prod., through 2006**			
Scenario	Two-sample t-test	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (7% discount)	p=0.0188	0.92	38,905	20,085	0	58,989	17,688	20,556	0	38,244
High Price	p=0.0000	0.90	36,958	18,649	0	55,607	16,478	20,099	0	36,577
Fixed Price	p=0.0418	0.92	65,479	35,440	0	100,919	48,219	31,826	0	80,045
Low Discount (2%)	p=0.0000	0.52	18,036	6,812	0	24,847	17,688	20,556	0	38,244
High Discount (10%)	p=0.0000	0.70	42,399	23,027	0	65,427	17,688	20,556	0	38,244
Steeper Cost Func.	p=0.0183	0.91	35,972	19,938	0	55,910	15,074	20,560	0	35,635
Shallower Cost Func.	p=0.0174	0.91	37,682	20,114	0	57,796	18,063	20,464	0	38,527
Hypothetical Tax Scenarios										
1) High Tax on Gross w/ ELF	p=0.0243	0.90	31,731	27,609	0	59,340	9,559	28,611	0	38,170
2) Tax on Gross w/ Credits	p=0.0376	0.91	33,165	25,874	2,607	59,039	12,414	25,784	3,210	38,198
3) Low Tax on Gross w/ Credits	p=0.0420	0.91	38,072	17,908	3,374	55,980	19,466	18,666	2,668	38,132
4) Tax on Gross w/ High Credits	p=0.1115	0.92	32,917	21,981	7,794	54,898	13,486	24,712	4,375	38,198
5) Tax on Net w/ Credits	p=0.0406	0.91	33,727	25,054	2,689	58,782	13,213	24,970	3,210	38,183
Reference (5% discount)	p=0.0001	0.82	116,392	0	0	116,392	118,482	0	0	118,482
High Price	p=0.0000	0.68	47,058	0	0	47,058	73,624	0	0	73,624
Reference Price	p=0.0000	0.81	50,484	0	0	50,484	75,221	0	0	75,221
Low Discount (2%)	p=0.0000	0.87	54,806	0	0	54,806	118,482	0	0	118,482
High Discount (10%)	p=0.0124	0.72	165,762	0	0	165,762	118,482	0	0	118,482
Steeper Cost Func.	p=0.0002	0.82	113,451	0	0	113,451	115,484	0	0	115,484
Shallower Cost Func.	p=0.0001	0.82	116,408	0	0	116,408	118,491	0	0	118,491
Taxes no ELF	p=0.0000	0.80	83,805	31,749	0	115,554	85,877	32,819	0	118,696
Taxes with ELF	p=0.0000	0.80	84,091	31,536	0	115,627	85,979	32,688	0	118,667

Table 12: Summary statistics for model results for Prudhoe Bay. State taxes include royalty and severance taxes only (excludes property tax, corporate income tax, and federal taxes, all of which are relatively small) and are net of credits refunded to producers (for applicable tax scenarios). Producer profits are for oil production from known fields, omitting exploration and overhead costs, are net of taxes, and include credits refunded by the state (for applicable tax scenarios). All quantities are in millions of barrels; all profits, taxes, and credits are in millions of 1982-84 US dollars. Production is assumed to end when production rate falls below 0.5% of historical maximum production for the field or when producer profits become negative, whichever comes first. For Prudhoe, Kuparuk and Colville, low production signals the end of production. For Endicott, negative profit signals the end of production. Northstar is a mixture of both.

	Best Fit	Hyp. Tax 1	Hyp. Tax 2	Hyp. Tax 3	Hyp. Tax 4	Hyp. Tax 5	Hist. Actual
Prod. End*	2084	2084	2084	2083	2083	2083	2006
Production							
Mean	25.67	25.76	25.87	25.90	26.22	25.95	33.02
Max. (yr)	52.10 (1987)	51.27 (1988)	53.19 (1987)	55.02 (1986)	57.49 (1985)	53.47 (1987)	51.85 (1986)
Min. (yr)	0.25 (2084)	0.26 (2084)	0.25 (2084)	0.26 (2083)	0.25 (2083)	0.26 (2083)	6.00 (2006)
Std. Dev.	15.97	15.96	16.21	16.75	17.43	16.29	13.03
Production Cost ($/bbl)							
Mean	1.96	2.04	2.06	1.96	2.07	2.04	
Max. (yr)	3.32 (2084)	3.56 (2084)	3.58 (2084)	3.32 (2083)	3.60 (2083)	3.55 (2083)	
Min. (yr)	0.43 (1985)	0.43 (1985)	0.44 (1985)	0.45 (1984)	0.46 (1984)	0.44 (1985)	
Std. Dev.	0.98	1.05	1.06	0.97	1.06	1.05	
Wellhead Value ($/bbl)							
Mean	35.23	35.23	35.23	34.81	34.81	34.81	12.19
Max. (yr)	79.93 (2084)	79.93 (2084)	79.93 (2084)	78.68 (2083)	78.68 (2083)	78.68 (2083)	27.90 (2006)
Min. (yr)	12.11 (1978)	12.11 (1978)	12.11 (1978)	12.11 (1978)	12.11 (1978)	12.11 (1978)	5.05 (1999)
Std. Dev.	20.47	20.47	20.47	20.10	20.10	20.10	5.6
Producer Profit ($ millions per year)							
Mean	1,266	1,063	1,070	1,236	1,058	1,100	
Max. (yr)	5,444 (1988)	4,697 (1989)	4,719 (1986)	5,799 (1987)	5,384 (1986)	4,859 (1987)	
Min. (yr)	165 (2084)	-1,226 (1992)	-157 (1979)	-444 (1991)	-1,467 (1979)	-185 (1979)	
Std. Dev.	1,187	1,081	1,030	1,236	1,112	1,047	
State Taxes** ($ millions per year)							
Mean	575	790	769	542	694	745	
Max. (yr)	2,215 (1987)	3,012 (1989)	3,064 (1987)	2,306 (1987)	3,181 (1987)	2,986 (1987)	
Min. (yr)	67 (2084)	92 (2084)	90 (2084)	67 (2083)	89 (2083)	89 (2083)	
Std. Dev.	565	776	723	513	657	704	
Adjustment Cost ($ millions in one year)							
Mean	17	17	17	22	24	18	
Max. (yr)	261 (1992)	403 (1992)	216 (1992)	402 (1991)	354 (1981)	219 (1992)	
Min.	0	0	0	0	0	0	
Std. Dev.	46	56	46	66	68	48	
State Credits ($ millions in one year)							
Mean	0	0	41	52	115	43	
Max. (yr)	0	0	519 (1992)	965 (1991)	1701 (1981)	526 (1992)	
Min.	0	0	0	0	0	0	
Std. Dev.	0	0	111	157	326	114	

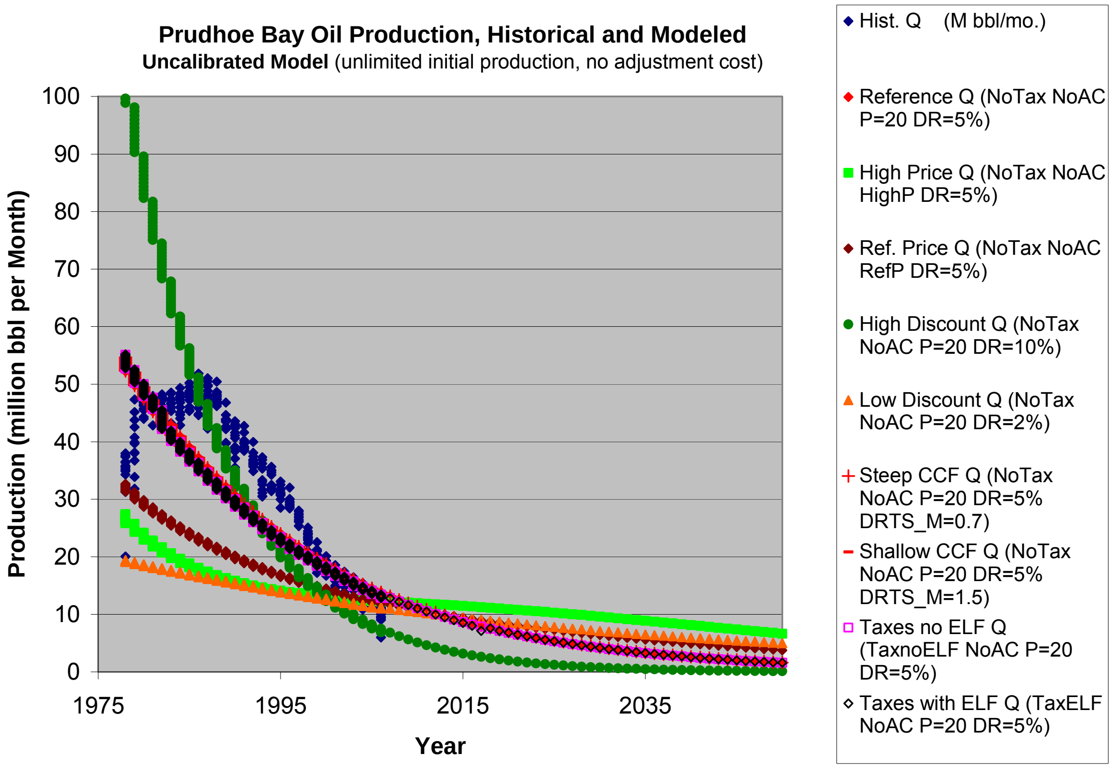


Figure 18: Sensitivity analysis for the uncalibrated model of Prudhoe Bay. Without initial production constraint or adjustment costs, model results (in a rainbow of colors) are negatively correlated with historical production (blue dots) pre-peak production, and positively correlated post-peak. The modeled production path is most sensitive to discount rate and is insensitive to changes in the cost function.

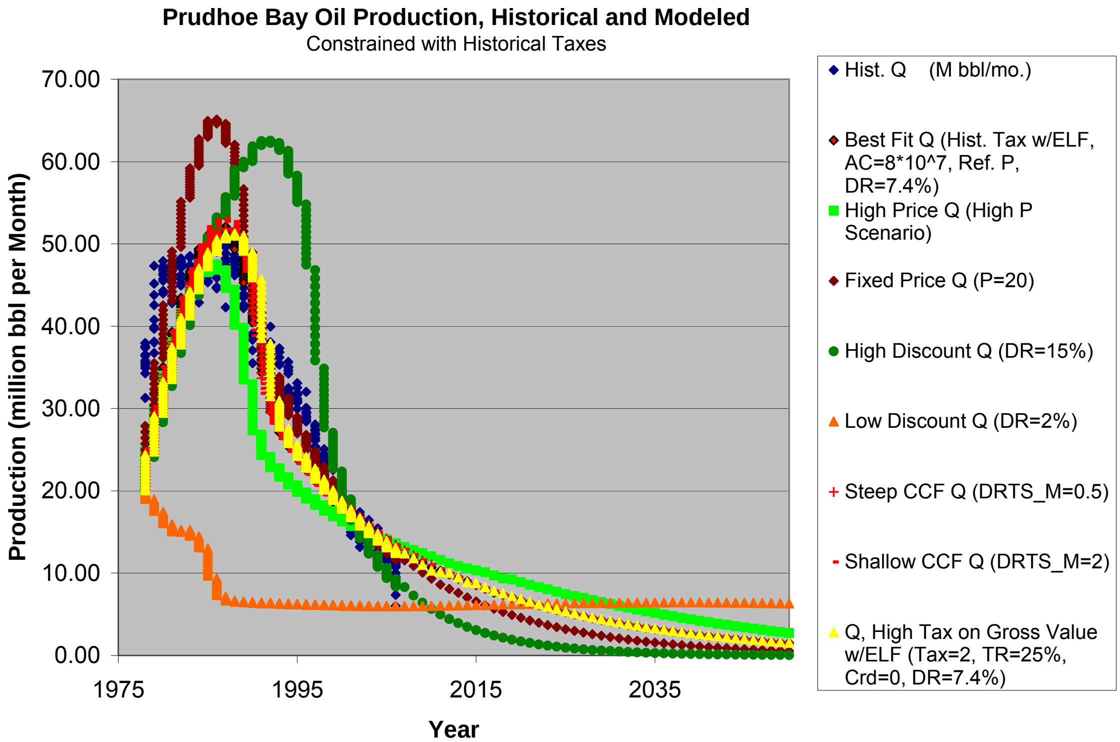


Figure 19: Sensitivity analysis of the "best fit" model for Prudhoe Bay. Model results fit historical production data well when initial production is constrained and adjustment costs are included. The model for Prudhoe Bay was calibrated with 7.4 percent discount rate. The calibrated model is most sensitive to discount rate and is insensitive to changes in the cost function. The application of the calibrated model is simulating alternative tax policies (see Figure 20).

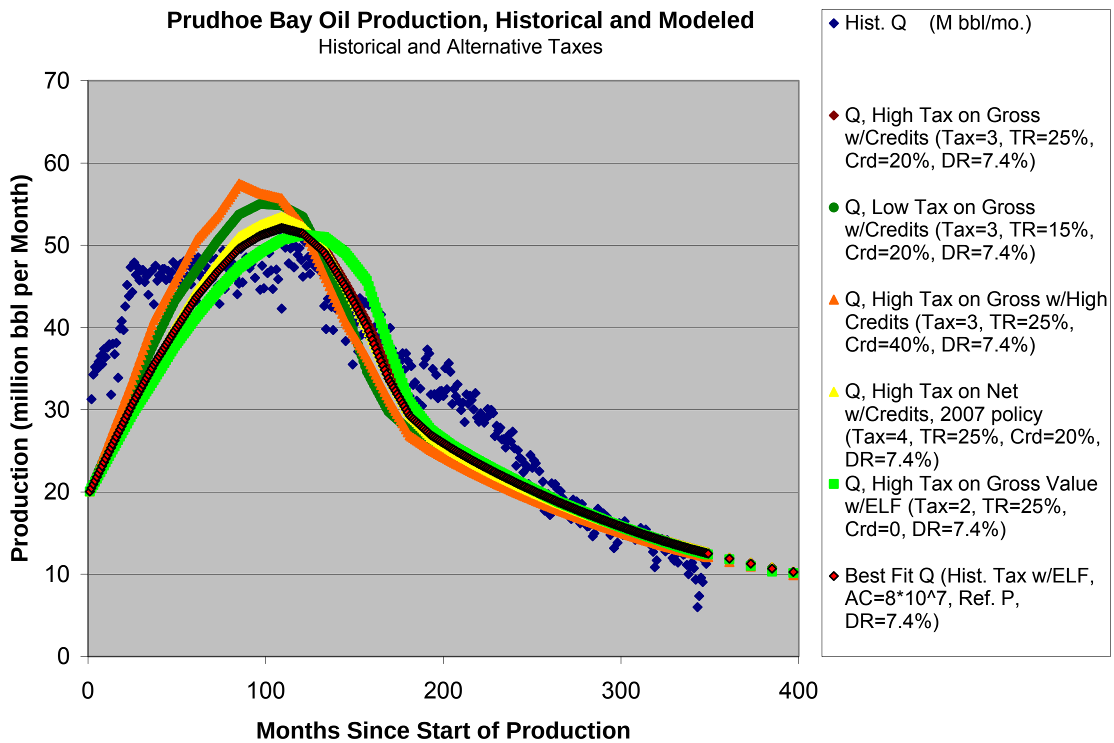


Figure 20: Best Fit and hypothetical tax scenario results for Prudhoe Bay. A "first-best" tax policy does not distort the dynamic optimization of oil production, thereby maximizing the total surplus (defined as producer profit plus tax revenue). In this case, a consistent tax rate on the gross value is first-best (the green path). The path induced by actual historical tax policy deviates slightly due to the increase in severance tax rate in 1981 and ELF adjustment factor, both of which acted to push production into earlier periods. Other hypothetical tax policies act to shift production into earlier periods as well by reimbursing a portion of adjustment costs, and change the allocation of surplus between producer profit and tax revenue but do not increase the total.

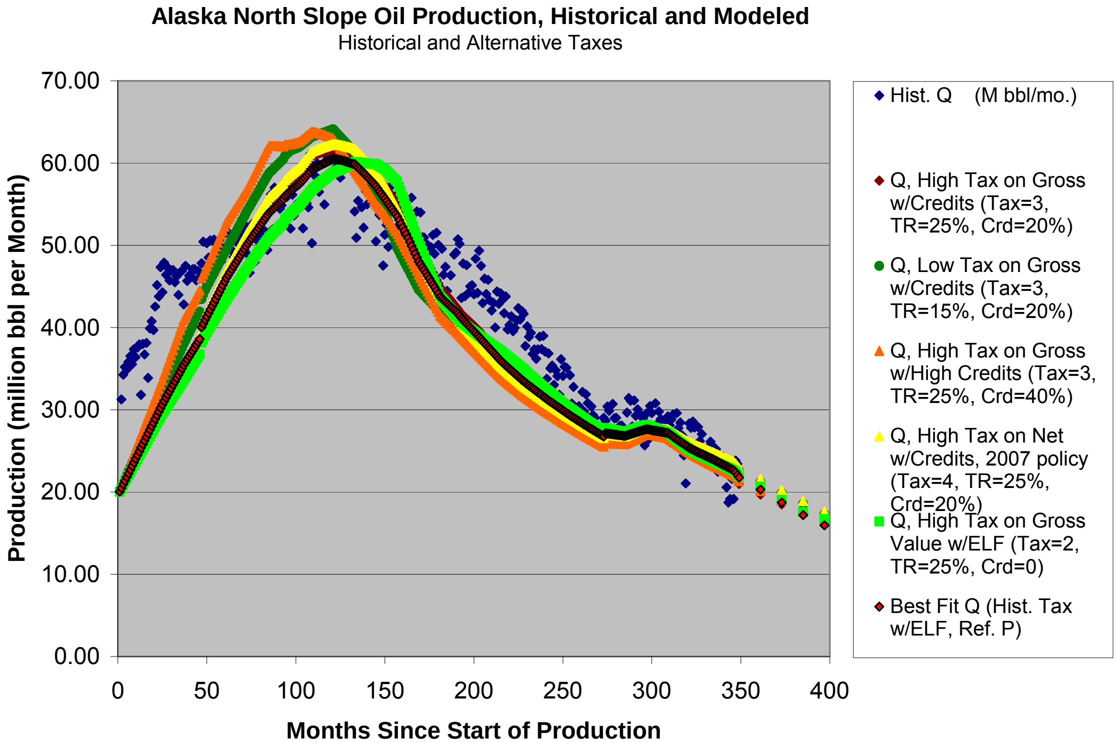


Figure 21: Best Fit and hypothetical tax scenario results for the North Slope, which is the sum of production from seven independently-optimized production units.

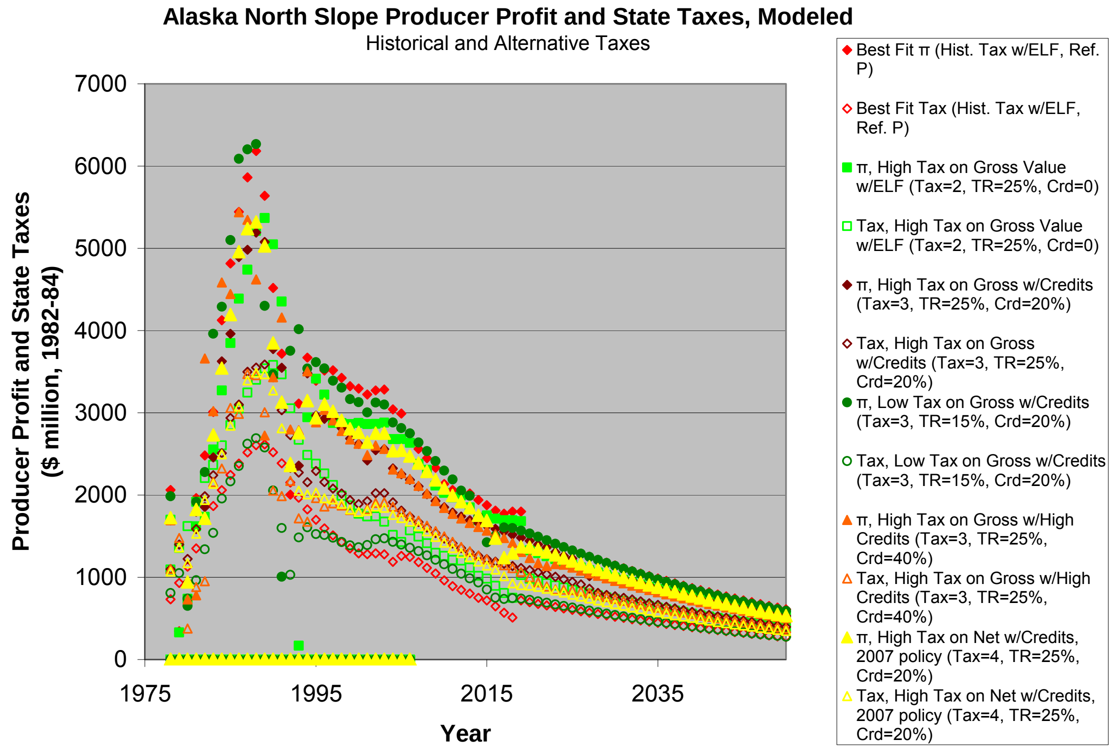


Figure 22: Producer profit and state tax revenue for the North Slope, which is the sum of production from seven independently-optimized production units, under historical and alternative tax policy scenarios.

5.4 Significance of Work

Our application of dynamic modeling methods to energy production decisions in a new location and with new data extends the general experience of this emerging field. Our novel method for estimating field-specific cost functions without direct production cost data may be applicable to other research efforts where data are limited. Our estimation of dynamically optimal extraction is of methodological interest; we have identified adjustment costs and constraint on initial production rate as features needed to bring a simple Hotelling model closer to reality.

Better understanding of the dynamic landscape in Alaska's oil industry will help avoid inefficiency in public policy and private investment decisions. Private industry is considering investing tens of billions of dollars in Alaska over the next decade, and both state and federal governments are considering financial involvement as well. Our empirical analysis of production decisions enables examination of whether the dynamic optimum predicted by theory actually occurs in practice and how tax policy can change these production decisions. The results may have implications for government tax and regulatory policy.

Our development of a model for understanding dynamic production behavior in the Alaska oil industry may provide a foundation for similar modeling of the potential Alaska natural gas industry, which may be an important component of the future domestic energy supply, and other low-carbon energy sources. Economics-based research is needed to complement systems optimization research in exploring emerging energy markets. Our research takes a step toward this goal by developing a flexible dynamic framework for considering the potential effect of policy on industry behavior that can be adapted to a variety of energy industries.

6 Extensions and Future Work

Dynamic modeling of oil production in Alaska proved too complex for numeric solution with an ODE boundary value approach, but tractable with a numeric value function maximization. Subject to uncertainty in the cost function estimation and calibration, the model can be used to simulate the impact of tax policies on optimum production paths, producer profits, and tax revenues, and can be used to evaluate the difference between economic theory and reality (although interpretation as sub-optimal production versus modeling error remains elusive).

Future work will diverge on two paths. On one hand, we will seek greater realism in modeling Alaska oil production by, for example, including a relationship between production rate and ultimate recovery, by making TAPS sizing endogenous to the modeling, and by formulating an integrated model of exploration, development, and production (see, for example, Kunce, 2003). On the other hand, we will seek further understanding of the benefits and limits of dynamic modeling of energy production decisions by applying these methods to distinctly different energy resources, like renewable rather than finite resources (e.g., wind and biomass).

6.1 Endogenous pipeline sizing

TAPS is the only means for delivering North Slope crude oil to market. As such, the pipeline capacity is intimately linked with oil production plans. Too little oil will leave a pipeline asset underutilized and will impact operations;[60] too much oil will exceed pipeline capacity, leaving the optimal production path capacity constrained. Although we did not find TAPS capacity to be a binding constraint on optimal production decisions in our modeling, future work may consider including the pipeline sizing decision as an endogenous component of dynamic production optimization. In general, the realized rate of oil production may be a combination of unit operator production decisions made in the context of the pipeline owners' design and construction decisions. Consequently, including the pipeline sizing as an endogenous part of the modeling may be important for evaluating future developments like the ongoing natural gas pipeline development.[61]

Endogenous pipeline sizing will require estimation of a pipeline cost function, similar to the research undertaken by Parker (2004). Our next step will be to incorporate

---

[60] For example, batching of gas-to-liquids products may be required for continued economical operation of TAPS if oil production becomes too low (Chukwu, 2002; Ejiofor et al., 2008).

[61] Recent Alaska Legislatures and Governors have proposed a variety of design constraints on the proposed natural gas pipeline that could impact gas production decisions made by unit operators in the context of the pipeline design and access regulations (Alaska Gas Pipeline website, 2007; Alaska Gasline Inducement Act website, 2007). Several alternative routes and project definitions have also been proposed for the natural gas pipeline, including a "highway" route to Alberta, Canada, an "all-Alaska" route to an LNG terminal at Valdez, Alaska, and a "Y-line" with branches to both Alberta and Valdez. If the configuration of TAPS infrastructure to bring oil to market is found to have significant effect on oil production decisions, it may be reasonable to suspect the design of natural gas transport infrastructure may influence natural gas production decisions as well. Finally, the primary three oil producers in Alaska—BP, ExxonMobil, and ConocoPhillips—built and operate TAPS (via subsidiary corporations), and may also build and operate the natural gas pipeline. An important question for the natural gas pipeline project is whether consistency in the entities making pipeline design and natural gas production decisions results in a more or less optimal system.

the pipeline cost function into the optimization problem for Prudhoe Bay production based on the assumption that TAPS was built for Prudhoe Bay and subsequent fields took this capacity as given. Thus, the results from the Prudhoe Bay optimization will be an optimal production path and optimal TAPS capacity. Our next step will be to consider the allocation of TAPS capacity among the seven North Slope production units. The simple assumption of 100 percent of TAPS capacity taken as given for the calculation of a "simple" optimal production path for each field is not realistic if the sum of production from all fields exceeds capacity. Consequently, modeling of joint allocation of TAPS capacity among all seven fields may be done assuming continuous bidding for TAPS capacity (i.e., an "Alaska Manager" seeks to maximize the sum of profits from all fields subject to the additional constraint that the sum of production from all fields cannot exceed the TAPS capacity) or discrete bidding for TAPS capacity (i.e., each field is allocated capacity shares in discrete intervals (e.g., 10 years), similar to FERC-regulated open access seasons, which they then hold until the next capacity allocation period).

6.2 The impact of tax changes

A salient question for policy makers looking forward is how changes in tax policy may impact future oil production behavior. For example, one way to consider the potential impact of the 2006/2007 severance tax change in Alaska is to model the effects of a similar change had it occurred at some point in the past. To this end, future work may consider the optimal production paths if Prudhoe Bay and Kuparuk River had operated under the old severance tax system while subsequent fields had operated under the new tax system.[62] Alternatively, we can model a tax change that applies to all fields at some point during the production history, with or without perfect foresight of the tax change. Note, however, that an integrated modeling framework is necessary for considering the effects of tax policy on the exploration and development phases of oil production (see section 6.8).

6.3 The impact of imperfect foresight

A fundamental assumption in the current modeling is that unit operators have perfect foresight of future oil prices and tax policies. This assumption is unrealistic. Consequently, future work will consider the impact of imperfect foresight on model results. One way to relax the assumption of perfect foresight is to combine two optimal paths. We begin by calculating the optimal path for a given price forecast and tax regime and then only take the first several years of it (e.g. 5 years) to represent the unit operator's optimization given its best understanding of price and taxes. Next, we take the result of the first optimal production path as the starting point in year six for a second optimization under a new set of tax and price conditions (i.e., conditions have changed and the producer re-optimizes). Note, this approach assumes the producer is able to re-

[62] In this approach Prudhoe Bay and Kuparuk River are used to "represent" current fields while other fields are used to "represent" future discoveries. Consequently, we would use USGS data on the potential distribution of future field sizes (Attanasi and Freeman, 2005) and then mimic this distribution in actual fields chosen to represent the future fields under the new policy. This would likely result in modeling more than the total number fields, which would then require scaling back the results accordingly. We would also need to discount all revenue and tax streams back to the same year whereas current field-specific results are discounted to the year of production start for each field.

optimize a field's production plan after it has been developed, which requires some notion of adjustment cost for transitioning from one path to the next. Finally, we can string these production paths together to produce an entire path for a unit operator struggling with imperfect foresight.[63]

6.4 The impact of carbon value and enhanced oil recovery

The potential for carbon to have economic value due to emissions caps implies injection of CO_2 into oil fields may produce revenue both by enhancing oil recovery and sequestering carbon. However, the optimal rate of CO_2 injection for enhanced oil recovery may differ from the optimal rate for carbon sequestration (DOE, 2008) and there are costs associated with CO_2 injection as well. Consequently, future work will seek to incorporate CO_2 injection as a control variable in our model of optimal producer behavior.

This modeling will require information on how enhanced oil recovery via CO_2 impacts production and the cost for CO_2 injection. We will assume unit operators are price takers for CO_2 credits, as they are for oil, so we will use a fixed CO_2 price (or forecasted function) as we did for oil. Comparison of model results when a field is managed for maximizing profits from oil sales versus when it is managed for maximizing profit from oil sales plus CO_2 storage revenues will provide insight into how future carbon values may affect oil production decisions.

6.5 Oil substitutes and backstop energy technologies

Other fossil fuels, nuclear, and renewable technologies provide energy sources that are substitutes for oil. Although the elasticity of substitution is likely to vary by application, with some enabling technologies required (e.g., batteries for electric cars), it may be reasonable to expect substitution away from oil to occur in such a way that the price of oil is effectively capped. In other words, alternatives become more competitive as the price of oil rises and the demand reduction that results from substitution away from oil imposes an upper bound on oil price. Previous researchers have used such scenarios in dynamic modeling of oil production (e.g., Berg et al., 1997 and Gao et al., 2008).[64]

Aside from our constant-price scenario, the price scenarios used in our modeling increase over time without an upper bound. Future work with our model may include a capped-price scenario to examine the impact of substitution away from oil on optimal oil production paths. We expect that both the imposition and magnitude of such a terminal stationary oil price, which is the uncertain result of technological development and learning, will influence optimal oil production paths.

---

[63] An alternative approach may be to define uncertainty in prices and tax regime with probability distribution functions and then use Monte Carlo analysis to examine the confidence bounds for optimal production paths.

[64] As Gao et al. explain, "one could imagine the price of oil rising continuously so that increasingly costly secondary recovery techniques become profitable. Our model is instead based on what we view as a far more likely scenario whereby, beyond some period, the energy market is dominated by a backstop technology (or technologies such as solar energy, nuclear fusion, or the so-called 'hydrogen economy') that controls demand for oil."

6.6 Hedging behavior in oilfield development

As mentioned in the sidebar in section 4.4, one driver for the gradual ramp-up in oil production observed in data may be hedging behavior against the risk and uncertainty in reservoir characteristics. Since any particular pattern of wells limits the range of possible future production adjustments, it is costly to miss-judge the reservoir characteristics and implement a development plan that is not optimized for the reservoir. Consequently, producers may intentionally ramp up production slowly, drilling in a dispersed pattern to gather more information about the reservoir with which to revise their development plan along the way.

Future work may investigate whether the spatial pattern of well drilling in Alaska has been more consistent with implementation of an established development plan or with gathering of reservoir information. With latitude and longitude coordinates for each well and the date production started, we can look at the spatial pattern of development. A pattern of collocation with collection points may indicate implementation of an established development plan while a dispersed pattern may indicate continued gathering of reservoir information during development.

6.7 A variable discount rate for capital investment recovery

Recovery of capital investments may be a primary consideration in a business where capital assets cost billions of dollars. To incorporate this consideration, future modeling may include a variable discount rate.

Our current models were built on economic theory, which says sunk costs are irrelevant for future production decisions. However, TAPS was built to bring Prudhoe Bay oil to market, at a cost in 1977 of $8 billion. There may be a feature of producer behavior that we are not modeling correctly: an emphasis on recouping sunk investments. Such emphasis can be incorporated in our modeling with a variable discount rate, which allows for heavier discounting in the near-term (i.e., emphasis on recovering infrastructure capital).

To incorporate a variable discount rate in the modeling, we turn to the present-value formulation of the Hamiltonian rather than the current value formulation (Arrow, 1964). The present-value Hamiltonian is

21 $$\text{Max}_{\{Q(t)\}} \int_0^\infty V(Q(t),S(t),t)dt$$

$$\text{s.t.} \quad \frac{dS(t)}{dt} = -Q(t)$$

$$Q(t) \geq 0$$

$$S(t) \geq 0$$

$$S(0) = S_o$$

Where V is the present discounted value of the profit function and $\frac{dS(t)}{dt}=-Q(t)$ is our specific form of the more general $\frac{dS(t)}{dt}=g(Q(t),S(t),t)$. Then, 22 $H = V(Q,S,t) + \mu(t)g(Q,S,t) = [P(t)Q_{it} - C(Q_{it},S_{it})]e^{-r(t)} + \mu(t)(-Q)$

where $P(t)Q_{it} - C(Q_{it},S_{it})$ is the profit function and $\mu(t)$ is the present discounted value of the shadow price (which equals $p(t)e^{-rt}$ or $p(t)e^{-r(t)t}$, note the discount rate *r* is now a function of time). From David Laibson's work on hyperbolic discounting (Laibson, 1997, 1998):

23 $e^{-r(t)t} = \beta\delta^t$ where β might be 0.9 and δ might be 0.9

Finally, solving for the discount rate *r* proceeds as follows:

24 $\ln(e^{-rt}) = \ln(\beta\delta^t) \Rightarrow -rt = \ln(\beta\delta^t) \Rightarrow -r = \frac{\ln(\beta\delta^t)}{t} \Rightarrow r = -\frac{\ln(\beta\delta^t)}{t}$

This formula yields the heaviest discount rate in the first period. The parameter β is the behavioral or non-rational or organizational parameter unique to each firm or field or situation (i.e., available for manipulation). The parameter δ is the economic parameter for discount rate. Although more fixed than β, δ could vary with firm-specific weighted cost of capital.

Finally, we can re-write $\mu(t)$ for a variable discount rate as

25 $\mu(t) = p(t)\beta\delta^t$ or $\mu(t) = p(t)\,e^{\frac{\ln(\beta\delta^t)}{t}t} = p(t)\,e^{\ln(\beta\delta^t)}$

6.8 An integrated model of exploration, development, and production

The change in severance tax policy enacted by the Alaska legislature in 2006/2007 was meant, in part, to induce more exploration and development activity. Our current models focus on the production phase only, leaving out exploration and development activities. Consequently, future work will seek to include these phases in an integrated model of exploration, development, and production like the one developed by Kunce (2003) while retaining the flexibility to model diverse tax policy structures.

6.9 Greater attention to engineering and reservoir geology

There is evidence of engineering and geologic constraints on optimal oil production paths. For example, Uhler's (1979) theoretical model integrated the physical behavior of reservoirs (via a reservoir pressure control variable) into the economics of petroleum exploration and production. He found the optimal extraction rate is often determined by pressure conditions within the reservoir.

Rather than the inverse production function we estimated for wells as a function of production rate and reserves remaining, Gao et al. (2008) used an "engineering computer model of dynamic fluid flow" to simulate reservoir data for the production quantity as a function of the number of wells. They then modeled the dynamic optimization problem with the number of wells as the control variable (see section 3.2.5 for further discussion).

Further integration of economic and engineering models may improve our understanding of producer behavior. For example, our current model does not capture the possible tradeoff between production rate and ultimate recovery (i.e., pumping faster may cause a drop in reservoir pressure that reduces ultimate recovery from the field; Bedrikovetsky, 1993).

6.10 Applications to other energy industries

When society recognizes global climate change, energy security costs, and peak oil as emergencies, we will want to quickly invest in energy conservation and efficiency and in large, new, indigenous energy supplies. Consequently, extension of our research to understanding producer behavior in alternative energy industries is important. For example, the Brazilian ethanol experience and wind industries in the United States and Germany may offer valuable case studies for development of dynamic models of renewable and low-carbon energy markets. Like our current model, these models can be used to analyze and simulate the effects of alternative policies and scenarios on production behavior, thereby strengthening the scientific basis for environmental management and policy decisions and practices, helping to avoid inefficiency in public policy and private investment decisions, and ensuring a smooth transition to desired outcomes. It is important to add such a decision-making context to system optimizations to consider whether industry will choose to build the optimal system or, alternatively, what conditions (regulatory, economic, or otherwise) are necessary to encourage them to build it.

6.11 Monte Carlo for Confidence Intervals

We do not present confidence intervals in the current research because some uncertainty could not be quantified. A Monte Carlo process may be used to estimate the confidence intervals associated with uncertainty in the price function and cost function estimation, but the resulting confidence intervals will certainly underestimate the true uncertainty in simulated optimal production paths.[65] Still, some estimate of uncertainty may prove informative.

6.12 Myopic Decision-Makers and Historical Discount Rates

The model also allows for simulating the statically optimal (but not dynamically optimal) production trajectory by simply setting the discount rate (ρ) equal to zero. In other words, a zero discount rate is equivalent to simulating myopic decision-making by Alaskan oil producers. Experimenting with the discount rate value is essentially equivalent to constraining producers to be more or less dynamic; matching the discount rate to historical fluctuations in discount rates or weighted costs of capital may prove interesting in terms of impact on the optimal production path.

---

[65] The Monte Carlo process begins with establishing a distribution for each parameter (normal, with mean equal to the point estimate and standard deviation equal to estimated standard error). Then, we randomly select coefficient values from these distributions to run the Monte Carlo analysis.

6.13 Social Optimality with Environmental Costs

Incorporating environmental costs into the profit function would allow consideration of social optimality rather than just profit maximization from the producer's point of view. One approach for doing so is to add a term for the probability of infrastructure failure as a function of cumulative production, to reflect the aging infrastructure. Combined with data on the distribution of cleanup costs for the spills that have occurred in Alaska, this would yield an expectation of future environmental cleanup costs as a function of cumulative oil production. A similar approach may be taken for incorporating the greater hazard of an accident with higher production volume (e.g., via the larger number of tanker ships moving) by adding a risk term to the profit function. Then the total expectation of future environmental costs is the sum of the expectation of infrastructure failure (and cost of such failure) plus the expectation of future accident (and cost of such accident).

However, the risk of environmental damage may increase with the scale of infrastructure development. More wells and associated pads and pipes, for example, offer more miles of infrastructure subject to failure, more opportunity of accidents, and may create more impact on migrating caribou (Cameron et al., 1992; Nellemann and Cameron, 1998). Thus, the environmental impact terms described previously may need to be a function of cumulative production but also a function of the total scale of oil production operations.

Thus, we may be able to wrap all three of the previous ideas into one term by making it a function of current production volume (to capture the risk from scale of operations), cumulative production (to capture the risk from aging infrastructure), and number of operating wells (to capture the risk from scale of infrastructure).

6.14 Alaska's Optimal Production

The modeling approaches described in this paper have focused on simulating the unit operator's optimal oil production path. It may also prove interesting to simulate a "North Slope Manager's" optimal production path (i.e., total production from all fields) or the Alaska government's optimal production path. Rather than profit maximization, the Alaska government goal may be to maximize tax revenue, perhaps with a high discount rate if re-election or funding infrastructure development are goals.[66] Such investigation of alternative objective functions in the dynamic optimization of oil production may be informative for considering the behavior of national oil companies, which now control most of world oil reserves (Jaffe, 2007).

[66] This objective function differs from the traditional social optimization of maximizing producer surplus plus consumer surplus for several reasons. First, actions by the Alaska government that would impact tax revenue (i.e., primarily tax policy) will not impact consumer surplus because Alaska oil does not directly supply Alaska consumers with gasoline, diesel, or other products and because Alaska oil is sold into the world market without influence on price (except some small effect on west coast refinery operations and thus Alaska gasoline and diesel prices, which is a second-order effect). Furthermore, although the Alaska government does pay attention to the jobs and economic vitality provided by the oil industry in Alaska, the oil producing firms are large multi-national corporations. Thus, it may be reasonable to think that the Alaska government is not concerned with producer surplus, per se, but rather with the amount of that surplus converted into tax revenue (and possibly jobs and in-state investment).

6.15 A Strategic Model of Dynamic Production Decisions

At first glance, conditions in Alaska appear conducive to imperfect competition and strategic interactions. Generally, perfect competition requires many sellers, none large relative to the market, perfect information, free entry and exit, and homogenous products. The Alaska oil industry, however, is characterized by few sellers (the three primary producers are BP, ExxonMobil and ConocoPhillips) each of which controls approximately 1/3 of production, and barriers to entry due to sunk costs, proprietary information, and access to the single oil pipeline for bringing oil to market (TAPS). For production decisions, however, there is a force that may overwhelm these conditions: unitization is required by law prior to production.

There are several interesting consequences of the legal requirement for unitization, which is somewhat unique to Alaska. First, it effectively eliminates strategic interactions such as information and extraction externalities in the exploration and development stages of oil production that have been documented elsewhere (Lin, 2007). Second, there is a significant component of negotiation strategy to the formation of these unit operating agreements (personal communication, Simon Harrison, BP Exploration Alaska, July 2, 2007). However, these agreements are formed prior to the production phase, so this source of strategic interaction is irrelevant for the modeling of production decisions described in this paper.

It is important to note, however, that unit management decisions are not without conflict since incentive incompatibility cannot be completely eliminated in the unit operating agreement (Libecap & Smith, 1999; Libecap & Wiggins, 1985). For example, there is often discrepancy in the distribution of oil production shares and gas production shares. The resulting incentive incompatibility has played out in unit management disputes over gas processing (i.e., how much to commercialize via processing into NGL for shipment down TAPS and how much to re-inject to boost oil production). However, each company may also have an interest in gas commercialization proportional to its production share of gas under current unit agreements. For example, since BP has minority share (14%) of the gas cap production share for Prudhoe Bay and Exxon and ConocoPhillips each have 42% share, we might expect BP to "drag its feet" in commercializing natural gas relative to the others. Thus, there might be some interesting results of digging deeper into unit contract structures, looking for systematic differences in production decisions based on the unit operator or oil share.

The "duty to produce" is a related topic of interest for Alaska policy-makers. The claim has been made that lease-holders (especially for gas) are dragging their feet in developing economically viable resources. For example, delaying building a natural gas pipeline when the gas is no longer "stranded" (i.e., uneconomical to bring to market) due to recent price increases. Another example may be delaying investments in heavy oil development. If true, this would imply the producers are not behaving as dynamic optimizers when looking at Alaska only, but perhaps are dynamically optimal when strategic considerations are included (e.g., preventing new entrants) or when the global context is considered. Why might oil producers "drag their feet?" One explanation may be they are considering a global portfolio of investment alternatives and allocating a relatively fixed quantity of capital to alternative investments according to rate of return, longevity of the play (discretionary vs. non-discretionary) and a host of other considerations including CEO judgment (e.g., growth may be a primary goal rather than profit maximization). Conversely, the Alaska government is interested only in Alaska, and is looking for optimum investment decisions in this context. So an investment that does not make the producer's list may still offer an attractive rate of return to Alaska. Such discussions are interesting for Alaska policy because the leases in Alaska are structured such that Alaska and the oil companies are "partners" in the venture of producing oil and the "duty to produce" clause in these contracts says oil companies must produce reserves that are economically viable (i.e., would deliver a reasonable rate of return). In this context, having better options for investment elsewhere is not an excuse for not producing in Alaska and, in fact, is reason for Alaska to reclaim the leases and find new business partners who will act in a simple dynamically optimal way (i.e., considering only the Alaska industry and not other global interests).

References

Adelman, M.A. (1993) "Modeling World Oil Supply," *The Energy Journal*, 14, 1-31.

Alaska Department of Revenue (ADR), Tax Division (2007) *Crude Oil and Natural Gas Prices – History*, table B-1A, available at http://www.tax.state.ak.us/sourcesbook/OilGasPrices.pdf (accessed July, 2008).

Alaska Department of Revenue (ADR), website (2007) www.revenue.state.ak.us/ (accessed 10 August, 2007).

Alaska Gasline Inducement Act, website (2007) www.gov.state.ak.us/agia/ (accessed 20 November, 2007).

Alaska Gas Pipeline, website (2007) www.gov.state.ak.us/gasline/ (accessed 10 August, 2007).

Alaska State Legislature, website (2007) "SCS CSHB 3001(NGD)," bill passed in the third special session of the 24th Legislature on 19 August, 2006, www.legis.state.ak.us/basis/get_bill.asp?bill=HB3001&session=24 (accessed 10 August, 2007).

Alaska State Legislature, website (2007) "HB 2001," bill passed in the first special session of the 25th Legislature on 16 November, 2007, www.legis.state.ak.us/basis/get_bill.asp?session=25&bill=HB2001 (accessed 21 November, 2007).

Alaska Oil and Gas Conservation Commission (AOGCC) (2008) *Pool Statistics*, available at http://aogcc.alaska.gov/annual/2004/Oil_Pools/ (accessed July, 2008).

Allain, Louis John Joseph (1979) *Capital investment models of the oil and gas industry: a systems approach*, New York: Arno Press.

Alyeska Pipeline Service Company, website (2007) www.alyeska-pipe.com (accessed June 16, 2008).

American Petroleum Institute (1969 – 2004) *Joint Association Survey of the U.S. Oil and Gas Producing Industry, Section 1: Drilling Costs*, 1801 K Street, N.W. (1969-72), 2101 L Street, N.W. (1973-82), 1220 L Street N.W. (1983-2004), Washington, D.C.

Arrow, K.J. (1964) "Optimal Capital Policy, the Cost of Capital, and Myopic Decision Rules," *Annals of the Institute of Statistical Mathematics* 16(1-2):21-30.

Attanasi, Emil D. and Philip A. Freeman, United States Geological Survey (2005) "Economics of Undiscovered Oil and Gas in the Central North Slope, Alaska," Open-File Report 2005-1276.

Bedrikovetsky, Pavel (1993) *Mathematical theory of oil and gas recovery: with applications to ex-USSR oil and gas fields*, Boston: Kluwer Academic.

Berg, Elin, Snorre Kverndokk and Knut Einar Rosendahl (1997) "Gains from Cartelisation in the Oil Market," *Energy Policy*, 25(13), 1075-1091.

BP (2006) "Prudhoe Bay Fact Sheet," available at http://www.dec.state.ak.us/SPAR/PERP/response/sum_fy06/060302301/factsheets/060302301_factsheet_PB.pdf (accessed 19 August, 2008).

Cameron, R.D., Reed, D.J, Dau, J.R. and Smith, W.T. (1992) "Redistribution of Calving Caribou in Response to Oil-Field Development on the Arctic Slope of Alaska," *Arctic* 45(4): 338-342.

Campbell, Colin J. and Jean H. Laherrere (1998) "The End of Cheap Oil: Global production of conventional oil will begin to decline sooner than most people think, probably within 10 years," *Scientific American*, March, pp. 78-84.

Chakravorty, U., Roumasset, J., and Tse, K. (1997) "Endogenous substitution among energy resources and global warming," *The Journal of Political Economy* 105(6), 1201—1234.

Chukwu, Godwin A. (2002) "Study of Transportation of GTL Products from Alaskan North Slope (ANS) to Markets," Petroleum Development Laboratory, University of Alaska Fairbanks, prepared for US Department of Energy, National Energy Technology Laboratory, Contract FC26-98FT40016.

Dasgupta, P. and Heal, G. (1979) *Economic theory and exhaustible resources*, New York: Oxford University Press.

Davidsen, P.I., Sterman, J.D. and Richardson, G.P. (1990) "A petroleum life cycle model for the United States with endogenous technology, exploration, recovery, and demand," *System Dynamics Review*, 6, 66-93.

Deacon, R., S. DeCanio, H. E. Frech III and M. B. Johnson (1990) *Taxing Energy: Oil Severance Taxation and the Economy*, New York: Holmes & Meier.

Deacon, R. (1993) "Taxation, Depletion, and Welfare: A Simulation Study of the U.S. Petroleum Resource," *Journal of Environmental Economics and Management* 24, 159-187.

Dismukes, D.E., Olatubi, W.O., Mesyanzhinov, D.V., Pulsipher, A.G. (2003) "Modeling the economic impacts of offshore oil and gas activities in the Gulf of Mexico: methods and applications," OCS Study MMS 2003-018 prepared by the Center for Energy Studies, Louisiana State University, Baton Rouge, Louisiana for the U.S. Department of the Interior, Minerals Management Service, Gulf of Mexico OCS Region, New Orleans, Louisiana.

Dorfman, R. (1969) "An economic interpretation of optimal control theory," *The American Economic Review* 59, 817-831.

Ejiofor, N., Patil, S.L., Chukwu, G.A., Reynolds, D., Khataniar, S. and Dandekar, A.Y. (2008) "Economic Appraisal of Transporting Gas-to-Liquids Products through the Trans-Alaska Pipeline System (TAPS)," *Energy Sources, Part B: Economics, Planning, and Policy* 3(2): 196-202.

Energy Information Administration, EIA (2007) Filename PET_PRI_WCO_K_W.XLS, updated on 8/29/2007, downloaded on 4 September, 2007 from http://tonto.eia.doe.gov/dnav/pet/pet_pri_spt_s1_d.htm.

Energy Information Administration, EIA (2007) *Annual Energy Outlook 2007 with Projections to 2030*, U.S. Department of Energy, report # DOE/EIA-0383, table 14: oil and gas supply, year-by-year tables (2004-2030), www.eia.doe.gov/oiaf/aeo/index.html (accessed 4 September, 2007).

Energy Information Administration, EIA (2008) *Annual Energy Outlook 2008*, U.S. Department of Energy, report # DOE/EIA-0383(2008), http://www.eia.doe.gov/oiaf/aeo/ (accessed 19 August, 2008).

Energy Information Administration, EIA (2008b) "U.S. Crude Oil Supply and Disposition," available at http://tonto.eia.doe.gov/dnav/pet/pet_sum_crdsnd_adc_mbbl_m.htm (accessed 19 August, 2008).

Favero, C. (1992) "Taxation and the Optimization of Oil Exploration and Production: The UK Continental Shelf," *Oxford Economic Papers* 44, 187-208.

Feraille, Mathieu, Emmanual Manceau, Isabelle Zabalza-Mezghani, Frederic Roggero, Lin-Ying Hu and Leandro Costa Reis (2003) "Integration of dynamic data in a mature field reservoir model to reduce the uncertainty on production forecasting," paper presented at the AAPG Annual Convention, Salt Lake City, Utah, May 11-14.

Gao, W., P.R. Hartley and R.C. Sickles (2008) "Optimal Dynamic Production from a Large Oil Field in Saudi Arabia," Working paper, Rice University

Griffin, J.M. (1985) "OPEC behavior: a test of alternative hypotheses," *American Economic Review*, 75, 954-963.

Hendricks, K. and D. Kovenock (1989) "Asymmetric information, information externalities, and efficiency: The case of oil exploration," *RAND Journal of Economics*, 20(2):164-182.

Hendricks, K. and R. Porter (1993) "Determinants of the timing and incidence of exploratory drilling on offshore wildcat tracts," *NBER Working Paper Series* (Working Paper No. 4605), Cambridge, MA.

Hendricks, K. and R. Porter (1996) "The timing of incidence of exploratory drilling on offshore wildcat tracts," *The American Economic Review*, 86(3):388-407.

Hotelling, H. (1931) "The economics of exhaustible resources," *The Journal of Political Economy* 39(2): 137-175.

Horwich, G. and Weimer, D.L. (1984) *Oil prices shock, market response, and contingency planning*. Washington, DC: American Enterprise Institute.

Jaffe, Amy Myers (2007) "Energy Security, Climate and Your Car: US Energy Policy and Beyond," presentation at the Asilomar Conference on Transportation and Climate Policy, Pacific Grove, California, August 21-24, 2007.

Johnson, N., C. Yang, J. Ni, J. Johnson, D.Z. Lin, J.M. Ogden (2005) "Optimal Design of a Fossil Fuel-Based Hydrogen Infrastructure with Carbon Capture and Sequestration: Case Study in Ohio," University of California at Davis, report UCD-ITS-RP-05-07.

Kaufmann, R. (1991) "Oil production in the Lower 48 States: Reconciling curve-fitting and econometric models," *Resources and Energy*, 1, 111-127.

Kumins, Larry (2005) "West Coast and Alaska Oil Exports," Congressional Research Service Report for Congress, Order Code RS22141, May 6.

Kunce, M. (2003) "Effectiveness of Severance Tax Incentives in the U.S. Oil Industry," *International Tax and Public Finance* 10, 565-587.

Laibson, D. (1997) "Golden eggs and hyperbolic discounting," *Quarterly Journal of Economics* 112 (2): 443-477.

Laibson, D. (1998) "Life-cycle consumption and hyperbolic discount factors," *European Economic Review* 42 (3-5): 861-871.

Leighty, W.W. (2007) "Alaska natural gas pipeline," presentation, Sustainable Transportation Energy Pathways Seminar, University of California at Davis, 13 February 2007.

Leighty, Wayne, Ben Sharpe, Ryohei Hinokuma (2007) "Carbon Capture and Sequestration for Coal Use in a Carbon Constrained World: A technical and economic feasibility scoping study," unpublished, University of California at Davis.

Libecap G.D. and J.L. Smith (1999) "The self-enforcing provisions of oil and gas unit operating agreements: Theory and evidence," *Journal of Law Economics and Organization* 15(2): 526-548.

Libecap G.D. and J.L. Wiggins (1985) "The influence of private contractual failure on regulation – the case of oil-field unitization," *Journal of Political Economy* 93(4): 690-714.

Lin, C.-Y. C. (2005) “The investment timing game in petroleum production: An econometric model,” *Physica A*, 355:62-68.

Lin, C.-Y. C. (2007) “Do firms interact strategically?: A structural model of the multi-stage investment timing game in offshore petroleum production,” working paper, University of California at Davis.

Lin, C.-Y.C. (2008) “Insights from a Simple Hotelling Model,” working paper, University of California at Davis, http://www.des.ucdavis.edu/faculty/Lin/ hotelling_sim_paper.pdf.

Lin, C.-Y. C. and G. Wagner (2007) “Steady-state growth in a Hotelling model of resource extraction,” *Journal of Environmental Economics and Management* 54: 68-83.

McCarthy, R.W., Yang, C. and J.M. Ogden (2008) "Interactions between hydrogen pathways and electricity supply: A stochastic representation of the California power sector," Proceedings of the NHA Annual Hydrogen Conference 2008, Sacramento, CA, March 30-April 3.

McCarthy, R.W., Yang, C. and J.M. Ogden (2007) "Impacts of electric-drive vehicles on California's energy system," Proceedings of the 23rd Electric Vehicle Symposium, Anaheim, CA, December 2-5.

McMains, Stephen E, Alaska Oil and Gas Conservation Commission (2007) “Endicott Monthly Production 2007-6-29,” Excel file extracted from the AOGCC_DataExtract Access database.

Moroney, J. (1997) *Exploration, Development, and Production: Texas Oil and Gas, 1970-1995*. Greenwich, Connecticut: JAI Press.

Moschini, Giancarlo and Karl Meilke (1989) “Modeling the Pattern of Structural Change in U.S. Meat Demand,” *American Journal of Agricultural Economics* 71(2), 261-353.

Nellemann, C. and Cameron, R.D. (1998) “Cumulative impacts of an evolving oil-field complex on the distribution of calving caribou,” *Canadian Journal of Zoology* 76(8): 1425-1430.

Parker, N.C. (2004). “Using Natural Gas Transmission Pipeline Costs to Estimate Hydrogen Pipeline Costs,” Research Report, Institute of Transportation Studies, University of California at Davis, UCD-ITS-RR-04-035.

Parker, N.C. (2007) “Optimizing the Design of Biomass Hydrogen Supply Chains Using Real-World Spatial Distributions: A Case Study Using California Rice Straw,” Research Report, University of California at Davis, UCD-ITS-RR-07-04.

Petroleum Production Tax, website (2007) www.gov.state.ak.us/oiltax/ppt_report.php (accessed 10 August, 2007).

Pesaran, M.H. (1990) "An Econometric Analysis of Exploration and Extraction of Oil in the U.K. Continental Shelf," *Economic Journal* 100, 367-390.

Pindyck, R.S. (1978) "The optimal exploitation and production of nonrenewable resources," *Journal of Political Economy,* 86(5), 841-861.

Pindyck, R.S. (1982) "OPEC oil pricing, and the implications for producers and consumers," in J.M. Griffin and D.J. Teece (Eds.), *OPEC behavior and world oil prices* (pp. 175-185) London: George Allen and Unwin.

Pontryagin, L.S., Boltyanskii, V.G., Gamkrelidzc, R.V., & Mishchenko, E.F. (1962). *The mathematical theory of optimal processes* (K.N. Trirogoff, Trans.) New York: Interscience.

Ramanathan, Ramu (2002) *Introductory Econometrics with Applications, Fifth Edition*, Mason, Ohio: Thompson Learning.

Rao, R.D. (2000) "An integrated modeling framework for exploration and extraction of petroleum resources," *Resources Policy 26*, 133-143.

Rao, R.D. (2002) "Modeling optimal exploitation of petroleum resources in India," *Resources Policy* 28, 133-144.

Reiss, Peter and Frank Wolak (2007) "Structural Econometric Modeling: Rationales and Examples from Industrial Organization," in J.J. Heckman and E.E. Leamer (Eds.), *Handbook of Econometrics, volume 6A* (pp. 4277-4415) Elsevier.

Rogner, H-H. (1997) "An Assessment of World Hydrocarbon Resources," *Annual Review of Energy and the Environment* 22:217-262.

Ruth, M. and Cleveland, C.J. (1993) "Nonlinear dynamic simulation of optimal depletion of crude oil in the Lower 48 United States," *Computers, Environment and Urban Systems*, 17, 425-435.

Salehi-Isfahani, D. and J. Cremer (1991) "Models of the Oil Market," in *Fundamentals of Pure and Applied Economics*, 44, London: Harwood Academic Publishers.

Shampine, L.F., M.W. Reichelt, and J. Kierzenka (2008) "Solving Boundary Value Problems for Ordinary Differential Equations in MATLAB with bvp4c," available at http://www.mathworks.com/bvp_tutorial (accessed August, 2008).

Uhler, R.S. and Bradley, P.G. (1970) "A Stochastic Model for Determining Economic Prospects of Petroleum Exploration over Larger Regions," *Journal of the American Statistical Association* 65(330), 623.

Uhler, R.S. (1976) "Costs and Supply in Petroleum Exploration – Case of Alberta," *Canadian Journal of Economics* 9(1), 72-90.

Uhler, R.S. (1979) "The Rate of Petroleum Exploration and Extraction," *Advances in the Economics of Energy Resources* vol. 2. JAI Press, Greenwich, CT.

United States Department of Energy (DOE), website (2008) "Oil Recovery R&D/CO2 Injection," www.fe.doe.gov/programs/oilgas/eor/ (accessed 19 August, 2008).

United States Department of Labor, Bureau of Labor Statistics (2007) "Consumer Price Index, All Urban Consumers (CPI-U), U.S. City Average, All Items, 1982-84 = 100," Washington, D.C., available at ftp://ftp.bls.gov/pub/special.requests/cpi/cpiai.txt (accessed 19 August, 2008).

United States Geological Survey (2005) "Oil and gas assessment of central North Slope, Alaska, 2005," USGS Fact Sheet 2005-3043, available at http://pubs.usgs.gov/fs/2005/3043/ (accessed 10 August, 2007).

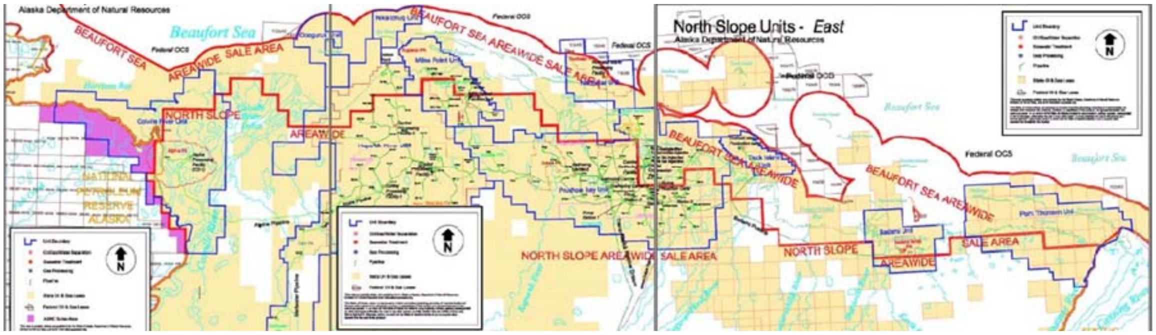
vanRensburg, W.C.J. (2000) "The choice of an appropriate discount rate in economic evaluations of petroleum and mineral projects," *IN SITU* 24(2-3): 163-182.

Williams, Brett D. and Kenneth S. Kurani (2006) "Estimating the Early Household Market for Light-Duty Hydrogen-Fuel-Cell Vehicles and other 'Mobile Energy' Innovations in California: A Constraints Analysis," *Journal of Power Sources* (in press), University of California at Davis report UCD-ITS-RP-06-10.

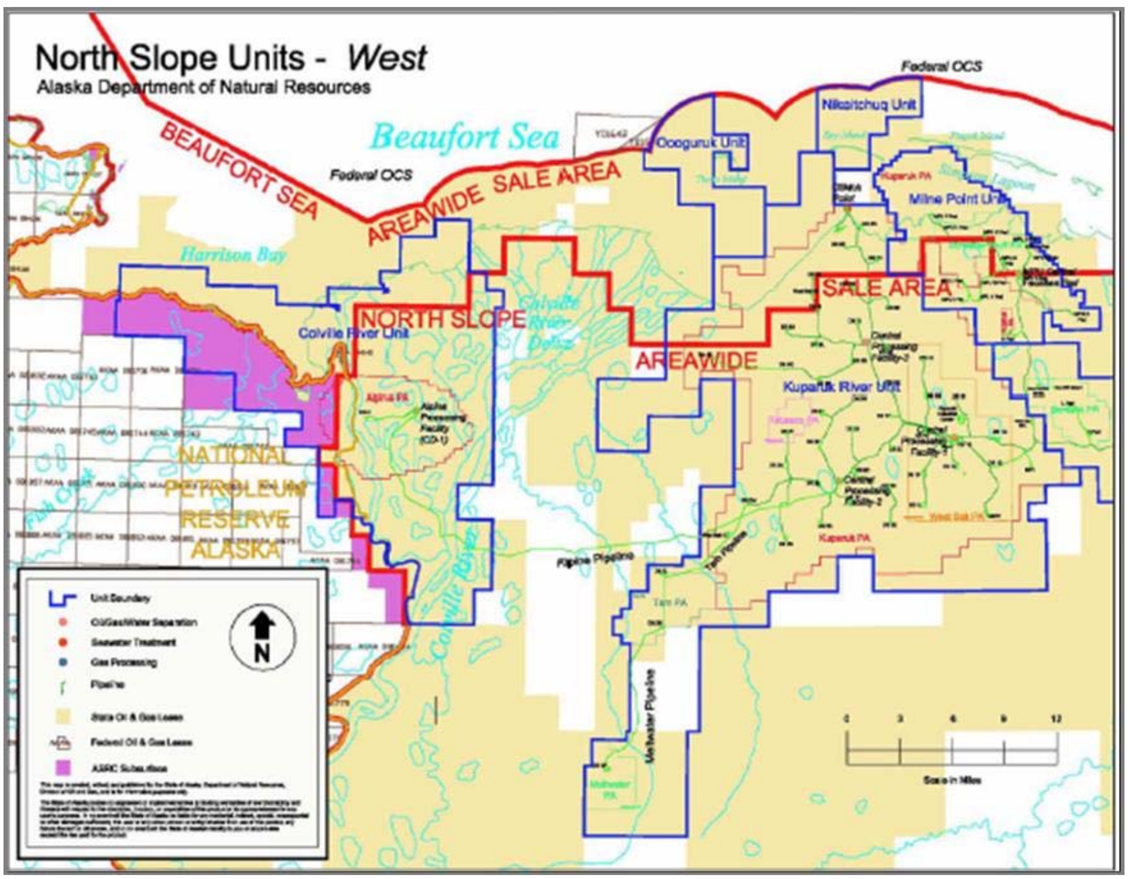
Wilson, J. (1999) "Theories of Tax Competition," *National Tax Journal* 52, 269-304.

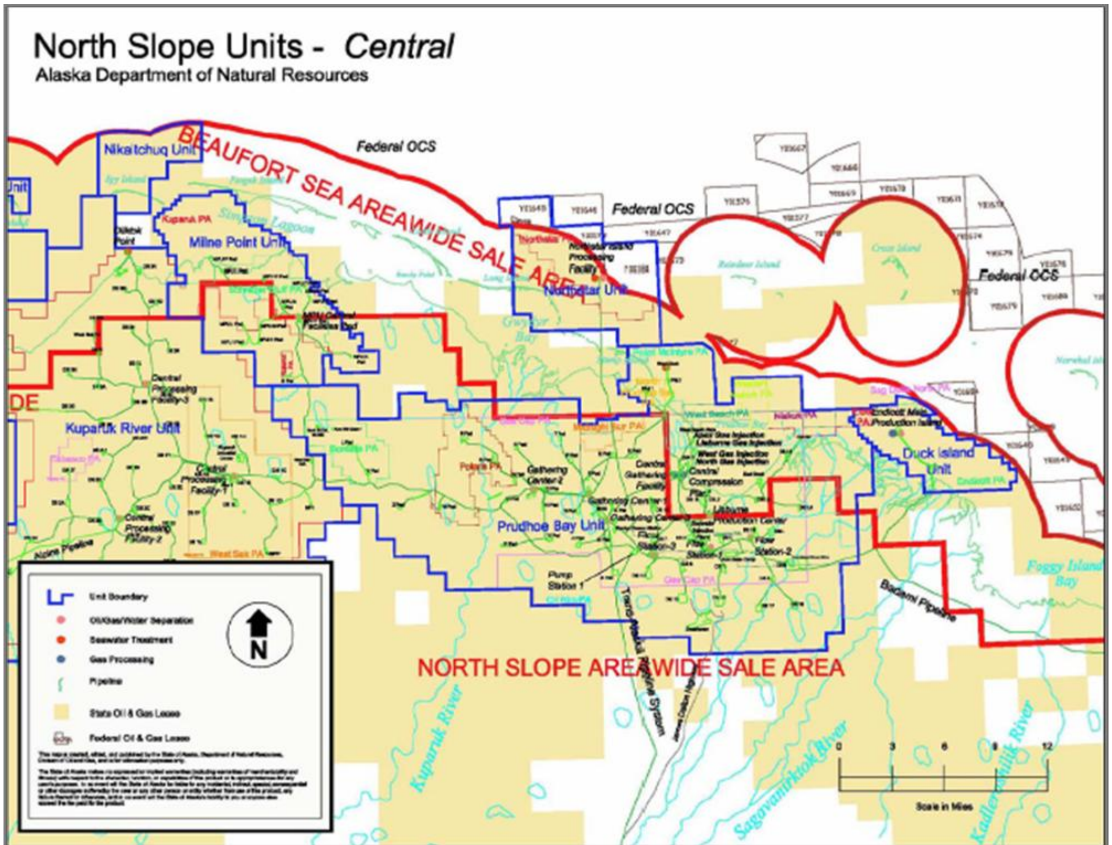
Yucel, M. (1989) "Severance Taxes and Market Structure in an Exhaustible Resource Industry," *Journal of Environmental Economics and Management* 16, 134-148.

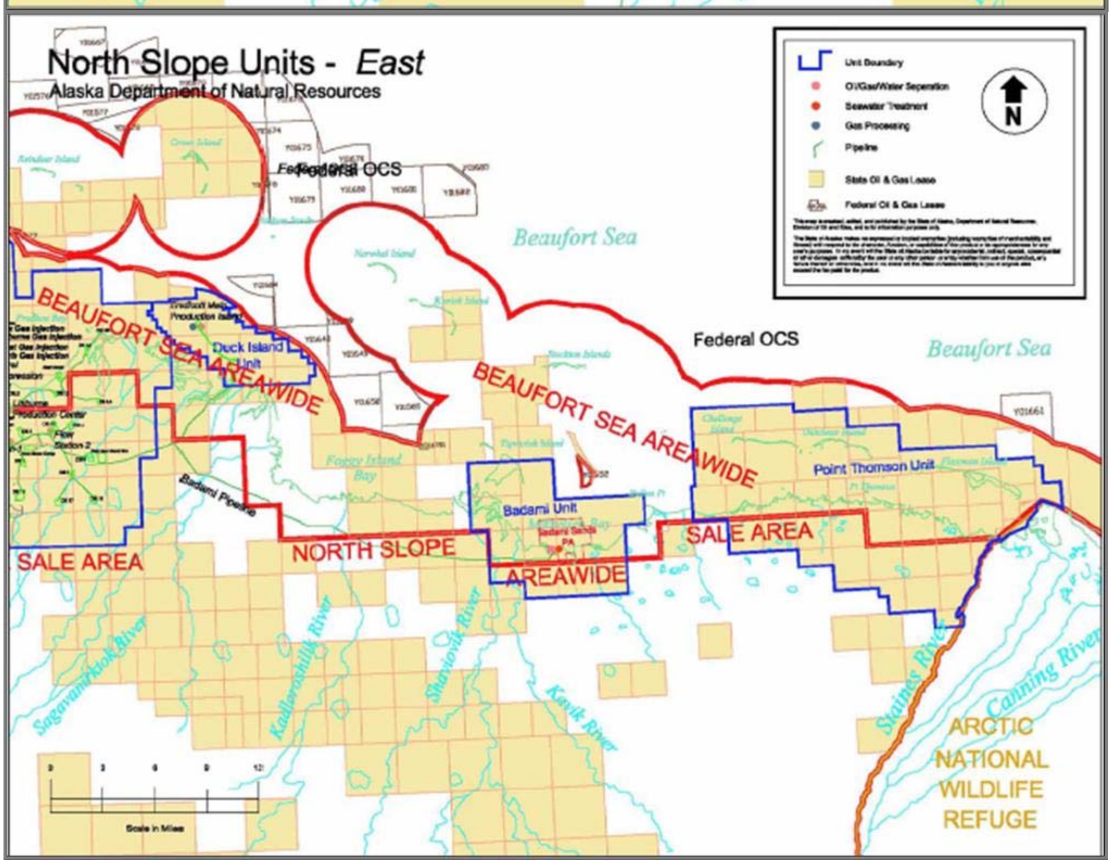
Appendix A: North Slope Production Units



Seven production units on Alaska's North Slope, from west to east: Colville River, Kuparuk River, Milne Point, Prudhoe Bay, Northstar, Endicott (Duck Island), and Badami (Alaska Department of Natural Resources). Notice the National Petroleum Reserve Area (NPRA) on the western fringe and the Arctic National Wildlife Refuge (ANWR) on the eastern fringe. Larger maps for detail follow.

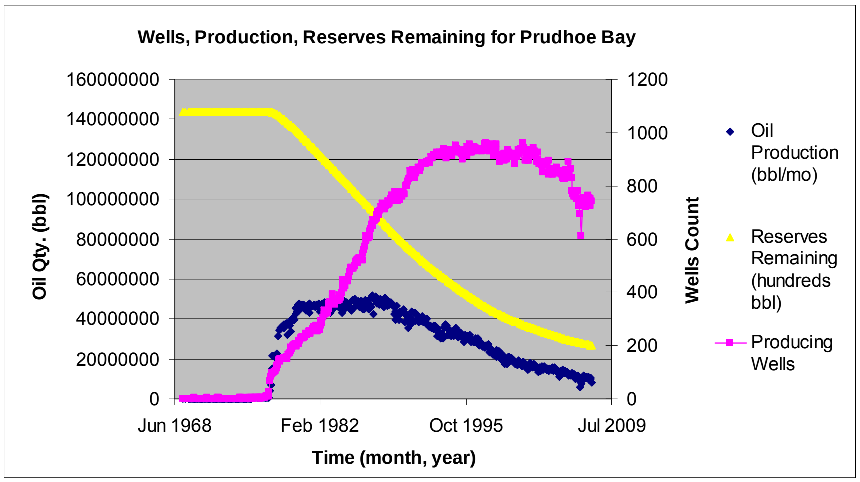


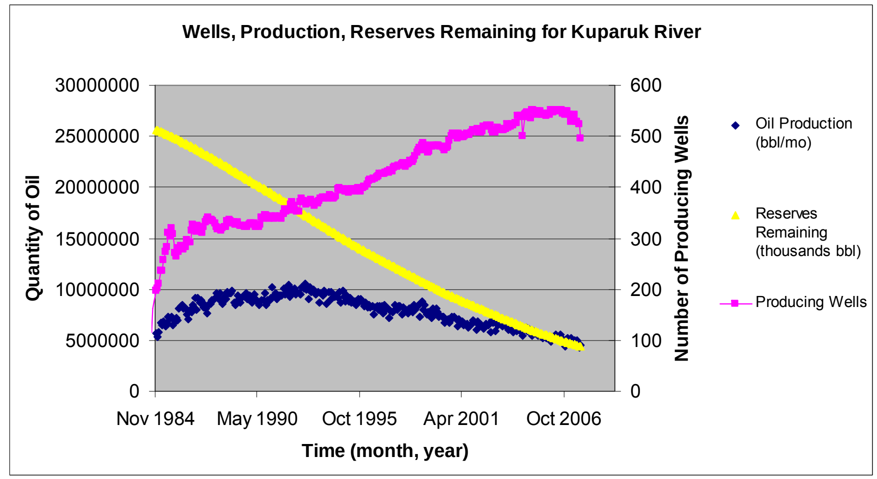

North Slope Units - Central
Alaska Department of Natural Resources
Federal OCS
BEAUFORT SEA AREAWIDE SALE AREA
Nikaitchuq Unit
Milne Point Unit
Northstar Unit
Kuparuk River Unit
Duck Island Unit
Prudhoe Bay Unit
NORTH SLOPE AREAWIDE SALE AREA
Badami Pipeline
Kuparuk River
Sagavanirktok River
Kadleroshilik River
Foggy Island Bay
Unit Boundary
Oil/Gas/Water Separation
Seawater Treatment
Gas Processing
Pipeline
State Oil & Gas Lease
Federal Oil & Gas Lease
Scale in Miles

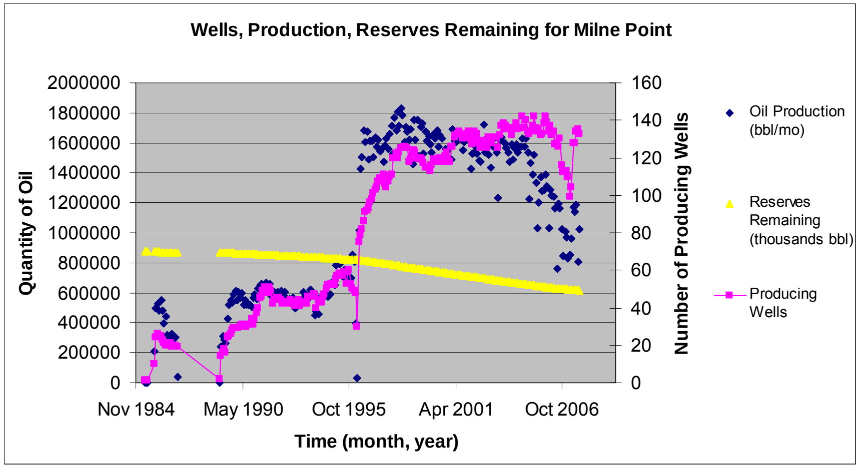

North Slope Units - East
Alaska Department of Natural Resources
Federal OCS
Beaufort Sea
BEAUFORT SEA AREAWIDE
Duck Island Unit
Badami Unit
Point Thomson Unit
SALE AREA
NORTH SLOPE
AREAWIDE
Badami Pipeline
Sagavanirktok River
Kadleroshilik River
Shaviovik River
Kavik River
Staines River
Canning River
ARCTIC NATIONAL WILDLIFE REFUGE
Foggy Island Bay
Unit Boundary
Oil/Gas/Water Separation
Seawater Treatment
Gas Processing
Pipeline
State Oil & Gas Lease
Federal Oil & Gas Lease
Scale in Miles

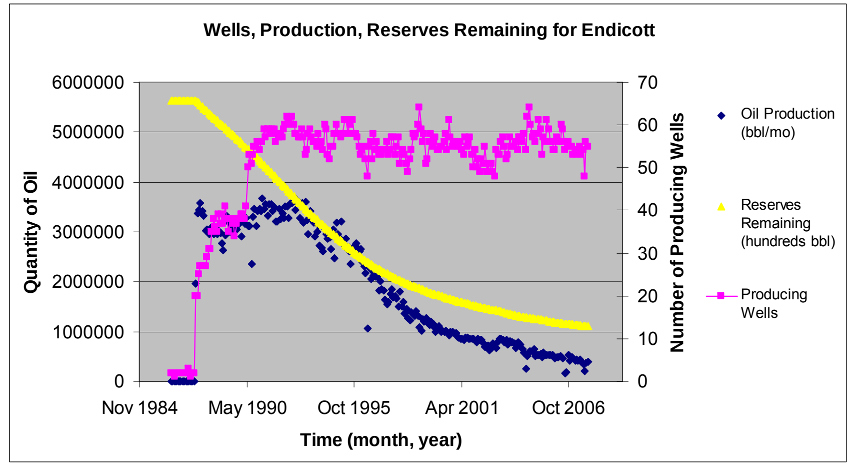
Appendix B: Data plots by field: wells, production, reserves remaining

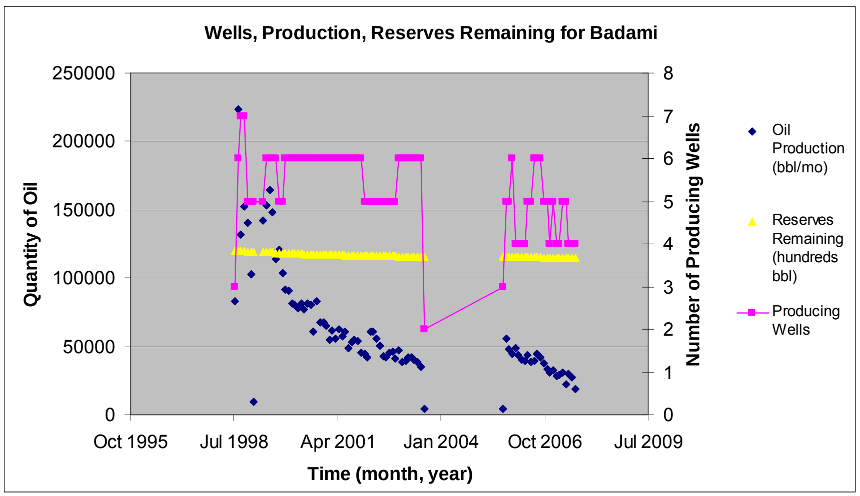
Plots of oil production (barrels/month), reserves remaining (hundreds or thousands of barrels), and the number of producing wells generally show the number of wells increasing in order to maintain a certain production rate while reserves remaining decline, although the history is more hectic for some units (e.g., Badami).

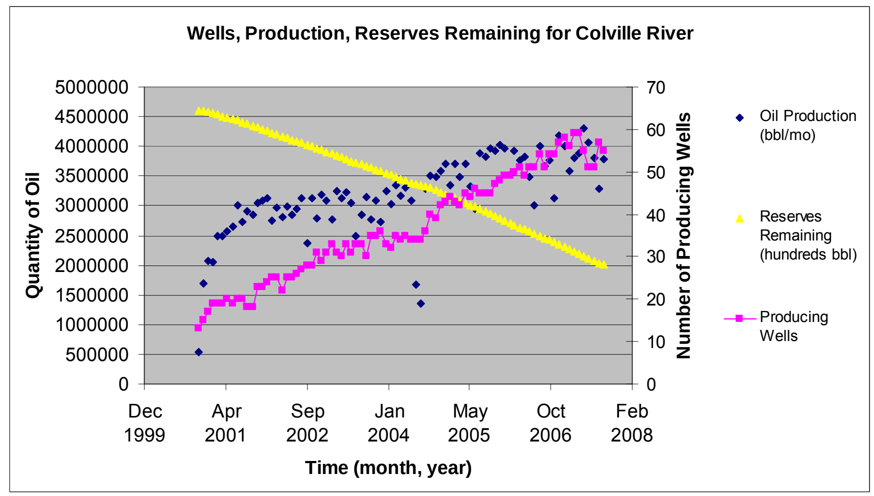


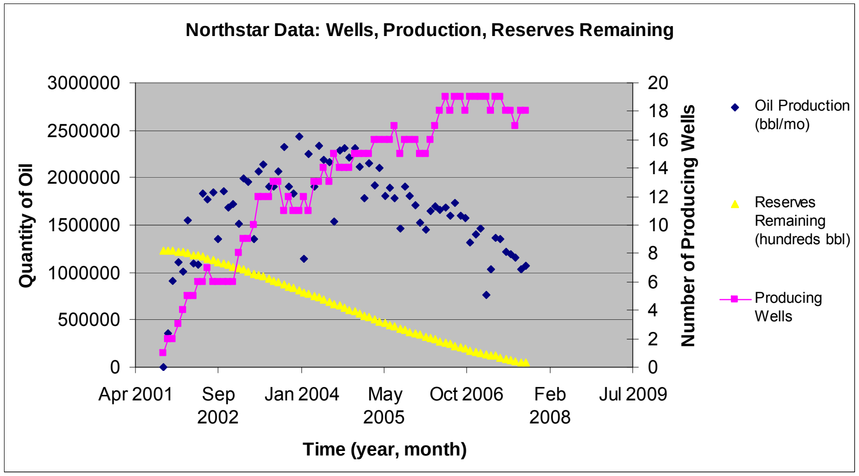



Wells, Production, Reserves Remaining for Milne Point
2000000
1800000
1600000
1400000
1200000
1000000
800000
600000
400000
200000
0
160
140
120
100
80
60
40
20
0
Quantity of Oil
Number of Producing Wells
Nov 1984
May 1990
Oct 1995
Apr 2001
Oct 2006
Time (month, year)
Oil Production (bbl/mo)
Reserves Remaining (thousands bbl)
Producing Wells


Wells, Production, Reserves Remaining for Endicott
6000000
5000000
4000000
3000000
2000000
1000000
0
70
60
50
40
30
20
10
0
Quantity of Oil
Number of Producing Wells
Nov 1984
May 1990
Oct 1995
Apr 2001
Oct 2006
Time (month, year)
Oil Production (bbl/mo)
Reserves Remaining (hundreds bbl)
Producing Wells

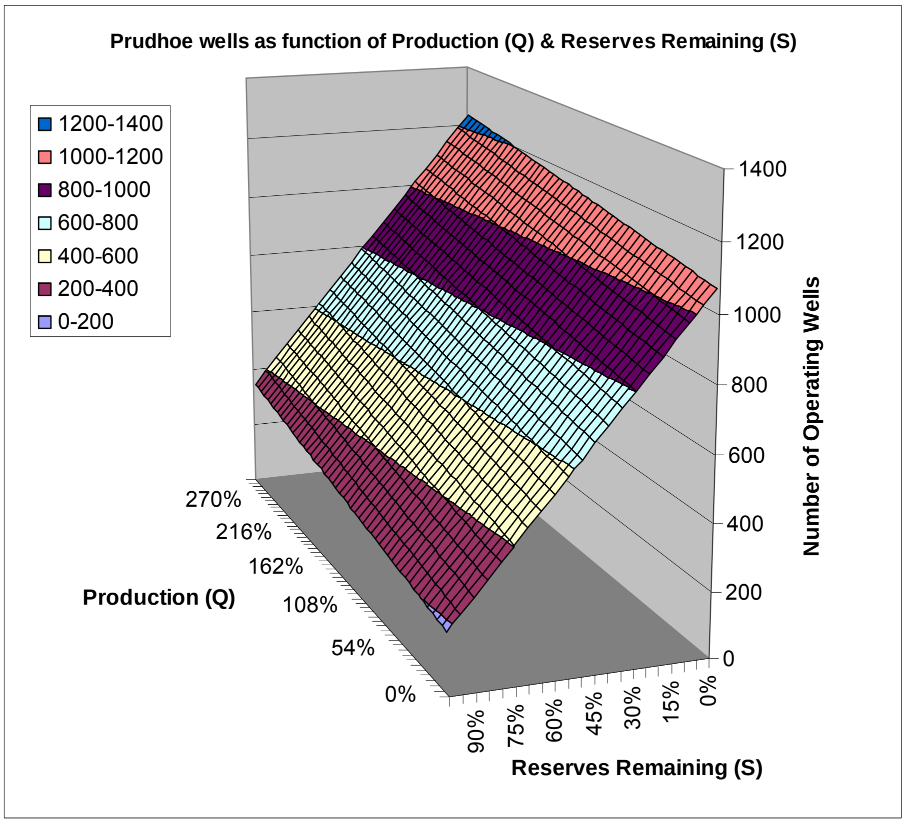

Wells, Production, Reserves Remaining for Badami
250000
200000
150000
100000
50000
0
8
7
6
5
4
3
2
1
0
Quantity of Oil
Number of Producing Wells
Oct 1995
Jul 1998
Apr 2001
Jan 2004
Oct 2006
Jul 2009
Time (month, year)
Oil Production (bbl/mo)
Reserves Remaining (hundreds bbl)
Producing Wells

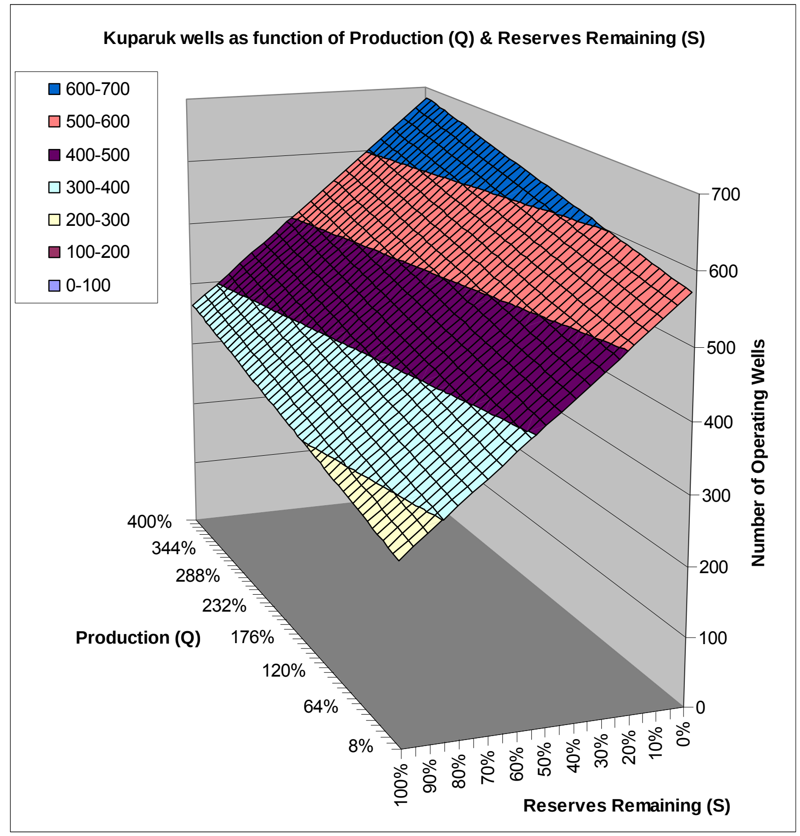

Wells, Production, Reserves Remaining for Colville River
5000000
4500000
4000000
3500000
3000000
2500000
2000000
1500000
1000000
500000
0
70
60
50
40
30
20
10
0
Quantity of Oil
Number of Producing Wells
Dec 1999
Apr 2001
Sep 2002
Jan 2004
May 2005
Oct 2006
Feb 2008
Time (month, year)
Oil Production (bbl/mo)
Reserves Remaining (hundreds bbl)
Producing Wells

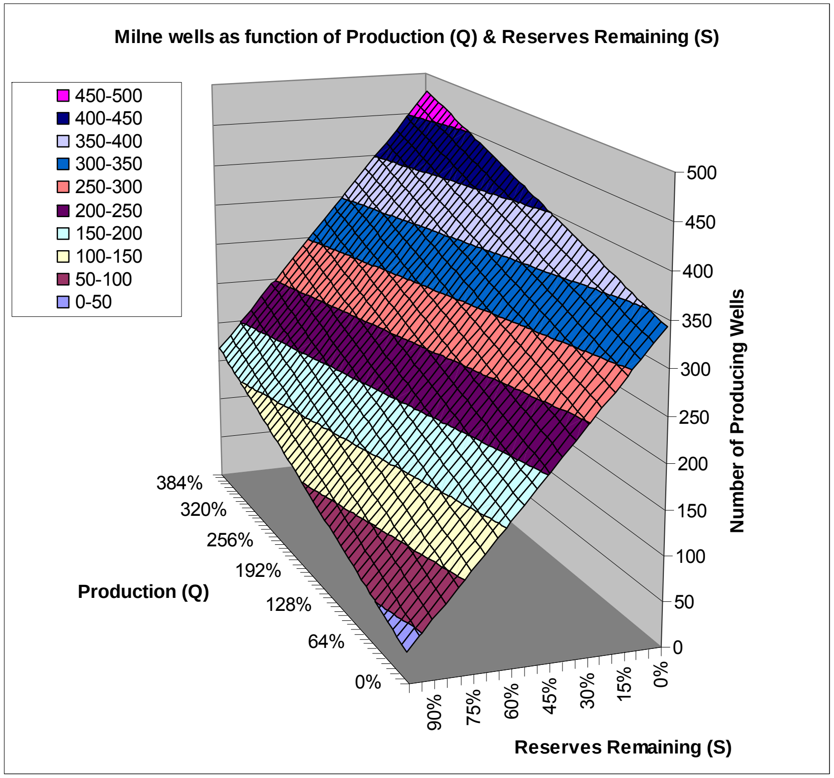

Northstar Data: Wells, Production, Reserves Remaining
Quantity of Oil
3000000
2500000
2000000
1500000
1000000
500000
0
Number of Producing Wells
20
18
16
14
12
10
8
6
4
2
0
Apr 2001
Sep 2002
Jan 2004
May 2005
Oct 2006
Feb 2008
Jul 2009
Time (year, month)
Oil Production (bbl/mo)
Reserves Remaining (hundreds bbl)
Producing Wells

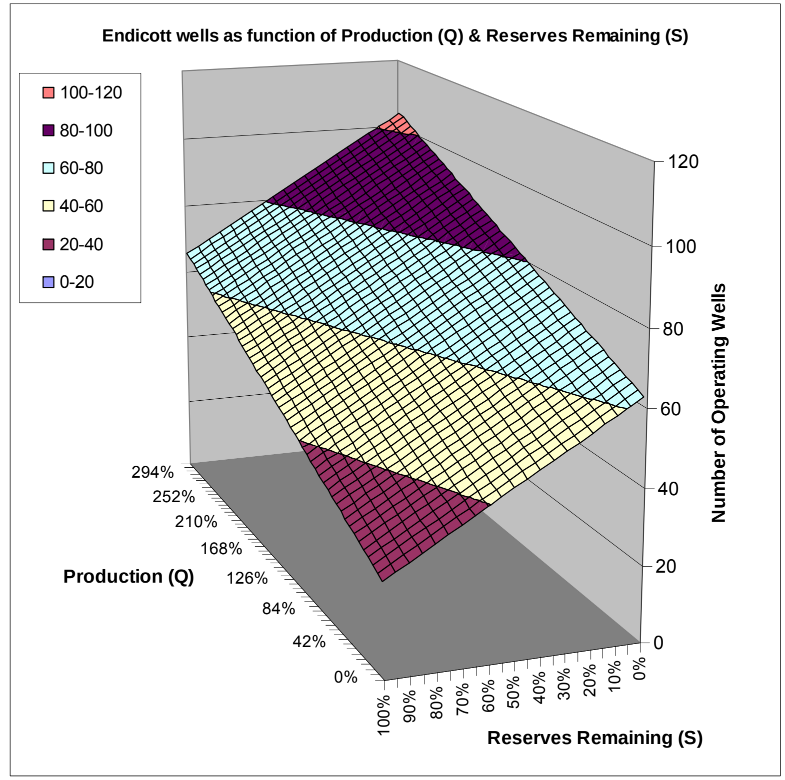
Appendix C: Constant Returns to Scale Planes

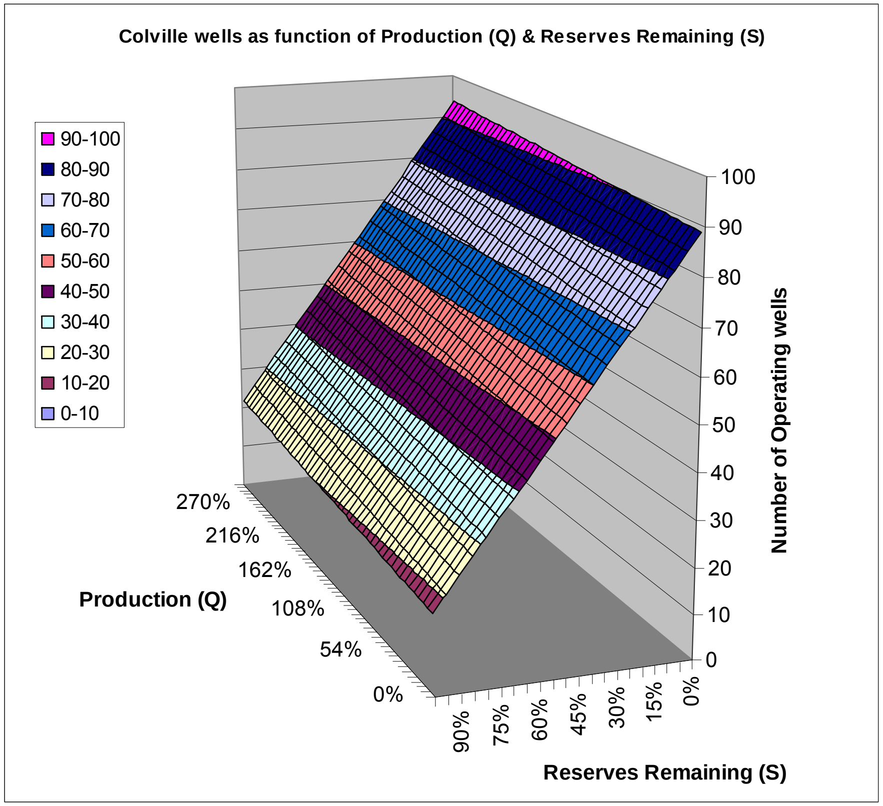
The constant returns wells planes for each field. The axis for reserves remaining extends from the original quantity of technically recoverable oil to zero. The production axis ranges from zero to three times the maximum historical rate of production. The vertical axis is the number of operating wells.

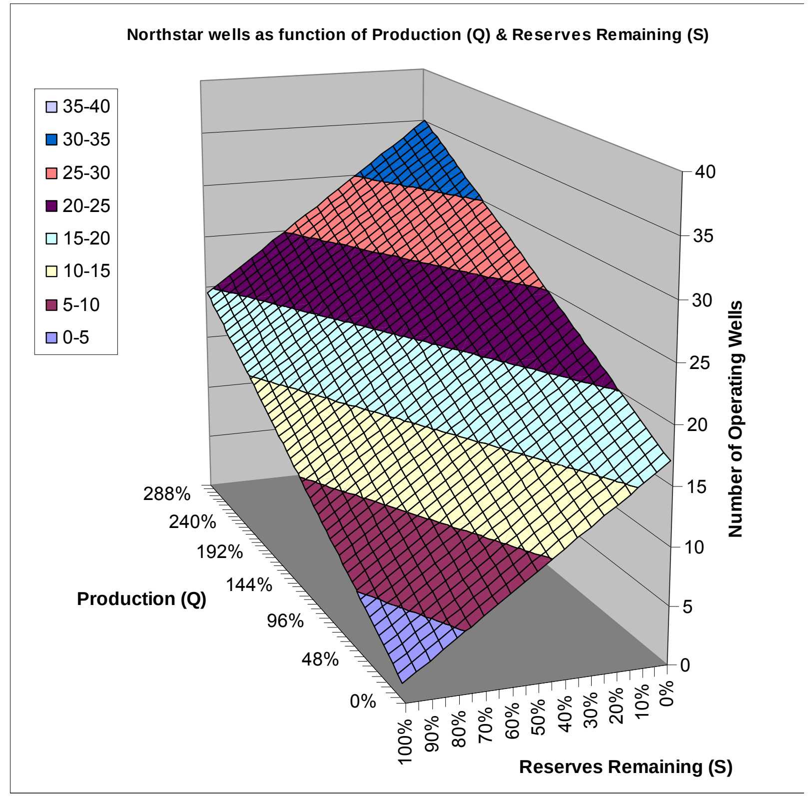





Milne wells as function of Production (Q) & Reserves Remaining (S)
450-500
400-450
350-400
300-350
250-300
200-250
150-200
100-150
50-100
0-50
Number of Producing Wells
0
50
100
150
200
250
300
350
400
450
500
Production (Q)
0%
64%
128%
192%
256%
320%
384%
Reserves Remaining (S)
90%
75%
60%
45%
30%
15%
0%

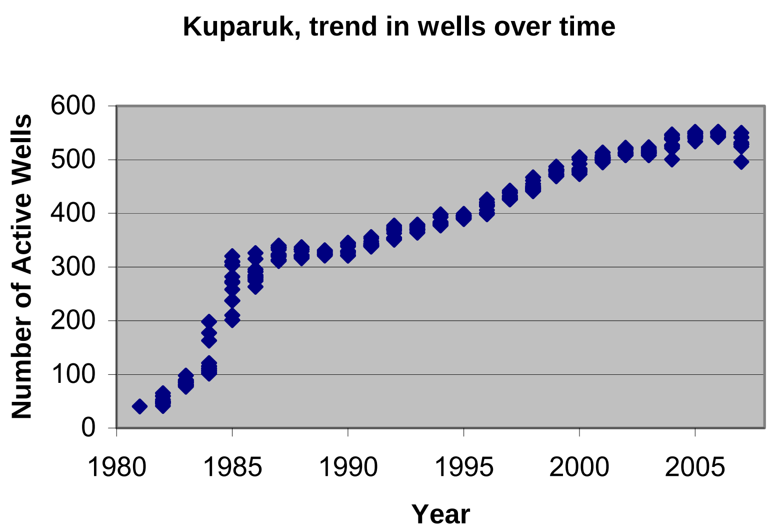

Endicott wells as function of Production (Q) & Reserves Remaining (S)
100-120
80-100
60-80
40-60
20-40
0-20
Number of Operating Wells
0
20
40
60
80
100
120
Production (Q)
0%
42%
84%
126%
168%
210%
252%
294%
Reserves Remaining (S)
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%


Colville wells as function of Production (Q) & Reserves Remaining (S)
90-100
80-90
70-80
60-70
50-60
40-50
30-40
20-30
10-20
0-10
Number of Operating wells
0
10
20
30
40
50
60
70
80
90
100
Production (Q)
0%
54%
108%
162%
216%
270%
Reserves Remaining (S)
90%
75%
60%
45%
30%
15%
0%


Northstar wells as function of Production (Q) & Reserves Remaining (S)
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
Number of Operating Wells
0
5
10
15
20
25
30
35
40
Production (Q)
0%
48%
96%
144%
192%
240%
288%
Reserves Remaining (S)
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%

Appendix D: Specification of Unit-Specific Decreasing-Returns Well Functions

The relationship between production quantity and number of wells for Kuparuk River shows peak production prior to peak wells, with decreasing production as the number of wells continued to climb (Figure 23). This relationship demonstrates the impact of reserves on the number of wells needed to produce at a given rate, and therefore the need to include reserves remaining in the model specification for estimating well functions. Consequently, we included both quantity of production (Q) and reserves remaining (S) as relevant regressors in our estimation of wells functions.



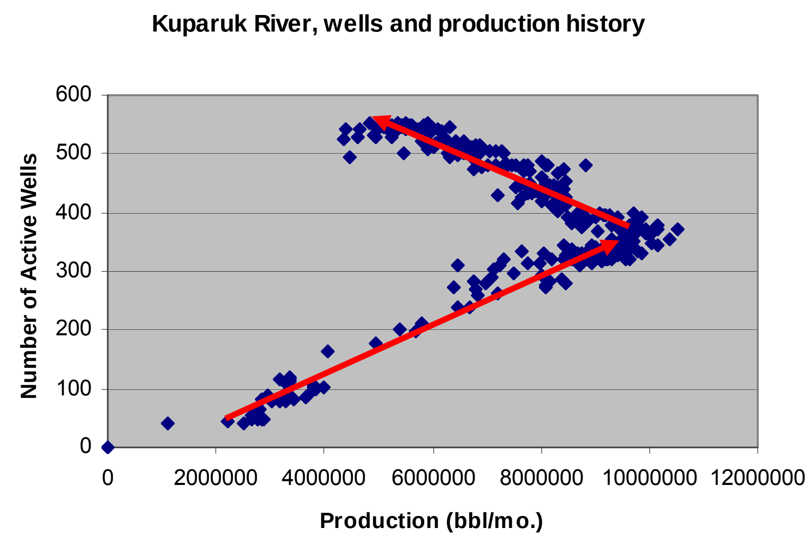


Figure 23: The left panel shows that the increase in wells in the Kuparuk field has been nearly monotonic. Thus, we can interpret the progression in production data over time as indicated by the red arrows in the right panel. The continued increase in number of wells after peak production rate shows the need for an increasing number of wells to maintain production as reserves are depleted since more wells are needed to encourage higher flow rate from a smaller reserve.

Including a time variable

We considered whether to include a time variable in the regression specifications for estimating wells functions. Adding a time regressor can account for events in the economy, developing well technology, and events in each field. Although we cannot separate these effects, it is not important to do so for the intended use of these regressions. We defined two time variables as the following.

year_index = (Year-1968)+(month-1)/12
yearsincestart_index = (year - year of first production)+(month-1)/12

These formulas index time to the nearest month by using decimals for the months, referenced to the beginning of each month (e.g., January 1970 is 2.0 and April 1970 is 2.25). We subtracted 1968 from the year index to normalize it close to one (the first field to start production, Prudhoe Bay, started production in 1969).

For pooled data it may make sense to use both time indexes, one for industry-wide effects and one for field-specific changes. However, for the field-specific regressions we developed, the time indexes will be collinear. Furthermore, the time index will be correlated with the reserves remaining variable. Finally, including time in the well functions complicates derivation of the dynamic model when using the ODE boundary value approach. For these reasons, we decided not to include a time variable in the well function specifications.

Including interaction terms

Adding interaction terms can allow the reserves remaining to impact the slope and shape of the curve rather than just shifting it as a fixed effect (which is the result of adding S-terms alone). The assumption that production cost is a decreasing function of stock size is common in the economic literature (e.g., Farrow, 1985; Hartwick, 1982; Pindyck, 1978; Ruth and Cleveland, 1993), which implies a squared term for the reserves remaining in well function specifications. For the decreasing returns wells surfaces, a cubic term for the quantity of production is necessary. Consequently, we defined interaction terms for all combinations of Q, Q^2, Q^3, S, and S^2.

Creation of data for interaction terms revealed the need for normalizing production and reserves remaining values close to one to avoid overflow errors (i.e., millions of barrels of production and billions of barrels of reserves remaining cause overflow errors when interacted if both are measured in barrels). Consequently, we defined production and reserves remaining indexes as follows.

Prodoil_index = Prodoil (bbl) / 1,000,000 = millions bbl oil per month
Resrem_index = Resrem (bbl) / 1,000,000,000 = billions bbl oil technically recoverable

From these data, we defined variables for Q^2, Q^3, S^2, QS, Q^2S, Q^3S, QS^2, Q^2S^2, and Q^3S^3 for pooled data as well as for each field individually.

The most complete regression model

The most complete regression, including all variables in a “kitchen sink” approach, was the following.

$$\text{Wells} = c_1+c_2Q+c_3Q^2+c_4Q^3+c_5S+c_6S^2+c_7QS+c_8QS^2+c_9Q^2S+c_{10}Q^2S^2+c_{11}Q^3S+c_{12}Q^3S^2$$

The penalty for omitting truly important regressors is serious (bias and inconsistency) while the penalty for including extraneous regressors is less severe (inefficiency leading to potentially inaccurate hypothesis testing) (Ramanathan, 2002). Since hypothesis testing is not central to our objectives, an approach of erring on the side of using more regressors may be valid. However, many of the regressors are correlated since they are interactions of two pieces of information, Q and S. Consequently, we tried to use variable selection techniques to define the "best" model specification for each field.

Variable Selection

We used forward, backward, stepwise, and Cp techniques for variable selection. These procedures evaluate the significance of each regressor individually, either while removing regressors from the "kitchen sink" specification or while adding them one at a time to the model specification. We set the arbitrary cutoff for inclusion in the model at a p-value of 0.1.

The variable selection procedure yields different regression specifications for each field, which we believe to be an acceptable result since field-specific specifications as well as coefficient estimates may be justified by the unique conditions (geologic and otherwise) present for each field.

Since the stepwise procedure includes both the forward and backward procedures, we focused on these results. Results were deemed "acceptable" if the function was well-behaved in monotonicity and curvature and displayed the anticipated decreasing returns to production rate and reserves remaining (i.e., increasing wells needed for higher production and lower reserves remaining; $\frac{\partial Wells}{\partial Q} > 0$, $\frac{\partial^2 Wells}{\partial Q^2} > 0$ and $\frac{\partial Wells}{\partial S} < 0$ $\frac{\partial^2 Wells}{\partial S^2} < 0$). We found acceptable results for Badami, Colville, Endicott, Milne, and Northstar. However, results for the largest fields, Kuparuk and Prudhoe, were unacceptable.

Iterative Model Re-specification

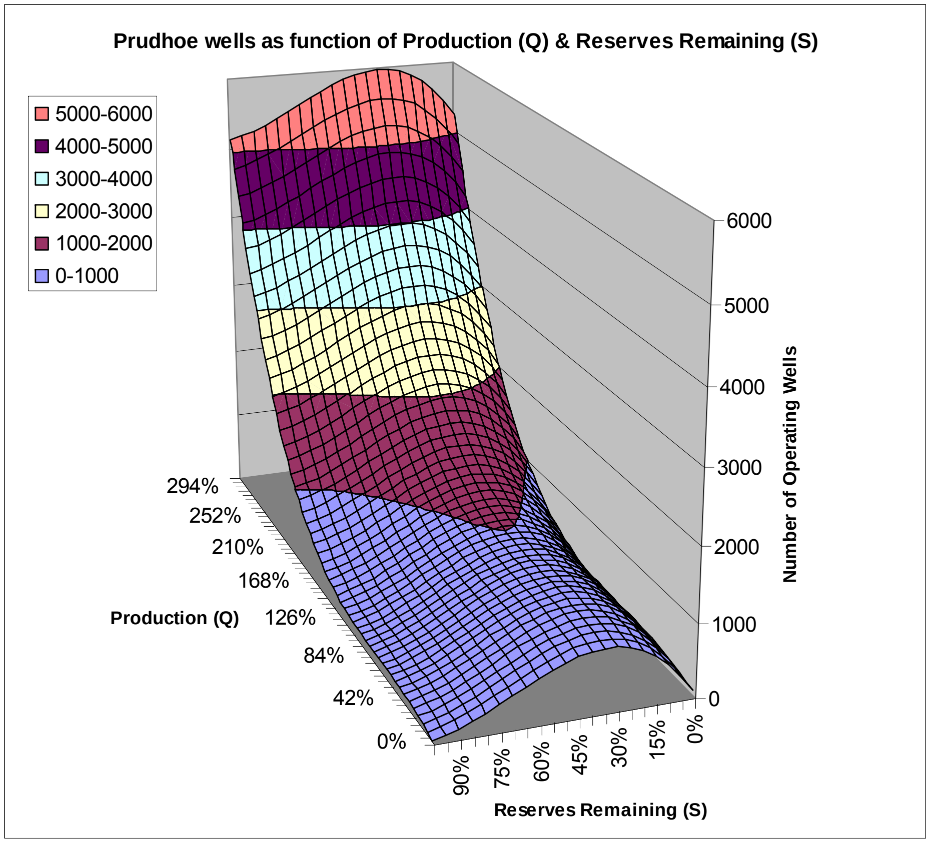
We rely on economic theory and reservoir geology to argue for a decreasing-returns wells surface, and to justify iterative model re-specification to find "acceptable" regression results. Our goal for this component of our overall research was to identify a plausible wells function that has the decreasing returns to scale we expect. Thus, "tampering" with the wells regression by evaluating results during the model specification process may be a legitimate approach. We were not searching for the "correct" wells function, but rather for a plausible one (i.e., fits the data) that matched our economic and geologic theory (i.e., structure) that we wanted to impose in our modeling. Finally, we did not use the standard errors for hypothesis testing (although future work may use them in monte-carlo simulation), so the fact that tampering has modified the standard errors is not a concern.

With this logic in mind, we evaluated possible well regression specifications, looking for results that showed the expected decreasing returns to scale. However, with a complete list of 11 potential independent variables (Q, Q^2, Q^3, S, S^2, QS, QS^2, Q^2S, Q^2S^2, Q^3S, Q^3S^2), the total number of possible model specifications is 2,047.[67] Thus, an automated process of finding the "best" specification is ideal. Such a process would first examine the regression results to determine whether they exhibit the declining returns to scale that we expect, and within bounds that are reasonable (i.e., rate of increase in wells is not too steep or too gradual). Then, the process would use a variable selection procedure to select the "best" model specifications from among to subset of acceptable forms. We did not, however, develop such an automated process, relying instead on judgment in iterative model re-specification to arrive at the final field-specific specifications described in this paper and shown in appendix E.

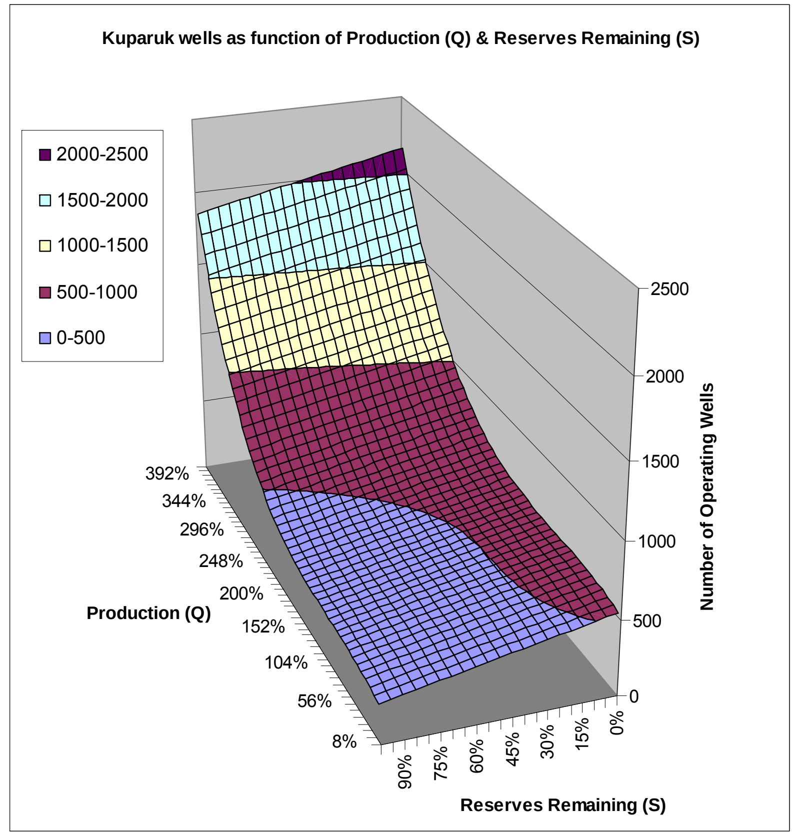
[67] The formula for calculating the number of possible combinations for a model specification of a particular size (e.g., 4 variables) is n!/[k!(n-k)!, where n is the number of variables from which to choose (11 in this case) and k is the number of variables to be specified in the model (4 in this case). Consequently, calculation of all the possible combinations involves summing this formula over all models of size 1 to 11 variables. That is, $\sum_{k=1}^{11} \frac{11!}{k!(11-k)!} = 2{,}047$.

Appendix E: Decreasing Returns to Scale Surfaces

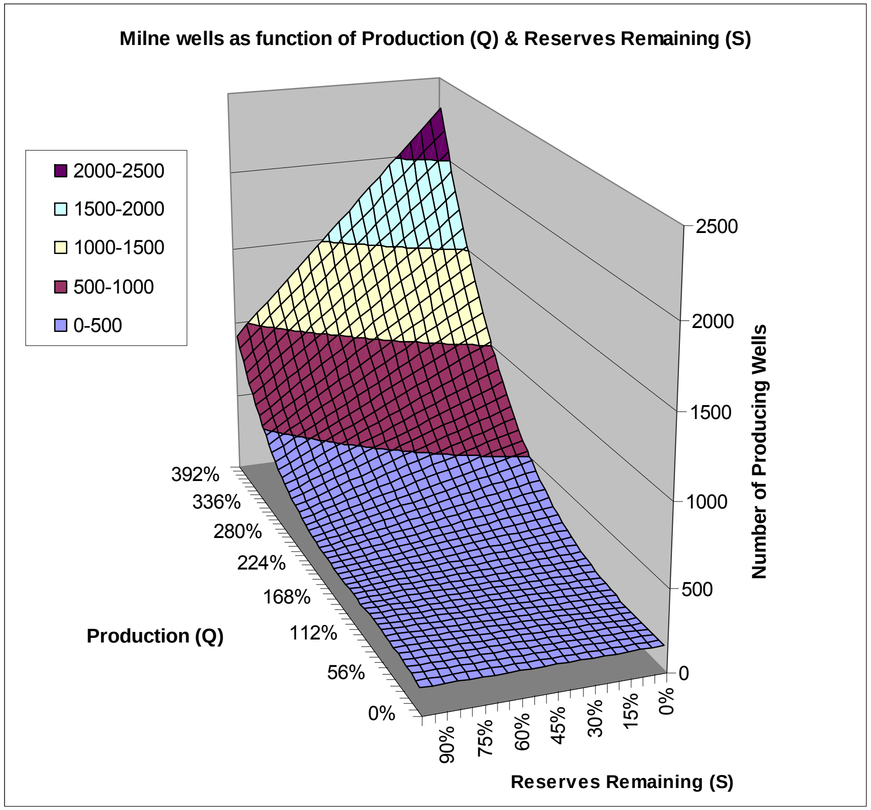
The decreasing returns to scale wells surfaces for each field. The axis for reserves remaining extends from the original quantity of technically recoverable oil to zero. The production axis ranges from zero to three times the maximum historical rate of production. The vertical axis is the number of operating wells.

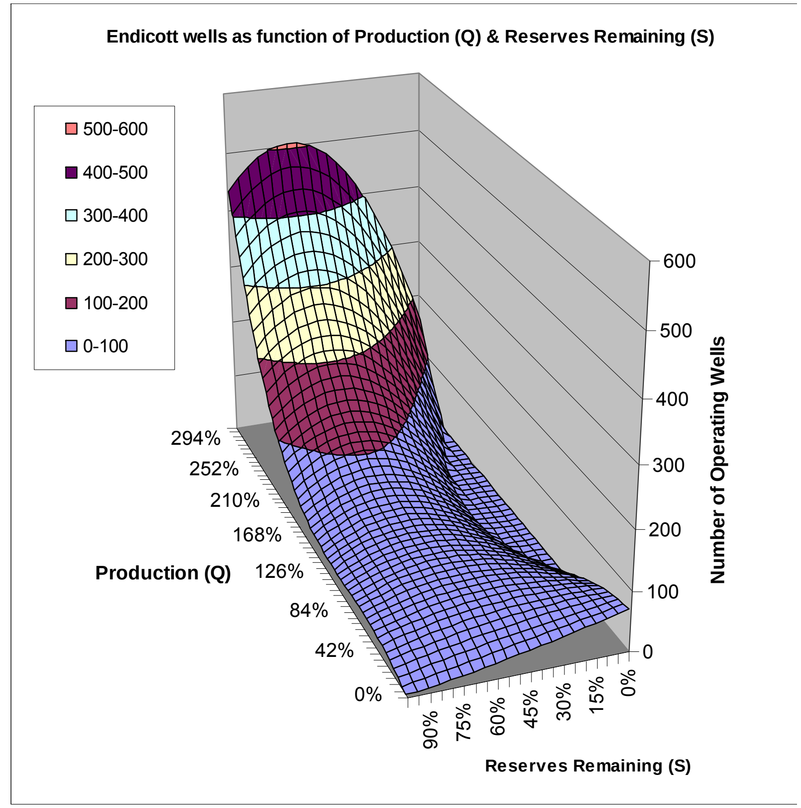


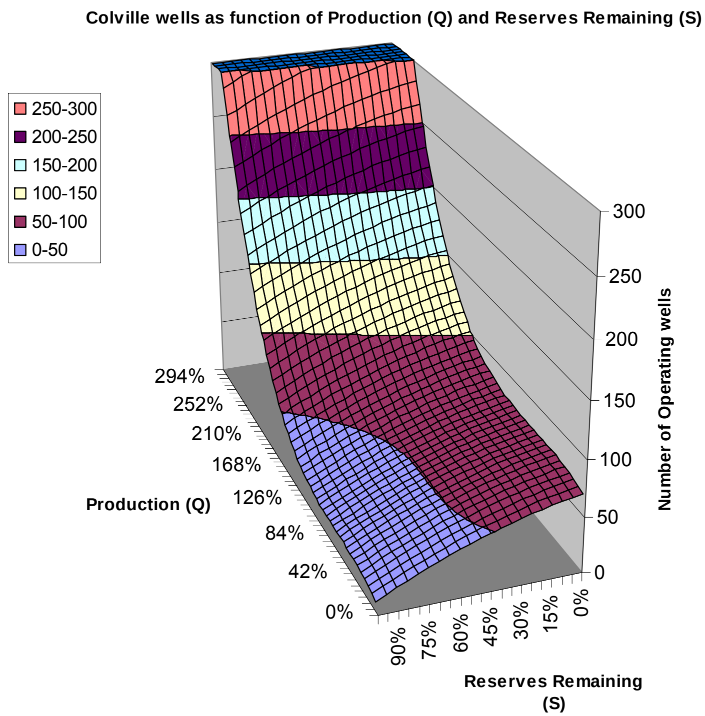
Prudhoe wells as function of Production (Q) & Reserves Remaining (S)

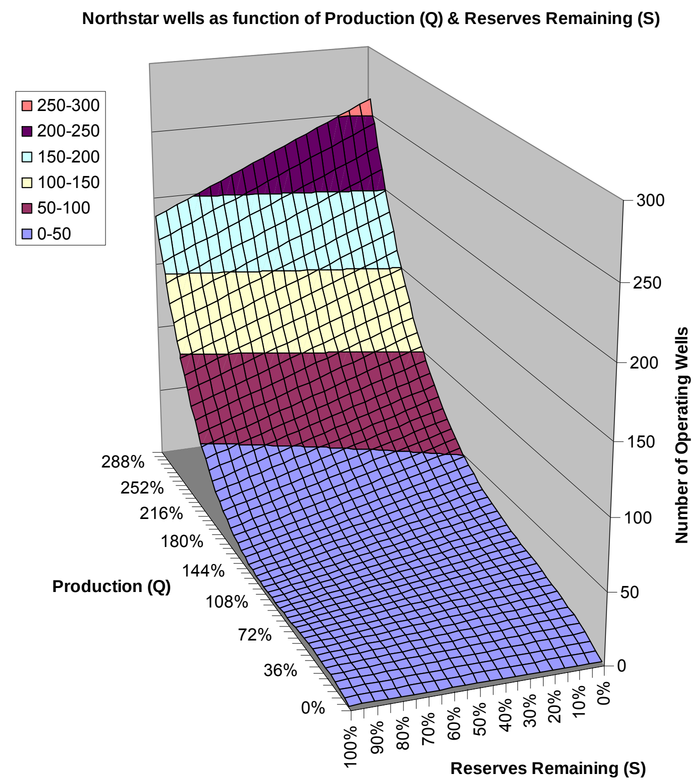


Kuparuk wells as function of Production (Q) & Reserves Remaining (S)


Milne wells as function of Production (Q) & Reserves Remaining (S)
2000-2500
1500-2000
1000-1500
500-1000
0-500
Number of Producing Wells
2500
2000
1500
1000
500
0
Production (Q)
392%
336%
280%
224%
168%
112%
56%
0%
Reserves Remaining (S)
90%
75%
60%
45%
30%
15%
0%

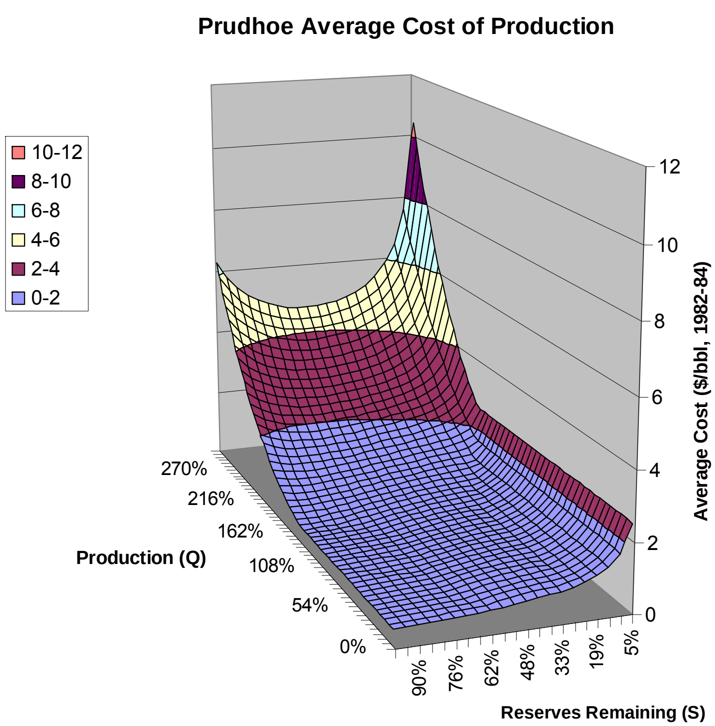

Endicott wells as function of Production (Q) & Reserves Remaining (S)
500-600
400-500
300-400
200-300
100-200
0-100
Number of Operating Wells
600
500
400
300
200
100
0
Production (Q)
294%
252%
210%
168%
126%
84%
42%
0%
Reserves Remaining (S)
90%
75%
60%
45%
30%
15%
0%

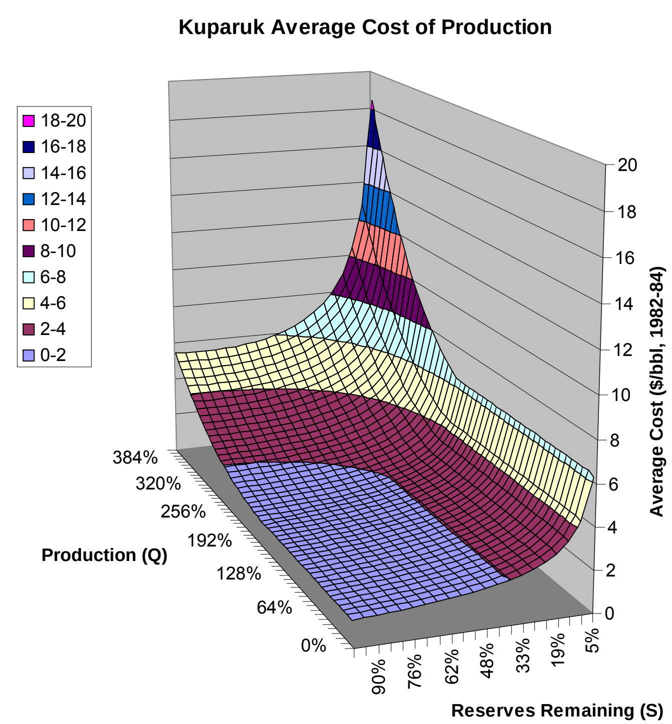

Colville wells as function of Production (Q) and Reserves Remaining (S)
250-300
200-250
150-200
100-150
50-100
0-50
Production (Q)
294%
252%
210%
168%
126%
84%
42%
0%
90%
75%
60%
45%
30%
15%
0%
Reserves Remaining (S)
Number of Operating wells
0
50
100
150
200
250
300

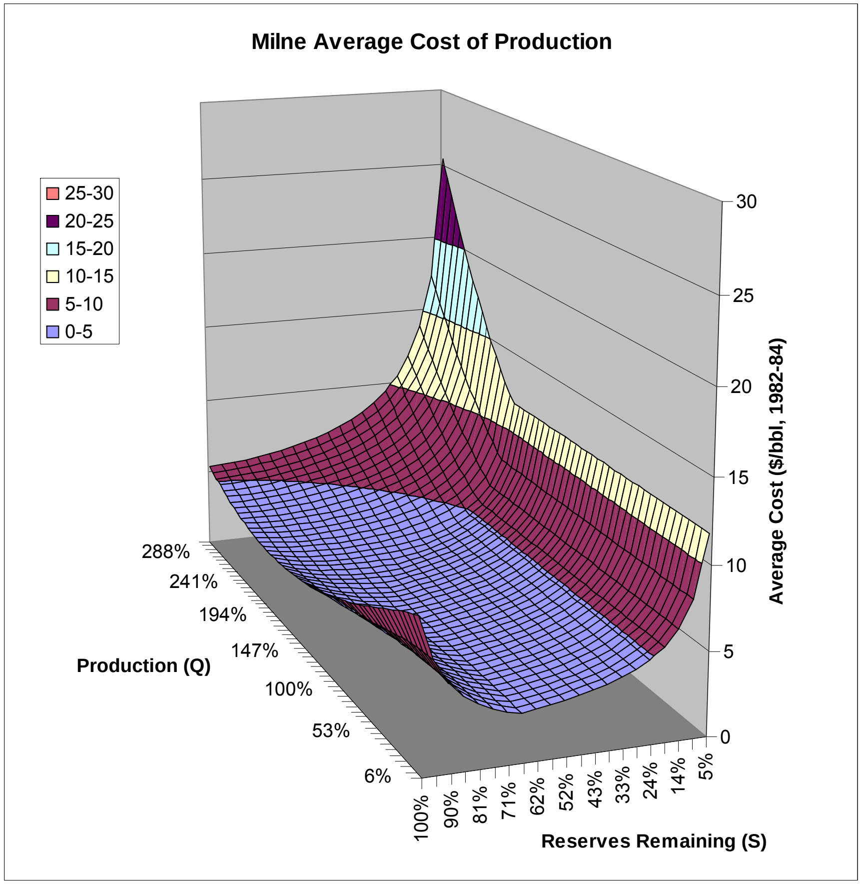

Northstar wells as function of Production (Q) & Reserves Remaining (S)
250-300
200-250
150-200
100-150
50-100
0-50
Production (Q)
288%
252%
216%
180%
144%
108%
72%
36%
0%
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Reserves Remaining (S)
Number of Operating Wells
0
50
100
150
200
250
300

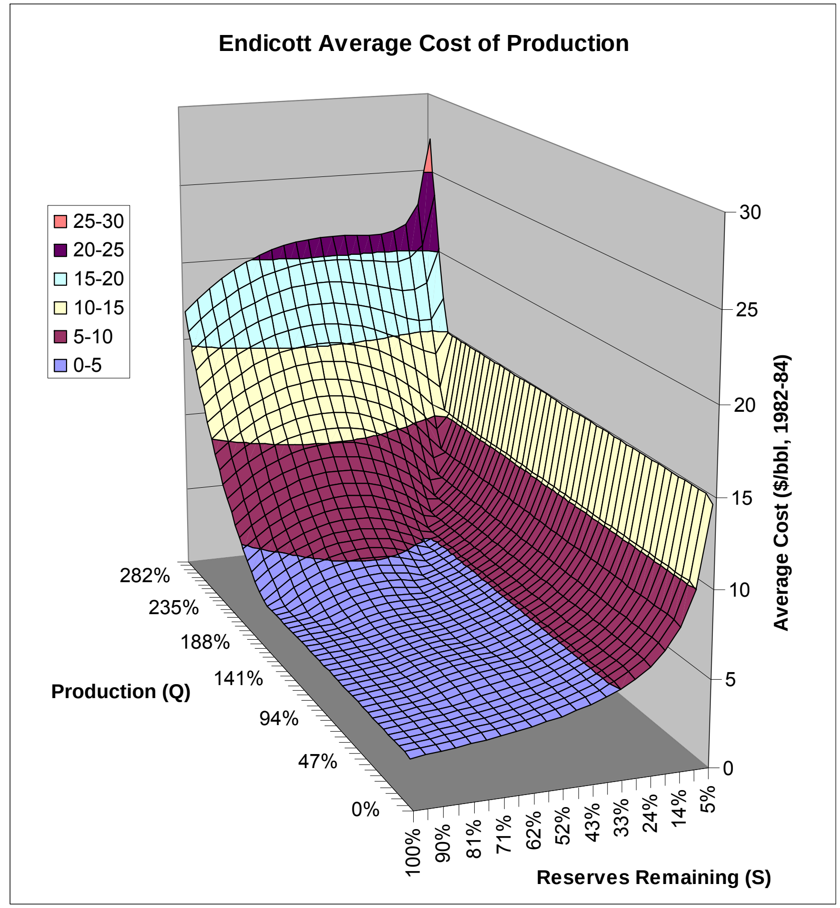
Appendix F: Composite Cost Functions

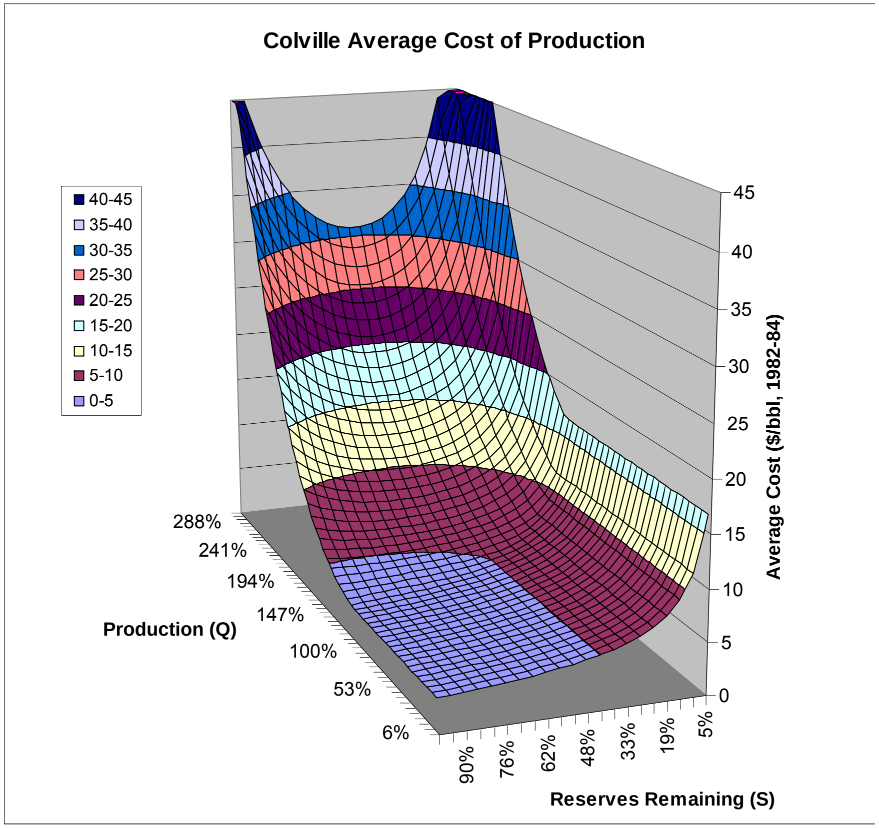
The composite cost functions by field, shown here as average cost ($/barrel, 1982-84 dollars) rather than total cost for ease of interpretation. The axis for reserves remaining extends from the original quantity of technically recoverable oil to zero. The production axis ranges from zero to three times the maximum historical rate of production. The vertical axis is the average cost of production.

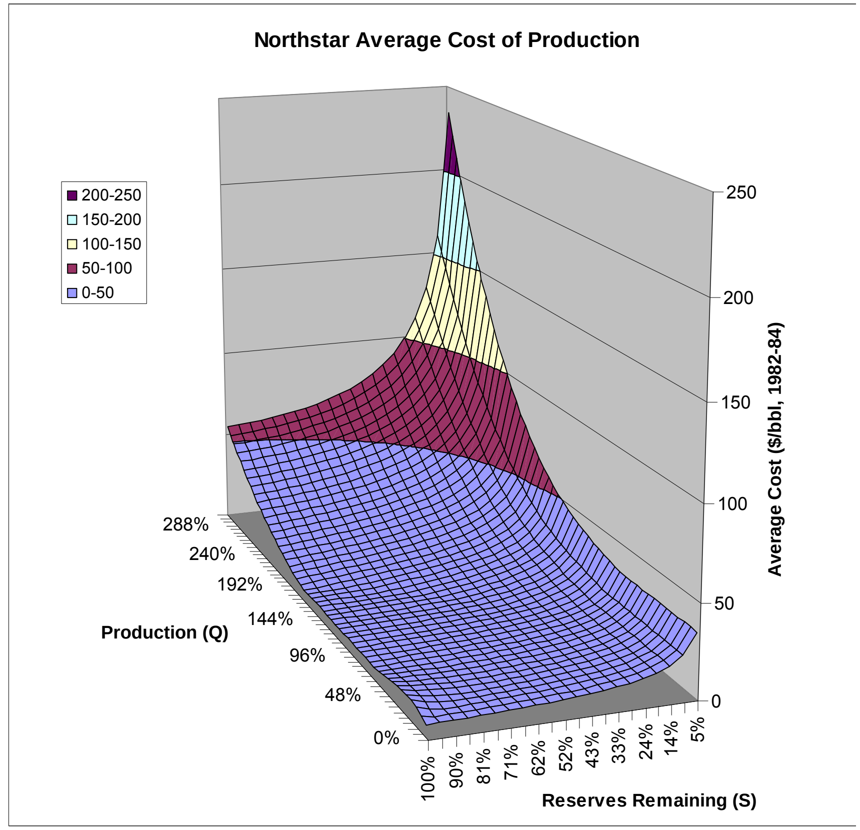





Milne Average Cost of Production
25-30
20-25
15-20
10-15
5-10
0-5
Average Cost ($/bbl, 1982-84)
0
5
10
15
20
25
30
Production (Q)
288%
241%
194%
147%
100%
53%
6%
Reserves Remaining (S)
100%
90%
81%
71%
62%
52%
43%
33%
24%
14%
5%


Endicott Average Cost of Production
25-30
20-25
15-20
10-15
5-10
0-5
Average Cost ($/bbl, 1982-84)
0
5
10
15
20
25
30
Production (Q)
282%
235%
188%
141%
94%
47%
0%
Reserves Remaining (S)
100%
90%
81%
71%
62%
52%
43%
33%
24%
14%
5%


Colville Average Cost of Production
40-45
35-40
30-35
25-30
20-25
15-20
10-15
5-10
0-5
Average Cost ($/bbl, 1982-84)
45
40
35
30
25
20
15
10
5
0
Production (Q)
288%
241%
194%
147%
100%
53%
6%
Reserves Remaining (S)
90%
76%
62%
48%
33%
19%
5%


Northstar Average Cost of Production
200-250
150-200
100-150
50-100
0-50
Average Cost ($/bbl, 1982-84)
250
200
150
100
50
0
Production (Q)
288%
240%
192%
144%
96%
48%
0%
Reserves Remaining (S)
100%
90%
81%
71%
62%
52%
43%
33%
24%
14%
5%

Appendix G: Modeling with the Ordinary Differential Boundary Value Problem approach

Modeling the dynamic oil production decision as a system of ordinary differential equations boundary value problem is based on the work of Harold Hotelling (1939) and Cynthia Lin (2008). We build upon this prior work by developing a model of perfect competition with exogenous price rather than the previous endogenous price models. We also push the limits of complexity in tax policy added to the model structure.

Exogenous price means the transversality condition of price equal to marginal cost, implying shadow price equal to zero, will not work. Additionally, since modeling with a finite horizon imposes an unrealistic a priori end to production, we use an infinite horizon. This essentially amounts to pinning down the front end of the production path with historical first-period production rather than the endpoint with a transversality condition. However, our inability to impose non-negativity constraints on production and reserves remaining in coding the model in Matlab ultimately precluded use of this method.[68]

The exogenous inputs for our modeling are summarized in Table 13. We use the composite cost function, which is comprised of a basic cost function that is scaled by a drilling cost scalar and wells scalar when appropriate, and include royalty and severance taxes. The unit operator's objective function (profit function) is given in equation 26.

26 Objective Function: $\pi(Q_{it}) = P(t)Q_{it}(1 - LR_{it} - ST_{it}ELF_i(Q_i(t)) - CCF(Q_i(t), S_i(t))$

Where Q_{it} is the quantity of production in barrels per month, LR_{it} is the average royalty rate for all leases in the production unit, ST_{it} is the severance tax rate, which is adjusted by the ELF factor that is a function of production rate, and CCF is the composite cost function that is a function of production rate and reserves remaining.

Then the producer's optimal control problem is to choose the production profile {Q(t)} to maximize the present discounted value of the entire stream of profits, given the initial stock S(0) and given the relationship between production Q(t) and the remaining stock S(t), and subject to the constraints that both production and stock are nonnegative. Mathematically, this is written as follows

27 $\text{Max}_{\{S(t)\}} \int_0^{\infty} [P(t)Q_{it}(1 - LR_{it} - ST_{it}ELF_i(Q_i(t))) - CCF(Q_i(t),S_i(t))]\, e^{-\rho t}\, dt$

s.t. $dS(t)/dt = -Q(t) \qquad : p(t)$

$Q(t) \geq 0$

$S(t) \geq 0$

$S(0) = S_o$

---

[68] Mathworks, the makers of Matlab, confirmed that non-negativity constraints cannot be imposed with the bvp4c procedure used to solve ordinary differential boundary value problems. Consequently, we were unable to impose constraints to prevent the production path from pushing reserves into negative territory, an illogical result (i.e. producing more oil than is available) that causes negative numbers raised to fractional exponents in the underlying equations (i.e., imaginary numbers).

Where p(t) is a multiplier associated with the equation of motion for the remaining stock S(t). In other words, p(t) is the value of relaxing this constraint; if the quantity of stock remaining in the ground is increased by one unit (i.e., one barrel is not pumped in the current period), then the value of this change is exactly the shadow price of the reserve or p(t).

The three first-order conditions (FOC) for dynamic optimality (from the Maximum Principle) are the following.

1.) Static optimality in the current period implies price equal to marginal cost for perfect competition (i.e., price takers). Alternatively, the shadow price p(t) must equal price minus marginal cost of production.

$$p(t) = P(t) - MC$$

$$p(t) = P(t) - \frac{\partial CCF(S(t), Q(t))}{\partial Q} - P(t)(LR_i + ST_i ELF(Q_i(t))) - ST_i Q_{it} P(t) \frac{\partial ELF}{\partial Q}$$

28 $$p(t) = P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - ST_i Q_{it} P(t) \frac{\partial ELF}{\partial Q} - \frac{\partial CCF}{\partial Q}$$

Note, the term ($LR_i + ST_i ELF(Q_i(t))$) is an approximation only of "total government take" (a percentage) since we do not include state property tax, state corporate income tax, or federal corporate income tax.

2.) Evolution of the shadow price over time, to ensure inter-period optimality over all finite sub-periods.[69]

29 $$\frac{dp(t)}{dt} = \frac{\partial CCF(S(t), Q(t))}{\partial S} + \rho p(t) = \frac{\partial CCF}{\partial S} + \rho [P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - \frac{\partial CCF}{\partial Q}]$$

3.) The transversality condition, required for optimality over an infinite time horizon.[70]

30 $$\lim_{t \to \infty} p(t) S(t) e^{-\rho t} = 0$$

Rewriting the problem as a boundary value problem with differential equations for Q_{it} (instead of P_{it}) and S_{it} proceeds as follows. Following the methodology from Lin (2008), the Hotelling problem can be reformulated into the following ordinary differential equation boundary value problem.

Step 1: combine FOC 1 and 2 (equations 28 and 29)

$$p(t) = P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - \frac{\partial CCF}{\partial Q} - ST_i Q_{it} P(t) \frac{\partial ELF}{\partial Q}$$ (from FOC 1, equation 28)

---

[69] There is a sign change when modeling with reserves remaining (S) rather than cumulative stock extracted (X). Since the change in cumulative stock extracted (X) is the negative of the change in reserves remaining (i.e., a 1-unit increase in X equals a 1-unit decrease in S), then dX/dS = -1. Then dCCF(X)/dS = (dCCF/dX)(dX/dS) = -(dCCF/dX) and dp(t)/dt = dCCF/dS + ρp(t) (without a negative sign on the dC/dS).

[70] The limit goes to zero because the unit operator wants to fully monetize the resource (i.e., the present discounted value of the shadow price times remaining resources is zero).

$$p(t) = P(t) - P(t)LR_i - P(t)ST_iELF(Q(t)) - \frac{\partial C}{\partial Q} - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q}$$

$$\frac{dp(t)}{dt} = \frac{d}{dt}P(t) - \frac{d}{dt}(P(t)LR_i) - \frac{d}{dt}(P(t)ST_iELF(Q_i(t))) - \frac{d}{dt}\frac{\partial C}{\partial Q} - \frac{d}{dt}[ST_iQ_i(t)P(t)\frac{\partial ELF}{\partial Q}]$$

$$\frac{dp(t)}{dt} = \frac{d}{dt}P(t) - \frac{d}{dt}(P(t)LR_i) - \frac{d}{dt}(P(t)ST_iELF(Q_i(t))) - \frac{d}{dt}\frac{\partial CCF}{\partial Q}$$
$$- [ST_i\frac{\partial Q}{\partial t}P(t)\frac{\partial ELF}{\partial Q} + ST_iQ_i(t)\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} + ST_iQ_i(t)P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q}]$$

31 $$\frac{dp(t)}{dt} = \frac{d}{dt}P(t) - LR_i\frac{d}{dt}P(t) - [ST_iP(t)\frac{d}{dt}(ELF(Q_i(t))) + ELF(Q_i(t))ST_i\frac{d}{dt}P(t)]$$
$$- \frac{d}{dt}\frac{\partial C}{\partial Q} - [ST_i\frac{\partial Q}{\partial t}P(t)\frac{\partial ELF}{\partial Q} + ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} + ST_iQ_{it}P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q}]$$

The following derivatives are known (see Table 14)

$$\frac{d}{dt}(ELF(Q_i(t))) = \frac{\partial ELF}{\partial Q}\frac{\partial Q}{\partial t}$$

$$\frac{d}{dt}\frac{\partial CCF}{\partial Q} = \frac{d^2CCF}{dQ^2}\frac{\partial Q}{\partial t} + \frac{d^2CCF}{dSdQ}\frac{\partial S}{\partial t}$$

Then, the previous equation for $\frac{dp(t)}{dt}$ (equation 31) can be re-written as follows.

32 $$\frac{dp(t)}{dt} = (1-LR_i)\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{\partial Q}{\partial t} - ELF(Q_i(t))ST_i\frac{d}{dt}P(t) - \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t}$$
$$- ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{\partial Q}{\partial t} - ST_iQ_i(t)\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_i(t)P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q}$$

Now, we substitute equation 32 into the left side of FOC 2 (equation 29)

$$(1-LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{\partial Q}{\partial t} - \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t} - ST_i\,P(t)\frac{\partial ELF}{\partial Q}\frac{\partial Q}{\partial t}$$
$$- ST_iQ_i(t)\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_i(t)P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q} = \frac{\partial C}{\partial S} + \rho[P(t)(1-LR_i-ST_iELF(Q_i(t))) - \frac{\partial C}{\partial Q}]$$

However, before using algebra to isolate $\frac{\partial Q}{\partial t}$ on the left side, it is necessary to write out the entire expression for $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ since it contains $\frac{\partial Q}{\partial t}$ terms (see appendix H). We then expand the expression above in appendix I to derive the following expression for $\frac{\partial Q}{\partial t}$.

33 $$\frac{\partial Q}{\partial t} = \frac{KA - LD + E}{H + IA + GB + FC + JD}$$

Where E = $(1-LR_i-ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t} - \frac{\partial C}{\partial S}$

$- \rho[P(t)(1-LR_i-ST_iELF(Q_i(t))) - \frac{\partial C}{\partial Q}]$

and all other letters in the equation are as given in appendix I, and remain consistent across cost specifications and constant/variable discount rates. Equation 33 is one of the differential equations in our boundary value problem; it contains the information from FOC 1 and 2 (equations 28 and 29).

Step 2: combine FOC 1 and 3 (equations 28 and 30), making sure the limit contains a Q term so the boundary conditions will pin it down.

FOC 1: $p(t) = P(t)(1-LR_i - ST_iELF(Q_i(t))) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}$

FOC 3: $\lim_{t \to \infty} p(t)S(t)e^{-\rho t} = 0$

34 $$\lim_{t \to \infty} [P(t)(1 - LR_i - ST_iELF(Q_i(t))) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}]S(t)e^{-\rho t} = 0$$

Equation 34 is one boundary condition in our boundary value problem; it contains the information from FOC 1 and 3, and the term $\frac{\partial C}{\partial Q}$ does contain a Q term.

Step 3: Define the second differential equation and second boundary condition from the maximization constraints.

The second differential equation comes from the fact that the rate of change in reserves remaining (S(t)) is equal to the negative rate of production (Q(t)). Thus, we have,

35 $$\frac{d}{dt}S(t) = -Q(t)$$

The second boundary condition comes directly from the constraints to which the producer's maximization problem is subject. That is,

36 $$S(0) = S_o$$

Finally, the solution to the boundary value problem specified by the two differential equations (equations 33 and 35) and two boundary conditions (equations 34 and 36) is equivalent to the original producer's optimal control problem. The boundary value problem is solved with the software package Matlab with the bvp4c routine. The derivations necessary for this modeling are summarized in Table 14.

Table 13: Exogenous Inputs to Modeling

Functions that are consistent across all fields

Estimated Functions

Price$_t$ = $P(t) = c_4 YR^2 + c_5 YR + c_6$

$c_4 = 0.0517781$ $c_5 = -206.4746$ $c_6 = 205{,}846.7$

Baseline Total Cost

$= BC_i = C(Q_i(t), S_i(t)) = c_1 Q_i^{c2} S_i^{c3}$

$c_1 = 178187.2068$ $c_2 = 1.000529$ $c_3 = -0.548916$

API Drilling Cost Scalar

$DCS = c_7 + c_8 YrIL + c_9 YrIL^2 + c_{10} YrIL^3 + c_{11} YrIL^4 + c_{12} YrIL^5 + c_{13} YrIL^6$

$c_7 = 1.413501$ $c_8 = -0.5839932$ $c_9 = 0.161024$ $c_{10} = -0.0175783$

$c_{11} = 0.0008877$ $c_{12} = -0.0000211$ $c_{13} = 0.000000192$

Dampened Drilling Cost Scalar

DDCS = 1+(DCS-1)/Dmp

Explanations

Price = wellhead value, 1982-84 $/bbl
Yr = year (date, e.g., 1982)
Total facilities cost of production
Total cost ($, 1982-84), ($/bbl)*Q

Applied only from 1969 to 2004
Indexed Year = YRI = (Year – 1968)
Optional lag: Lag in years (e.g., 1)
Indexed & Lagged Year
= YRIL = (Year – 1968 – lag)
The Dampener (Dmp) reduces the magnitude of scalar deviations around 1 (e.g., dampener = 2 cuts magnitude in half).

Field-Specific Elements

Estimated Functions

Historical Wells = $HW = c_{14} + c_{15}{*}QI + c_{16}{*}SI$

Badami (BHW):	$c_{14} = 4.235744$	$c_{15} = 8.210379$	$c_{16} = 51.36989$
Colville (CHW):	$c_{14} = 83.06196$	$c_{15} = 1.718612$	$c_{16} = -151.6971$
Endicott (EHW):	$c_{14} = 62.86919$	$c_{15} = 9.693985$	$c_{16} = -107.5757$
Kuparuk (KHW):	$c_{14} = 1882.539$	$c_{15} = 18.81515$	$c_{16} = -176.0778$
Milne (MHW):	$c_{14} = 238.7935$	$c_{15} = 49.09711$	$c_{16} = -277.0513$
Northstar (NHW):	$c_{14} = 16.87012$	$c_{15} = 2.481977$	$c_{16} = -125.87012$
Prudhoe (PHW):	$c_{14} = 1087.458$	$c_{15} = 5.359837$	$c_{16} = -78.11036$

Explanations

"Historical Wells" functions define flat planes in the Q,S,Wells space (i.e., constant returns to scale)

Q Index = QI = Q/1,000,000
= million bbl/mo.
S Index = SI = S/1,000,000,000
= billlion bbl res. Rem.

Wells (W)

"Wells" functions define surfaces in the Q, S, Wells space with decreasing returns to scale.

BadamiWells = BW

$= c_{17} + c_{18}*QI + c_{19}*SI^2 + c_{20}*QI^2*SI^2 + c_{21}*QI^3$

$c_{17} = 2.823319$ $c_{18} = 48.87226$ $c_{19} = 5693.26$ $c_{20} = -2447862$ $c_{21} = 1616.327$

ColvilleWells = CW

$= c_{17} + c_{18}QI + c_{19}QI^2 + c_{20}QI^3 + c_{21}SI + c_{22}SI^2$

$c_{17} = 70.13352$ $c_{18} = 5.993816$ $c_{19} = -1.729683$ $c_{20} = 0.2007895$

$c_{21} = -83.5993$ $c_{22} = -105.38$

EndicottWells = EW

$= c_{17}+c_{18}QI*SI+c_{19}QI*SI^2+c_{20}QI+c_{21}QI^2+c_{22}QI^3+c_{23}SI$

$c_{17} = 66.16589$ $c_{18} = 179.3657$ $c_{19} = -214.7895$ $c_{20} = -7.839296$

$c_{21} = -9.65297$ $c_{22} = 1.21689$ $c_{23} = -114.1997$

KuparukWells = KW

$= c_{17} + c_{18}QI + c_{19}QI^2 + c_{20}QI^3 + c_{21}SI$

$c_{17} = 1345.958$ $c_{18} = 139.626$ $c_{19} = -12.91675$ $c_{20} = 0.3473245$ $c_{21} = -151.1711$

MilneWells = MW

$= c_{17} + c_{18}QI*SI + c_{19}QI^2*SI + c_{20}QI^3$

$c_{17} = -5.005079$ $c_{18} = 222.55$ $c_{19} = -210.2137$ $c_{20} = 62.8936$

NorthstarWells = NW

$= c_{17} + c_{18}QI + c_{19}QI^2 + c_{20}QI^3 + c_{21}QI*SI$

$c_{17} = 0.8447826$ $c_{18} = 21.46647$ $c_{19} = -7.329782$ $c_{20} = 1.265286$ $c_{21} = -81.79707$

PrudhoeWells = PW

$= c_{17} + c_{18}Q + c_{19}Q^2 + c_{20}Q^3 + c_{21}S + c_{22}S^2 + c_{23}S^3$

$c_{17} = 31.68469$ $c_{18} = 0.0000143$ $c_{19} = -4.99*10\text{^}(-13)$ $c_{20} = 5.75*10\text{^}(-21)$

$c_{21} = 3.69*10\text{^}(-7)$ $c_{22} = -5.14*10\text{^}(-17)$ $c_{23} = 1.78*10\text{^}(-27)$

Note the Prudhoe Wells function was estimated with Q and S rather than the indexed variables QI and SI.

Table 14: Derivations

Derivations that are consistent across all fields

Derivations

Wells Scalar (WS)

WS = 1 if (HW*DRTS_M) > W
WS = W/(HW*DRTS_M) otherwise

Badami = BWS, Colville = CWS, Endicott = EWS, Kuparuk = KWS, Milne = MWS, Northstar = NWS, Prudhoe = PWS

Composite Cost Function (CCF): Variant 1: CCF = BC

- The year is after 2004, so the drilling cost scalar is not used
- The HW*DRTS_M > W, so the WS function = 1 (omitted)

$$\frac{\partial CCF(Q(t),S(t))}{\partial Q} = \frac{\partial CCF}{\partial Q} = c_1 c_2 Q_i^{\,c_2-1} S_i^{\,c_3}$$

$$\frac{\partial CCF(Q(t),S(t))}{\partial S} = \frac{\partial CCF}{\partial S} = c_1 c_3 Q_i^{\,c_2} S_i^{\,c_3-1}$$

$$\frac{d}{dt}\frac{\partial CCF}{\partial Q} = \frac{\partial \frac{\partial CCF}{\partial Q}}{\partial Q}\frac{\partial Q}{\partial t} + \frac{\partial \frac{\partial CCF}{\partial Q}}{\partial S}\frac{\partial S}{\partial t} = \frac{d^2 CCF}{dQ^2}\frac{\partial Q}{\partial t} + \frac{d^2 CCF}{dSdQ}\frac{\partial S}{\partial t}$$

$$\frac{d^2 CCF}{dQ^2} = (c_2 - 1) c_1 c_2 Q_i^{\,c_2-2} S_i^{\,c_3} = \frac{(c_2-1)\frac{\partial CCF}{\partial Q}}{Q}$$

$$\frac{d^2 CCF}{dSdQ} = c_1 c_2 c_3 Q_i^{\,(c_2-1)} S_i^{\,(c_3-1)} = \frac{c_2 \frac{\partial CCF}{\partial S}}{Q}$$

Notes and Explanations

This definition becomes field-specific since the HW and W functions are field-specific.

DRTS_M shifts the historical wells plan up or down, changing the margin above (or below) the historical number of wells at which decreasing returns to scale set in (i.e., when the W function exceeds the HW function).

The following Derivatives remain true for all four cost function variations:

$$\frac{dS}{dt} = -Q\,,\quad \frac{d}{dt}P(t) = 2c_4 YR + c_5$$

Composite Cost Function (CCF): Variant 2: CCF = BC * DDCS

- The year is pre-2005, so the drilling cost scalar is used
- The HW*DRTS_M > W, so the WS function = 1 (omitted)

$$\frac{\partial CCF}{\partial Q} = \frac{\partial BC}{\partial Q} DDCS + BC * \frac{\partial DDCS}{\partial Q} = \frac{\partial BC}{\partial Q} DDCS$$

$$\frac{\partial CCF}{\partial S} = \frac{\partial BC}{\partial S} DDCS + BC * \frac{\partial DDCS}{\partial S} = \frac{\partial BC}{\partial S} DDCS$$

$$\frac{d}{dt}\frac{\partial CCF}{\partial Q} = \frac{d^2 CCF}{dQ^2}\frac{\partial Q}{\partial t} + \frac{d^2 CCF}{dSdQ}\frac{\partial S}{\partial t} + \frac{\partial^2 CCF}{\partial YrIL \partial Q}\frac{\partial YrIL}{\partial t}$$

$$\frac{d^2 CCF}{dQ^2} = \frac{\partial}{\partial Q}(\frac{\partial BC}{\partial Q} DDCS) = \frac{\partial^2 BC}{\partial Q^2} DDCS + \frac{\partial BC}{\partial Q}\frac{\partial DDCS}{\partial Q} = \frac{\partial^2 BC}{\partial Q^2} DDCS$$

$$\frac{d^2 CCF}{dSdQ} = \frac{\partial}{\partial S}(\frac{\partial BC}{\partial Q} DDCS) = \frac{\partial^2 BC}{\partial S \partial Q} DDCS + \frac{\partial BC}{\partial Q}\frac{\partial DDCS}{\partial S} = \frac{\partial^2 BC}{\partial S \partial Q} DDCS$$

$$\frac{d^2 CCF}{dYrILdQ} = \frac{\partial^2 BC}{\partial YrIL \partial Q} DDCS + \frac{\partial DDCS}{\partial YrIL}\frac{\partial BC}{\partial Q} = \frac{\partial DDCS}{\partial YrIL}\frac{\partial BC}{\partial Q}$$

$$\frac{\partial DDCS}{\partial YrIL} = (\frac{1}{Dmp})(c_8 + 2c_9 YrIL + 3c_{10} YrIL^2 + 4c_{11} YrIL^3 + 5c_{12} YrIL^4 + 6c_{13} YrIL^5)$$

$$\frac{\partial YrIL}{\partial t} = 0.08333$$

$$\frac{\partial DDCS}{\partial Q} = 0$$

$$\frac{\partial DDCS}{\partial S} = 0$$

The term $\frac{\partial^2 CCF}{\partial YrIL \partial Q}\frac{\partial YrIL}{\partial t}$ is included because the DDCS function has the variable YrIL.

$$\frac{\partial DDCS}{\partial Q} = 0, \quad \frac{\partial DDCS}{\partial S} = 0$$

$$\frac{\partial^2 BC}{\partial YrIL \partial Q} = 0$$

YrIL is measured in years and *t* is measured in months, so the change in year for a change in month is (1/12) = 0.08333.

Composite Cost Function (CCF): Variant 3: CCF = BC * WS

- The year is after 2004, so the drilling cost scalar is not used
- The HW*DRTS_M < W, so the WS function <u>is</u> used

$$\frac{\partial CCF}{\partial Q} = \frac{\partial BC}{\partial Q} WS + BC * \frac{\partial WS}{\partial Q}$$

$$\frac{\partial CCF}{\partial S} = \frac{\partial BC}{\partial S} WS + BC * \frac{\partial WS}{\partial S}$$

$$\frac{d}{dt}\frac{\partial CCF}{\partial Q} = \frac{d^2 CCF}{dQ^2}\frac{\partial Q}{\partial t} + \frac{d^2 CCF}{dSdQ}\frac{\partial S}{\partial t} \quad \frac{d^2 CCF}{dQ^2} = \frac{\partial}{\partial Q}\left(\frac{\partial BC}{\partial Q} DCS + BC\frac{\partial WS}{\partial Q}\right)$$

$$= \frac{\partial^2 BC}{\partial Q^2} WS + \frac{\partial BC}{\partial Q}\frac{\partial WS}{\partial Q} + \frac{\partial BC}{\partial Q}\frac{\partial WS}{\partial Q} + BC\frac{\partial^2 WS}{\partial Q^2}$$

$$= \frac{\partial^2 BC}{\partial Q^2} WS + 2\frac{\partial BC}{\partial Q}\frac{\partial WS}{\partial Q} + BC\frac{\partial^2 WS}{\partial Q^2}$$

$$\frac{d^2 CCF}{dSdQ} = \frac{\partial}{\partial S}\left(\frac{\partial BC}{\partial Q} WS + BC\frac{\partial WS}{\partial Q}\right)$$

$$= \frac{\partial^2 BC}{\partial S \partial Q} WS + \frac{\partial BC}{\partial Q}\frac{\partial WS}{\partial S} + \frac{\partial BC}{\partial S}\frac{\partial WS}{\partial Q} + BC\frac{\partial^2 WS}{\partial S \partial Q}$$

Where,

$WS, \frac{\partial WS}{\partial Q}, \frac{\partial WS}{\partial S}, \frac{\partial^2 WS}{\partial Q^2}, and\ \frac{\partial^2 WS}{\partial S \partial Q}$

are field-specific

Composite Cost Function (CCF), Variant 4: CCF=BC*DDCS*WS

- The year is pre-2005, so the drilling cost scalar is used
- The HW*DRTS_M < W, so the WS function is used

$$\frac{\partial CCF}{\partial Q} = \frac{\partial BC}{\partial Q} \cdot DDCS \cdot WS + BC \cdot \frac{\partial DDCS}{\partial Q} \cdot WS + BC \cdot DDCS \cdot \frac{\partial WS}{\partial Q}$$

$$= \frac{\partial BC}{\partial Q} \cdot DDCS \cdot WS + BC \cdot DDCS \cdot \frac{\partial WS}{\partial Q}$$

$$\frac{\partial CCF}{\partial S} = \frac{\partial BC}{\partial S} \cdot DDCS \cdot WS + BC \cdot \frac{\partial DDCS}{\partial S} \cdot WS + BC \cdot DDCS \cdot \frac{\partial WS}{\partial S}$$

$$= \frac{\partial BC}{\partial S} \cdot DDCS \cdot WS + BC \cdot DDCS \cdot \frac{\partial WS}{\partial S}$$

$$\frac{d}{dt}\frac{\partial CCF}{\partial Q} = \frac{d^2 CCF}{dQ^2}\frac{\partial Q}{\partial t} + \frac{d^2 CCF}{dSdQ}\frac{\partial S}{\partial t} + \frac{\partial^2 CCF}{\partial YrIL \partial Q}\frac{\partial YrIL}{\partial t}$$

$$\frac{d^2 CCF}{dQ^2} = \frac{\partial}{\partial Q}\left(\frac{\partial BC}{\partial Q} \cdot DDCS \cdot WS + BC \cdot DDCS \cdot \frac{\partial WS}{\partial Q}\right)$$

$$= \frac{\partial^2 BC}{\partial Q^2} \cdot DDCS \cdot WS + \frac{\partial BC}{\partial Q}\frac{\partial DDCS}{\partial Q} \cdot WS + \frac{\partial BC}{\partial Q} \cdot DDCS \cdot \frac{\partial WS}{\partial Q}$$

$$+ \frac{\partial BC}{\partial Q} \cdot DDCS \cdot \frac{\partial WS}{\partial Q} + BC \cdot \frac{\partial DDCS}{\partial Q}\frac{\partial WS}{\partial Q} + BC \cdot DDCS \cdot \frac{\partial^2 WS}{\partial Q^2}$$

$$= \frac{\partial^2 BC}{\partial Q^2} \cdot DDCS \cdot WS + 2\frac{\partial BC}{\partial Q} \cdot DDCS \cdot \frac{\partial WS}{\partial Q} + BC \cdot DDCS \cdot \frac{\partial^2 WS}{\partial Q^2}$$

$$\frac{d^2 CCF}{dSdQ} = \frac{\partial}{\partial S}\left(\frac{\partial BC}{\partial Q} \cdot DDCS \cdot WS + BC \cdot DDCS \cdot \frac{\partial WS}{\partial Q}\right)$$

$$\frac{\partial DDCS}{\partial Q} = 0$$

$$\frac{\partial DDCS}{\partial S} = 0$$

The term $\frac{\partial^2 CCF}{\partial YrIL \partial Q}\frac{\partial YrIL}{\partial t}$ is included because the DDCS function has the variable YrIL.

$$\frac{\partial DDCS}{\partial Q} = 0$$

$$= \frac{\partial^2 BC}{\partial S \partial Q} \cdot DDCS \cdot WS + \frac{\partial BC}{\partial Q}\frac{\partial DDCS}{\partial S} \cdot WS + \frac{\partial BC}{\partial Q} \cdot DDCS \cdot \frac{\partial WS}{\partial S}$$

$$+ \frac{\partial BC}{\partial S} \cdot DDCS \cdot \frac{\partial WS}{\partial Q} + BC \cdot \frac{\partial DDCS}{\partial S}\frac{\partial WS}{\partial Q} + BC \cdot DDCS \cdot \frac{\partial^2 WS}{\partial S \partial Q}$$

$$= \frac{\partial^2 BC}{\partial S \partial Q} DDCS \cdot WS + \frac{\partial BC}{\partial Q}\frac{\partial WS}{\partial S} DDCS + \frac{\partial BC}{\partial S}\frac{\partial WS}{\partial Q} DDCS$$

$$+ \frac{\partial^2 WS}{\partial S \partial Q} BC \cdot DDCS$$

$$\frac{d^2 CCF}{dYrILdQ} = \frac{\partial BC}{\partial Q}\frac{\partial DDCS}{\partial YrIL} WS + BC \frac{\partial DDCS}{\partial YrIL}\frac{\partial WS}{\partial Q}$$

$$\frac{\partial DDCS}{\partial YrIL} = (\frac{1}{Dmp})(c_8 + 2c_9 YrIL + 3c_{10} YrIL^2 + 4c_{11} YrIL^3 + 5c_{12} YrIL^4 + 6c_{13} YrIL^5)$$

$$\frac{\partial YrIL}{\partial t} = 0.08333$$

$$\frac{\partial DDCS}{\partial S} = 0$$

where $WS, \frac{\partial WS}{\partial Q}, \frac{\partial WS}{\partial S}, \frac{\partial^2 WS}{\partial Q^2}, and \frac{\partial^2 WS}{\partial S \partial Q}$ are field-specific

$$\frac{\partial^2 BC}{\partial YrIL \partial Q} = 0, \ \frac{\partial WS}{\partial YrIL} = 0, \ \frac{\partial BC}{\partial YrIL} = 0, \ \frac{\partial^2 WS}{\partial YrIL \partial Q} = 0$$

YrIL is measured in years and *t* is measured in months, so the change in year for a change in month is (1/12) = 0.08333

For all fields, the following functional forms are consistent

HW = $c_{14} + c_{15}QI + c_{16}SI = c_{14} + (c_{15}/10^6)Q + (c_{16}/10^9)S$ ➔ $\frac{\partial HW}{\partial Q} = \frac{c_{15}}{10^6}$ and $\frac{\partial HW}{\partial S} = \frac{c_{16}}{10^9}$ and $\frac{\partial^2 HW}{\partial Q^2} = 0$ and $\frac{\partial^2 HW}{\partial S \partial Q} = 0$

WS = 1 if (HW*DRTS_M) > W
= W/(HW*DRTS_M) otherwise

Thus, if WS = 1, $\frac{\partial WS}{\partial Q} = \frac{\partial WS}{\partial S} = \frac{\partial^2 WS}{\partial Q^2} = \frac{\partial^2 WS}{\partial S \partial Q} = 0$ and Otherwise, $\frac{\partial WS}{\partial Q} = (\frac{1}{DRTS\_M})[\frac{\partial W}{\partial Q} HW^{-1} - \frac{\partial HW}{\partial Q} \cdot W \cdot HW^{-2}]$

$$\frac{\partial WS}{\partial S} = (\frac{1}{DRTS\_M})[\frac{\partial W}{\partial S} HW^{-1} - \frac{\partial HW}{\partial S} \cdot W \cdot HW^{-2}]$$

$$\frac{\partial^2 WS}{\partial Q^2} = (\frac{1}{DRTS\_M})[\frac{\partial^2 W}{\partial Q^2} HW^{-1} - 2\frac{\partial W}{\partial Q}\frac{\partial HW}{\partial Q} HW^{-2} + 2 \cdot W \cdot HW^{-3}(\frac{\partial HW}{\partial Q})^2]$$

$$\frac{\partial^2 WS}{\partial S \partial Q} = (\frac{1}{DRTS\_M})[\frac{\partial^2 W}{\partial S \partial Q} HW^{-1} - \frac{\partial W}{\partial Q}\frac{\partial HW}{\partial S} HW^{-2} - \frac{\partial W}{\partial S}\frac{\partial HW}{\partial Q} HW^{-2} + 2 \cdot W \cdot HW^{-3}\frac{\partial HW}{\partial Q}\frac{\partial HW}{\partial S}]$$

The economic limit factor (ELF) formula in Alaska Statute (AS 43.55.150) presented unique challenges for finding derivatives.

$$\text{ELF}(Q_i(t)) = (1 - \frac{300 * WELLS_{it} * DAYS_t}{Q_{it}})^{(\frac{150,000}{Q_{it}/DAYS_t})^{(\frac{460*WELLS_{it}*DAYS_t}{300*WELLS_{it}*DAYS_t})}} = (1 - \frac{300 * WELLS_{it} * DAYS_t}{Q_{it}})^{(\frac{150,000}{Q_{it}/DAYS_t})^{1.5333}}$$

$$\frac{\partial ELF(Q_i(t))}{\partial Q} = \frac{\partial ELF}{\partial Q} = [[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^v)(\frac{dv}{dQ})][\frac{\partial}{\partial Q} g(f(Q))][\frac{\partial}{\partial Q} f(Q)]$$ (See Appendix H)

$$\frac{d}{dt}\frac{\partial ELF}{\partial Q} = \{\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ})] + \frac{d}{dt}[(\ln u)(u^v)(\frac{dv}{dQ})]\}[\frac{\partial f(Q)}{\partial Q}] + \{\frac{d}{dt}[\frac{\partial f(Q)}{\partial Q}]\}[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^v)(\frac{dv}{dQ})]$$ (See Appendix H)

Supporting Field-Specific Derivations

Derivations

$BW = c_{17} + c_{18}*QI + c_{19}*SI^2 + c_{20}*QI^2*SI^2 + c_{21}*QI^3 = c_{17} + (c_{18}/10^6)Q + (c_{19}/10^{18})S^2 + (c_{20}/10^{30})Q^2S^2 + (c_{21}/10^{18})Q^3$

$$\frac{\partial BW}{\partial Q} = \frac{c_{18}}{10^6} + (\frac{2c_{20}}{10^{30}})QS^2 + (\frac{3c_{21}}{10^{18}})Q^2 \qquad \frac{\partial BW}{\partial S} = (\frac{2c_{19}}{10^{18}})S + (\frac{2c_{20}}{10^{30}})Q^2S$$

$$\frac{\partial^2 BW}{\partial Q^2} = (\frac{2c_{20}}{10^{30}})S^2 + (\frac{6c_{21}}{10^{18}})Q \qquad \frac{\partial^2 BW}{\partial S \partial Q} = (\frac{4c_{20}}{10^{30}})QS$$

$CW = c_{17} + c_{18}QI + c_{19}QI^2 + c_{20}QI^3 + c_{21}SI + c_{22}SI^2 = c_{17}+(c_{18}/10^6)Q+(c_{19}/10^{12})Q^2+(c_{20}/10^{18})Q^3+(c_{21}/10^9)S+(c_{22}/10^{18})S^2$

$$\frac{\partial CW}{\partial Q} = \frac{c_{18}}{10^6} + (\frac{2c_{19}}{10^{12}})Q + (\frac{3c_{20}}{10^{18}})Q^2 \qquad \frac{\partial CW}{\partial S} = (\frac{c_{21}}{10^9}) + (\frac{2c_{22}}{10^{18}})S$$

$$\frac{\partial^2 CW}{\partial Q^2} = (\frac{2c_{19}}{10^{12}}) + (\frac{6c_{20}}{10^{18}})Q \qquad \frac{\partial^2 CW}{\partial S \partial Q} = 0$$

$EW = c_{17}+c_{18}QI*SI+c_{19}QI*SI^2+c_{20}QI+c_{21}QI^2+c_{22}QI^3+c_{23}SI = c_{17} +(c_{18}/10^{15})QS+(c_{19}/10^{24})QS^2 +(c_{20}/10^6)Q+(c_{21}/10^{12})Q^2+(c_{22}/10^{18})Q^3 +(c_{23}/10^9)S$

$$\frac{\partial EW}{\partial Q} = (\frac{c_{18}}{10^{15}})S + (\frac{c_{19}}{10^{24}})S^2 + (\frac{c_{20}}{10^6}) + (\frac{2c_{21}}{10^{12}})Q + (\frac{3c_{22}}{10^{18}})Q^2 \qquad \frac{\partial EW}{\partial S} = (\frac{c_{18}}{10^{15}})Q + (\frac{2c_{19}}{10^{24}})QS + (\frac{c_{23}}{10^9})$$

$$\frac{\partial^2 EW}{\partial Q^2} = (\frac{2c_{21}}{10^{12}}) + (\frac{6c_{22}}{10^{18}})Q \qquad \frac{\partial^2 EW}{\partial S \partial Q} = (\frac{c_{18}}{10^{15}}) + (\frac{2c_{19}}{10^{24}})S$$

$KW = c_{17} + c_{18}QI + c_{19}QI^2 + c_{20}QI^3 + c_{21}SI = c_{17} + (c_{18}/10^6)Q + (c_{19}/10^{12})Q^2 + (c_{20}/10^{18})Q^3 + (c_{21}/10^9)S$

$$\frac{\partial KW}{\partial Q} = (\frac{c_{18}}{10^6}) + (\frac{2c_{19}}{10^{12}})Q + (\frac{3c_{20}}{10^{18}})Q^2 \qquad \frac{\partial KW}{\partial S} = (\frac{c_{21}}{10^9})$$

$$\frac{\partial^2 KW}{\partial Q^2} = (\frac{2c_{19}}{10^{12}}) + (\frac{6c_{20}}{10^{18}})Q \qquad \frac{\partial^2 KW}{\partial S \partial Q} = 0$$

MW = $c_{17} + c_{18}QI*SI + c_{19}QI^2*SI + c_{20}QI^3$ = $c_{17} + (c_{18}/10^{15})Q*S + (c_{19}/10^{21})Q^2*S + (c_{20}/10^{18})Q^3$

$$\frac{\partial MW}{\partial Q} = (\frac{c_{18}}{10^{15}})S + (\frac{2c_{19}}{10^{21}})QS + (\frac{3c_{20}}{10^{18}})Q^2 \qquad \frac{\partial MW}{\partial S} = (\frac{c_{18}}{10^{15}})Q + (\frac{c_{19}}{10^{21}})Q^2$$

$$\frac{\partial^2 MW}{\partial Q^2} = (\frac{2c_{19}}{10^{21}})S + (\frac{6c_{20}}{10^{18}})Q \qquad \frac{\partial^2 MW}{\partial S \partial Q} = (\frac{c_{18}}{10^{15}}) + (\frac{2c_{19}}{10^{21}})Q$$

NW = $c_{17} + c_{18}QI + c_{19}QI^2 + c_{20}QI^3 + c_{21}QI*SI$ = $c_{17} + (c_{18}/10^6)Q + (c_{19}/10^{12})Q^2 + (c_{20}/10^{18})Q^3 + (c_{21}/10^{15})Q*S$

$$\frac{\partial NW}{\partial Q} = (\frac{c_{18}}{10^6}) + (\frac{2c_{19}}{10^{12}})Q + (\frac{3c_{20}}{10^{18}})Q^2 + (\frac{c_{21}}{10^{15}})S \qquad \frac{\partial NW}{\partial S} = (\frac{c_{21}}{10^{15}})Q$$

$$\frac{\partial^2 NW}{\partial Q^2} = (\frac{2c_{19}}{10^{12}}) + (\frac{6c_{20}}{10^{18}})Q \qquad \frac{\partial^2 NW}{\partial S \partial Q} = (\frac{c_{21}}{10^{15}})$$

PW = $c_{17} + c_{18}Q + c_{19}Q^2 + c_{20}Q^3 + c_{21}S + c_{22}S^2 + c_{23}S^3$

$$\frac{\partial PW}{\partial Q} = c_{18} + 2c_{19}Q + 3c_{20}Q^2 \qquad \frac{\partial PW}{\partial S} = c_{21} + 2c_{22}S + 3c_{23}S^2$$

$$\frac{\partial^2 PW}{\partial Q^2} = 2c_{19} + 6c_{20}Q \qquad \frac{\partial^2 NW}{\partial S \partial Q} = 0$$

Appendix H: Derivations of ELF

In the ELF function, WELLS is a function of Q and S or X. Thus, we have

$$\text{ELF(Q,X)} = \left(1 - \frac{300 * WELLS(Q,X) * DAYS_t}{Q_{it}}\right)^{\left(\frac{150{,}000}{Q_{it}/DAYS_t}\right)^{1.5333}}$$

Estimate WELLS(Q,X) as WELLS = $c_7 Q^{c8} X^{c9}$

- nonlinear relationship between wells and Q since average well size (bbl/day) probably increases with Q since production is likely coming from a larger and/or more productive field where each well can produce more.
- Nonlinear relationship between wells and X since the need for additional wells to maintain production at some level likely increases nonlinearly as the field is depleted.
- Estimate with linear regression: Log(WELLS) = log(c_7) + $c_8$log(Q) + $c_9$log(X)

Then $\frac{\partial WELLS}{\partial Q} = c_7 c_8 Q^{c8-1} X^{c9}$

Let $\quad$ $f(Q) = \frac{WELLS}{Q} = (c_7 Q^{c8} X^{c9})/Q$

$$g(f(Q) = (1 - 300*DAYS*f(Q))$$

$$h(g(f(Q)) = [g(f(Q))]^{(150{,}000*DAYS*Q^{-1})^{1.5333}}$$

Chain Rule:

F = g(f(Q)) $\quad$ F' = g'(f(Q))f'(Q)

and

G = h(g(f(Q))) = ELF $\quad$ G' = h'(g(f(Q)))[g(f(Q))]'

then

$$G' = \frac{\partial ELF}{\partial Q} = h'(g(f(Q)))g'(f(Q))f'(Q)$$

Then,

$$f'(Q) = c_7(c_8-1)Q^{c8-2}X^{c9}$$

$$g'(f(Q)) = (1 - 300*DAYS*f(Q))' = 300*DAYS$$

$$h'(g(f(Q))) = \frac{\partial}{\partial Q}[g(f(Q))]^{(150{,}000*DAYS*Q^{-1})^{1.5333}}$$

if u, v are both functions of Q, then

$$\frac{\partial}{\partial Q}(u^{v}) = vu^{v-1}\frac{du}{dQ} + (\ln u)(u^{v})(\frac{dv}{dQ})$$

Let u = g(f(Q))

Let v = $(150{,}000*Days*Q^{-1})^{1.5333}$

Then $\frac{\partial u}{\partial Q} = 300 * DAYS$

And $\frac{\partial v}{\partial Q} = 1.5333(150{,}000 * DAYS * Q^{-1})^{0.5333}(-150{,}000 * DAYS * Q^{-2})$

Then $h'(g(f(Q))) = \frac{\partial}{\partial Q}(u^{v}) = vu^{v-1}\frac{du}{dQ} + (\ln u)(u^{v})(\frac{dv}{dQ})$

Thus, $\frac{\partial ELF}{\partial Q} = [[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^{v})(\frac{dv}{dQ})][\frac{\partial}{\partial Q}g(f(Q))][\frac{\partial}{\partial Q}f(Q)]$

Where $f(Q) = \frac{WELLS}{Q} = (c_7 Q^{c8} X^{c9})/Q = c_7 Q^{c8-1} X^{c9}$

$$\frac{\partial}{\partial Q}f(Q) = c_7(c_8 - 1)Q^{c_8 - 2}X^{c_9}$$

g(f(Q)) = (1 – 300*DAYS*f(Q))

$$\frac{\partial}{\partial Q}g(f(Q)) = 300 * DAYS$$

u = g(f(Q)) = (1 – 300*DAYS*f(Q))

v = $(150{,}000*Days*Q^{-1})^{1.5333}$

$$\frac{\partial u}{\partial Q} = 300 * DAYS$$

$$\frac{\partial v}{\partial Q} = 1.5333(150{,}000 * DAYS * Q^{-1})^{0.5333}(-150{,}000 * DAYS * Q^{-2})$$

Now we can calculate $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$. Since ELF is a function of Q and X (or S), each of which is a function of time, the derivative will be as follows.

$$\frac{d}{dt}\frac{\partial ELF}{\partial Q} =$$

$$\underbrace{\frac{d}{dt}\{[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^{v})(\frac{dv}{dQ})]\}}_{\text{Unknown \#1}}\underbrace{[\frac{\partial f(Q)}{\partial Q}]}_{\text{known}} + \underbrace{\frac{d}{dt}\{[\frac{\partial f(Q)}{\partial Q}]\}}_{\text{unknown \#2}}\underbrace{[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^{v})(\frac{dv}{dQ})]}_{\text{known}}$$

First, we derive two equations that will be needed later in the following derivations.

$$u = g(f(Q)) = (1 - 300*DAYS*f(Q(t))) = (1 - 300 * DAYS * c_7Q(t)^{c8-1}X(t)^{c9})$$

$$\frac{du}{dt} = -300 * DAYS * c_7(c_8 - 1)Q(t)^{c_8-2}X(t)^{c_9}\frac{dQ(t)}{dt} - 300 * DAYS * c_7Q(t)^{c_8-1}c_9X(t)^{c_9-1}\frac{dX(t)}{dt}$$

$$\frac{du}{dt} = [-300 * DAYS * c_7][(c_8 - 1)Q(t)^{c_8-2}X(t)^{c_9}\frac{dQ(t)}{dt} + c_9Q(t)^{c_8-1}X(t)^{c_9-1}\frac{dX(t)}{dt}$$

$$v = (150{,}000*Days*Q^{-1})^{1.5333}$$

$$\frac{dv}{dt} = 1.5333(150{,}000 * DAYS * Q(t)^{-1})^{0.5333}(-150{,}000 * DAYS * Q(t)^{-2})\frac{dQ(t)}{dt}$$

Taking the unknown portions of the $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ equation in turn, we have the following:

Unknown #1:

$$\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^{v})(\frac{dv}{dQ})] = \underbrace{\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ})]}_{\text{Unknown A}} + \underbrace{\frac{d}{dt}[(\ln u)(u^{v})(\frac{dv}{dQ})]}_{\text{Unknown B}}$$

Unknown A

$$\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ})] = (\frac{d}{dt}(vu^{v-1}))(\frac{du}{dQ}) + (\frac{d}{dt}(\frac{du}{dQ}))(vu^{v-1})$$

Unknown a | known | unknown b | known

Unknown a

$$\frac{d}{dt}(vu^{v-1}) = (v)(v-1)(u^{v-2})(\frac{du}{dt}) + (\ln(u))(u^{v-1})(\frac{dv}{dt})$$

Unknown b

$$\frac{du}{dQ} = 300 * DAYS \qquad => \qquad \frac{d}{dt}\frac{du}{dQ} = 0$$

Unknown B

$$\frac{d}{dt}[(\ln u)(u^{v})(\frac{dv}{dQ})] = (\frac{d}{dt}(\ln u))(u^{v})(\frac{dv}{dQ}) + (\ln u)(\frac{d}{dt}(u^{v}))(\frac{dv}{dQ}) + (\ln u)(u^{v})(\frac{d}{dt}\frac{dv}{dQ})$$

Unknown c | known | known | known | unknwn d | known | known | known | unknwn e

Unknown c

$$\frac{d}{dt}(\ln u) = \frac{1}{u}\frac{du}{dt}$$

Unknown d

$$\frac{d}{dt}(u^{v}) = vu^{v-1}\frac{du}{dt} + (\ln u)(u^{v})(\frac{dv}{dt})$$

Unknown e

$$\frac{dv}{dQ} = 1.533(150{,}000 * DAYS * Q(t)^{-1})^{0.533}(-150{,}000 * DAYS * Q(t)^{-2})$$

j(Q(t) | k(Q(t)

$$\frac{d}{dt}\frac{dv}{dQ} = 1.533[0.533(150{,}000 * DAYS * Q(t)^{-1})^{-0.466}(-150{,}000 * DAYS * Q(t)^{-2})(\frac{dQ(t)}{dt})(-150{,}000 * DAYS * Q(t)^{-2})$$
$$+ (300{,}000 * DAYS * Q(t)^{-3})(\frac{dQ(t)}{dt})(150{,}000 * DAYS * Q(t)^{-1})^{0.533}]$$

<u>Unknown C</u>

$$\frac{df(Q)}{dQ} = \mathrm{c}_7(\mathrm{c}_8 - 1)\mathrm{Q(t)}^{\mathrm{c}_8 - 2}\mathrm{X(t)}^{\mathrm{c}_9}$$

$$\frac{d}{dt}\frac{df(Q)}{dQ} = \mathrm{c}_7(\mathrm{c}_8 - 1)[(\mathrm{c}_8 - 2)(\mathrm{Q(t)}^{\mathrm{c}_8 - 3})(\mathrm{X(t)}^{\mathrm{c}_9})(\frac{dQ(t)}{dt}) + (c_9)(\mathrm{Q(t)}^{\mathrm{c}_8 - 2})(\mathrm{X(t)}^{\mathrm{c}_9 - 1})(\frac{dX(t)}{dt})$$

<u>**Unknown #2:**</u>

This unknown is the same as unknown 1-C, so we have the following,

$$\frac{d}{dt}\frac{df(Q)}{dQ} = \mathrm{c}_7(\mathrm{c}_8 - 1)[(\mathrm{c}_8 - 2)(\mathrm{Q(t)}^{\mathrm{c}_8 - 3})(\mathrm{X(t)}^{\mathrm{c}_9})(\frac{dQ(t)}{dt}) + (c_9)(\mathrm{Q(t)}^{\mathrm{c}_8 - 2})(\mathrm{X(t)}^{\mathrm{c}_9 - 1})(\frac{dX(t)}{dt})$$

And now we have $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ entirely in terms of known functions

$$\frac{d}{dt}\frac{\partial ELF}{\partial Q} = \{\underbrace{\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ})]}_{\text{Known (1, A.a, A.b)}} + \underbrace{\frac{d}{dt}[(\ln u)(u^{v})(\frac{dv}{dQ})]}_{\text{Known (1, B.c, B.d, B.e)}}\}\underbrace{[\frac{\partial f(Q)}{\partial Q}]}_{\text{known}} + \{\underbrace{\frac{d}{dt}[\frac{\partial f(Q)}{\partial Q}]}_{\text{Known (2)}}\}\underbrace{[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^{v})(\frac{dv}{dQ})]}_{\text{known}}$$

Appendix I: Derivation of Step 1 of Boundary Value Problem for Model Specifications including Royalty and Severance Tax

The term $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ contains the term $\frac{dQ}{dt}$. Thus, we need to expand $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ when doing the algebra to isolate $\frac{dQ}{dt}$ in step 1 of the Boundary Value Problem. In this appendix, we do this expansion for the variable discount rate case modeled with reserves remaining (S). Fortunately, all lettered terms other than "E" will remain the same for all other model specifications (i.e., constant discount rate and modeling with cumulative stock extracted (X) or no stock effects (no S or X). The term "E" unique to each model specification is defined in the main text of this report.

The lettered expressions u and v, and function f(Q) are defined in the appendix showing the derivation of $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$.

Expanding the $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ term in the equation we last had in the main text before coming to this appendix, we have:

$$(1\text{-}LR_i\text{-}ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{dQ}{dt} - ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial^2 C}{\partial Q^2}\frac{dQ}{dt} - \frac{\partial^2 C}{\partial S\partial Q}\frac{dS}{dt}$$

$$- ST_iQ_{it}P(t)\Big[\Big\{[(v)(v-1)(u^{v-2})\frac{du}{dt} + (\ln u)(u^{v-1})\frac{dv}{dt}][\frac{du}{dQ}] + (0)(vu^{v-1}) + [(\frac{1}{u})(\frac{du}{dt})(u^{v})(\frac{dv}{dQ}) + (\ln u)(vu^{v-1}\frac{du}{dt} + (\ln u)(u^{v})\frac{dv}{dt})(\frac{dv}{dQ})$$

$$+ (\ln u)(u^{v})(\frac{d}{dt}\frac{dv}{dQ})]\Big\}\Big\{\frac{df(Q)}{dQ}\Big\} + \Big\{\frac{d}{dt}\frac{df(Q)}{dQ}\Big\}\Big\{(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^{v})(\frac{dv}{dQ})\Big\}\Big] = \frac{dC}{dS} + (P(t) - \frac{dC}{dQ})(\frac{dr(t)}{dt}t + r(t))$$

Isolating terms with $\frac{dQ}{dt}$ on the left side yields:

$$ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{dQ}{dt} + ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q} + \frac{\partial^2 C}{\partial Q^2}\frac{dQ}{dt} + ST_iQ_{it}P(t)(v)(v\text{-}1)(u^{v-2})\frac{du}{dt}\frac{du}{dQ}\frac{df(Q)}{dQ} + ST_iQ_{it}P(t)(\ln u)(u^{v-1})\frac{dv}{dt}\frac{du}{dQ}\frac{df(Q)}{dQ}$$

$$+ ST_iQ_{it}P(t)(\frac{1}{u})(\frac{du}{dt})(u^v)(\frac{dv}{dQ})\frac{df(Q)}{dQ} + ST_iQ_{it}P(t)(\ln u)(vu^{v-1})\frac{du}{dt}\frac{dv}{dQ}\frac{df(Q)}{dQ} + ST_iQ_{it}P(t)(\ln u)(\ln u)(u^v)\frac{dv}{dt}\frac{dv}{dQ}\frac{df(Q)}{dQ}$$

$$+ ST_iQ_{it}P(t)(\ln u)(u^v)\frac{d}{dt}\frac{dv}{dQ}\frac{df(Q)}{dQ} + ST_iQ_{it}P(t)\frac{d}{dt}\frac{df(Q)}{dQ}(vu^{v-1})(\frac{du}{dQ}) + ST_iQ_{it}P(t)\frac{d}{dt}\frac{df(Q)}{dQ}(\ln u)(u^v)(\frac{dv}{dQ})$$

$$= (1\text{-}LR_i\text{-}ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial^2 C}{\partial S\partial Q}\frac{dS}{dt} - \frac{dC}{dS} - (P(t) - \frac{dC}{dQ})(\frac{dr(t)}{dt}t + r(t))$$

Collecting terms yields:

$$\frac{dQ}{dt}[2ST_iP(t)\frac{\partial ELF}{\partial Q} + \frac{\partial^2 C}{\partial Q^2}]$$

$$+ \frac{du}{dt}[ST_iQ_{it}P(t)\frac{df(Q)}{dQ}][(v)(v\text{-}1)(u^{v-2})\frac{du}{dQ} + (\frac{1}{u})(u^v)(\frac{dv}{dQ}) + (\ln u)(u^{v-1})\frac{dv}{dQ}]$$ **Brackets [] are Expression A**

$$+ \frac{dv}{dt}[ST_iQ_{it}P(t)\frac{df(Q)}{dQ}][(\ln u)(u^{v-1})\frac{du}{dQ} + (\ln u)(\ln u)(u^v)\frac{dv}{dQ}]$$ **Brackets [] are Expression B**

$$+ \frac{d}{dt}\frac{dv}{dQ}[ST_iQ_{it}P(t)\frac{df(Q)}{dQ}][(\ln u)(u^v)]$$ **Brackets [] are Expression C**

$$+ \frac{d}{dt}\frac{df(Q)}{dQ}[ST_iQ_{it}P(t)][(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^v)\frac{dv}{dQ}]$$ **Brackets [] are Expression D**

$$= (1\text{-}LR_i\text{-}ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial^2 C}{\partial S\partial Q}\frac{dS}{dt} - \frac{dC}{dS} - (P(t) - \frac{dC}{dQ})(\frac{dr(t)}{dt}t + r(t))$$ **Exp. E**

Known equations from the derivation of $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ include the following (see appendix G):

$$\frac{du}{dt} = -300 * DAYS * c_7(c_8 - 1)Q(t)^{c_8-2} X(t)^{c_9} \frac{dQ(t)}{dt} - 300 * DAYS * c_7 Q(t)^{c_8-1} c_9 X(t)^{c_9-1} \frac{dX(t)}{dt}$$

$$\frac{dv}{dt} = 1.5333(150{,}000 * DAYS * Q(t)^{-1})^{0.5333}(-150{,}000 * DAYS * Q(t)^{-2})\frac{dQ(t)}{dt}$$

All except $\frac{dQ(t)}{dt}$ is Expression G

$$\frac{d}{dt}\frac{dv}{dQ} = 1.533[0.533(150{,}000 * DAYS * Q(t)^{-1})^{-0.466}(-150{,}000 * DAYS * Q(t)^{-2})(\frac{dQ(t)}{dt})(-150{,}000 * DAYS * Q(t)^{-2})$$

$$+ (300{,}000 * DAYS * Q(t)^{-3})(\frac{dQ(t)}{dt})(150{,}000 * DAYS * Q(t)^{-1})^{0.533}]$$

$$= \frac{dQ}{dt}[0.8177(150{,}000 * DAYS * Q^{-1})^{-0.46}(150{,}000 * DAYS * Q^{-2})^2 + 1.5333(300{,}000 * DAYS * Q^{-3})(150{,}000 * DAYS * Q^{-1})^{0.5333}$$

All except $\frac{dQ(t)}{dt}$ is Expression F

$$\frac{d}{dt}\frac{df(Q)}{dQ} = c_7(c_8 - 1)[(c_8 - 2)(Q(t)^{c_8-3})(X(t)^{c_9})(\frac{dQ(t)}{dt}) + (c_9)(Q(t)^{c_8-2})(X(t)^{c_9-1})(\frac{dX(t)}{dt})$$

Then we can expand the differential terms in front of expressions A – D as follows:

$$\frac{dQ}{dt}\underbrace{[2ST_iP(t)\frac{\partial ELF}{\partial Q} + \frac{\partial^2 C}{\partial Q^2}]}_{\textbf{Expression H}} + \frac{dQ}{dt}\underbrace{[-300*Days*c_7(c_8-1)Q^{c8-2}X^{c9}]}_{\textbf{Expression I}}[\mathbf{A}] + \frac{dQ}{dt}[\mathbf{G}][\mathbf{B}] + \frac{dQ}{dt}[\mathbf{F}][\mathbf{C}] + \frac{dQ}{dt}\underbrace{[c7(c8-1)(c8-2)(Q^{c8-3})X^{c9}]}_{\textbf{Expression J}}[\mathbf{D}]$$

$$= \underbrace{[300*Days*c_7*c_9*Q^{c8-1}X^{c9-1}\frac{dX}{dt}]}_{\textbf{Expression K}}[\mathbf{A}] - \underbrace{[c_7(c_8-1)c_9Q^{c8-2}X^{c9-1}\frac{dX}{dt}]}_{\textbf{Expression L}}[\mathbf{D}] + \mathbf{E}$$

Then, $\frac{dQ}{dt}$ [H + IA + GB + FC + JD] = KA – LD + E

Finally, $\frac{dQ}{dt}$ can be written in terms of the expressions defined above as follows:

$$\frac{dQ}{dt} \; \frac{KA - LD + E}{H + IA + GB + FC + JD}$$

Appendix J: Summary statistics for model results by unit

This page intentionally left blank.

	Prudhoe Bay					
	Best Fit	Hyp. Tax1	Hyp. Tax2	Hyp. Tax3	Hyp. Tax4	Hyp. Tax5
Prod. End*	2084	2084	2084	2083	2083	2083
Production						
Mean	25.67	25.76	25.87	25.90	26.22	25.95
Max. (yr)	52.10	51.27	53.19	55.02	57.49	53.47
Min. (yr)	0.25	0.26	0.25	0.26	0.25	0.26
Std. Dev.	15.97	15.96	16.21	16.75	17.43	16.29
Production Cost ($/bbl)						
Mean	1.96	2.04	2.06	1.96	2.07	2.04
Max. (yr)	3.32	3.56	3.58	3.32	3.60	3.55
Min. (yr)	0.43	0.43	0.44	0.45	0.46	0.44
Std. Dev.	0.98	1.05	1.06	0.97	1.06	1.05
Wellhead Value ($/bbl)						
Mean	35.23	35.23	35.23	34.81	34.81	34.81
Max. (yr)	79.93	79.93	79.93	78.68	78.68	78.68
Min. (yr)	12.11	12.11	12.11	12.11	12.11	12.11
Std. Dev.	20.47	20.47	20.47	20.10	20.10	20.10
Producer Profit ($ millions per year)						
Mean	1,266	1,063	1,070	1,236	1,058	1,100
Max. (yr)	5,444	4,697	4,719	5,799	5,384	4,859
Min. (yr)	165	-	-157	-444	-	-185
Std. Dev.	1,187	1,081	1,030	1,236	1,112	1,047
State Taxes ($ millions per year)**						
Mean	575	790	769	542	694	745
Max. (yr)	2,215	3,012	3,064	2,306	3,181	2,986
Min. (yr)	67	92	90	67	89	89
Std. Dev.	565	776	723	513	657	704
Adjustment Cost ($ millions in one year)						
Mean	17	17	17	22	24	18
Max. (yr)	261	403	216	402	354	219
Min. (yr)	0	0	0	0	0	0
Std. Dev.	46	56	46	66	68	48
State Credits ($ millions in one						
Mean	0	0	41	52	115	43
Max. (yr)	0	0	519	965	1701	526
Min. (yr)	0	0	0	0	0	0
Std. Dev.	0	0	111	157	326	114

	Kuparuk River					
	Best Fit	Hyp. Tax1	Hyp. Tax2	Hyp. Tax3	Hyp. Tax4	Hyp. Tax5
Prod. End*	2076	2077	2075	2075	2075	2075
Production						
Mean	5.38	5.26	5.43	5.49	5.47	5.45
Max. (yr)	10.66	9.77	10.28	10.78	11.03	10.49
Min. (yr)	0.05	0.05	0.05	0.05	0.05	0.05
Std. Dev.	3.14	3.01	3.14	3.25	3.31	3.18
Production Cost ($/bbl)						
Mean	9.36	9.28	9.17	9.36	9.26	9.32
Max. (yr)	31.77	31.76	30.85	31.47	31.13	31.41
Min. (yr)	0.93	0.92	0.94	0.95	0.94	0.94
Std. Dev.	8.51	8.49	8.26	8.43	8.33	8.41
Wellhead Value ($/bbl)						
Mean	33.15	33.55	32.75	32.75	32.75	32.75
Max. (yr)	71.32	72.50	70.16	70.16	70.16	70.16
Min. (yr)	12.29	12.29	12.29	12.29	12.29	12.29
Std. Dev.	17.77	18.12	17.42	17.42	17.42	17.42
Producer Profit ($ millions per year)						
Mean	238	217	191	228	193	203
Max. (yr)	1176	960	940	1153	1002	993
Min. (yr)	-259	-129	-347	-392	-208	-437
Std. Dev.	264	242	229	271	236	236
State Taxes ($ millions per year)**						
Mean	121	160	172	121	156	153
Max. (yr)	470	579	637	480	638	615
Min. (yr)	12	16	17	12	16	11
Std. Dev.	113	145	160	113	146	152
Adjustment Cost ($ millions in one year)						
Mean	5	3	5	5	6	5
Max. (yr)	47	30	44	58	62	53
Min. (yr)	0	0	0	0	0	0
Std. Dev.	10	7	11	13	13	12
State Credits ($ millions in one						
Mean	0	0	11	13	26	12
Max. (yr)	0	0	106	140	264	126
Min. (yr)	0	0	0	0	0	0
Std. Dev.	0	0	26	31	60	28

	Milne Point					
	Best Fit	Hyp. Tax1	Hyp. Tax2	Hyp. Tax3	Hyp. Tax4	Hyp. Tax5
Prod. End*	2079	2083	2095	2079	2099	2085
Production						
Mean	0.83	0.82	0.87	1.18	0.98	1.08
Max. (yr)	1.46	1.55	1.90	2.49	2.05	2.42
Min. (yr)	0.01	0.01	0.01	0.02	0.00	0.01
Std. Dev.	0.50	0.51	0.54	0.75	0.63	0.70
Production Cost ($/bbl)						
Mean	3.91	4.15	15.69	15.47	10.56	18.38
Max. (yr)	7.50	7.50	52.76	54.59	19.75	69.02
Min. (yr)	2.92	3.01	3.04	2.54	2.72	2.66
Std. Dev.	0.64	0.63	13.06	14.05	5.66	17.89
Wellhead Value ($/bbl)						
Mean	35.25	36.97	42.51	35.25	44.48	37.85
Max. (yr)	74.80	79.72	95.62	74.80	101.29	82.26
Min. (yr)	12.65	12.65	12.65	12.65	12.65	12.65
Std. Dev.	18.65	20.12	24.86	18.65	26.56	20.87
Producer Profit ($ millions per year)						
Mean	40	42	61	63	57	57
Max. (yr)	194	224	223	295	215	271
Min. (yr)	-32	-33	-25	-216	-38	-155
Std. Dev.	44	53	70	87	63	77
State Taxes ($ millions per year)**						
Mean	24	34	67	47	56	52
Max. (yr)	79	115	195	160	190	188
Min. (yr)	2	3	5	5	0	2
Std. Dev.	20	27	60	43	55	53
Adjustment Cost ($ millions in one year)						
Mean	1	1	1	3	1	2
Max. (yr)	10	11	14	41	15	34
Min. (yr)	0	0	0	0	0	0
Std. Dev.	4	4	3	3	3	3
State Credits ($ millions in one						
Mean	0	0	2	2	5	2
Max. (yr)	0	0	42	39	78	38
Min. (yr)	0	0	0	0	0	0
Std. Dev.	0	0	7	7	15	7

Table 15: Summary statistics for model results by unit. Production is assumed to end when production rate falls below 0.5% of historical maximum production for the field or when producer profits become negative, whichever comes first. For Prudhoe, Kuparuk, Colville low production signals the end of production. For Endicott, negative profit signals the end of production. Northstar is a mixture of both.

	Endicott						Colville River						Northstar					
	Best Fit	Hyp. Tax1	Hyp. Tax2	Hyp. Tax3	Hyp. Tax4	Hyp. Tax5	Best Fit	Hyp. Tax1	Hyp. Tax2	Hyp. Tax3	Hyp. Tax4	Hyp. Tax5	Best Fit	Hyp. Tax1	Hyp. Tax2	Hyp. Tax3	Hyp. Tax4	Hyp. Tax5
Prod. End	2019	2014	2002	2020	2013	2025	2046	2058	2059	2055	2058	2031	2015	2016	2012	2026	2025	2028
Production																		
Mean	1.86	1.83	2.65	1.82	1.87	1.79	2.05	1.79	2.20	2.10	1.98	2.11	1.00	0.88	0.69	0.76	0.69	0.86
Max. (yr)	3.48	3.26	4.18	3.86	3.88	3.84	3.59	3.14	4.14	3.82	3.67	3.77	1.78	1.68	1.77	1.83	1.78	1.89
Min. (yr)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.02	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.00
Std. Dev.	0.93	0.80	1.16	1.00	1.00	1.01	1.39	1.20	1.63	1.53	1.48	1.48	0.44	0.49	0.62	0.61	0.61	0.59
Production Cost ($/bbl)																		
Mean	7.53	5.86	4.00	7.60	5.88	9.18	14.22	13.79	15.21	15.49	14.43	16.07	11.78	11.79	13.00	15.01	13.24	15.80
Max. (yr)	16.72	12.87	9.41	17.07	12.59	21.76	20.62	20.16	24.20	24.35	22.99	25.07	16.97	16.45	16.16	19.69	16.63	20.27
Min. (yr)	1.98	1.96	1.99	1.99	2.00	1.99	2.40	2.40	2.40	2.40	2.40	2.40	5.13	5.13	5.13	5.13	5.13	5.13
Std. Dev.	4.91	3.75	2.23	4.93	3.64	6.22	6.08	6.05	6.45	6.72	6.08	7.29	3.92	3.65	2.83	3.87	2.99	4.33
Wellhead Value ($/bbl)																		
Mean	16.88	15.98	14.21	17.08	15.81	18.09	24.89	25.82	26.79	26.14	27.45	25.20	17.64	18.03	22.48	22.48	22.48	21.17
Max. (yr)	23.09	20.70	16.17	23.60	20.26	26.34	37.29	39.64	42.10	40.45	43.79	38.06	19.52	20.37	30.82	30.82	30.82	27.66
Min. (yr)	12.73	12.73	12.73	12.73	12.73	12.73	15.67	15.67	15.67	15.67	15.67	15.67	15.94	15.94	15.94	15.94	15.94	15.94
Std. Dev.	3.17	2.45	1.09	3.32	2.32	4.14	6.52	7.21	7.94	7.45	8.44	6.74	1.19	1.44	4.52	4.52	4.52	3.59
Producer Profit ($ millions per year)																		
Mean	106	115	144	90	87	73	70	64	35	55	39	56	21	15	3	6	3	8
Max. (yr)	309	253	295	338	274	279	337	250	306	341	274	316	95	57	60	96	70	97
Min. (yr)	0	0	-29	0	-52	-17	-12	-15	-32	0	1	1	0	-10	-12	0	-12	0
Std. Dev.	105	78	112	98	91	86	112	89	81	108	81	101	28	17	12	20	13	21
State Taxes** ($ millions per year)																		
Mean	51	68	156	61	95	60	43	48	62	47	59	51	29	28	19	16	19	14
Max. (yr)	145	180	257	152	215	193	192	219	295	220	285	236	91	113	123	92	120	87
Min. (yr)	0	0	0	0	0	0	0	0	4	3	4	2	0	0	0	0	0	0
Std. Dev.	42	53	78	43	58	55	64	70	93	71	88	79	28	38	30	22	28	23
Adjustment Cost ($ millions in one year)																		
Mean	1	1	2	1	1	1	1	0	1	1	1	1	0	0	0	0	0	0
Max. (yr)	7	6	10	8	13	10	10	7	18	12	12	11	3	4	4	4	4	4
Min. (yr)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Std. Dev.	2	2	3	2	3	2	2	1	3	2	2	2	1	1	1	1	1	1
State Credits ($ millions in one year)																		
Mean	0	0	4	2	5	2	0	0	2	2	3	2	0	0	0	0	1	0
Max. (yr)	0	0	24	19	63	23	0	0	44	30	59	26	0	0	10	9	19	8
Min. (yr)	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Std. Dev.	0	0	7	5	14	5	0	0	8	5	10	5	0	0	2	2	4	2

Table 16: Summary statistics for model results by unit (continued). Production is assumed to end when production rate falls below 0.5% of historical maximum production for the field or when producer profits become negative, whichever comes first. For Prudhoe, Kuparuk, Colville low production signals the end of production. For Endicott, negative profit signals the end of production. Northstar is a mixture of both.

Appendix K: Present Discounted Values and Correlation Coefficients for Model Results and Historical Production by Unit and North Slope Total, 5% fixed discount rate

This page intentionally left blank.

Prudhoe Bay

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Hist. Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (7.4% discount)	**0.92**	49,780	25,039	0	**74,819**	25,687	24,413	0	50,099
High Price	**0.90**	51,082	24,523	0	**75,605**	24,546	23,964	0	48,510
Fixed Price	**0.92**	80,033	42,494	0	**122,527**	60,895	37,465	0	98,360
Low Discount (2%)	**0.52**	25,744	9,305	0	**35,049**	25,687	24,413	0	50,099
High Discount (15%)	**0.70**	51,598	28,506	0	**80,104**	25,687	24,413	0	50,099
Steeper Cost Func.	**0.91**	45,952	24,820	0	**70,772**	22,443	24,419	0	46,862
Shallower Cost Func.	**0.91**	48,207	25,111	0	**73,317**	26,187	24,280	0	50,467
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.90**	40,777	34,410	0	**75,187**	16,158	33,868	0	50,026
2) Tax on Gross w/ Credits	**0.91**	42,427	32,479	2,972	**74,906**	19,251	30,821	3,468	50,072
3) Low Tax on Gross w/ Credits	**0.91**	48,634	22,481	3,821	**71,116**	27,678	22,311	2,915	49,989
4) Tax on Gross w/ High Credits	**0.92**	42,105	27,834	8,709	**69,938**	20,540	29,532	4,851	50,072
5) Tax on Net w/ Credits, 2007 Policy	**0.91**	43,175	31,399	3,061	**74,574**	20,209	29,842	3,468	50,051
Reference (5% discount)	**0.82**	137,388	0	0	**137,388**	138,513	0	0	138,513
High Price	**0.68**	70,106	0	0	**70,106**	87,200	0	0	87,200
Reference Price	**0.81**	64,967	0	0	**64,967**	88,747	0	0	88,747
Low Discount (2%)	**0.87**	69,282	0	0	**69,282**	138,513	0	0	138,513
High Discount (10%)	**0.72**	185,698	0	0	**185,698**	138,513	0	0	138,513
Steeper Cost Func.	**0.82**	133,622	0	0	**133,622**	134,776	0	0	134,776
Shallower Cost Func.	**0.82**	137,409	0	0	**137,409**	138,523	0	0	138,523
Taxes no ELF	**0.80**	98,495	37,638	0	**136,133**	100,232	38,512	0	138,743
Taxes with ELF	**0.80**	98,909	37,274	0	**136,183**	100,378	38,321	0	138,699

Table 17: Present discounted values and correlation coefficients for Prudhoe Bay model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Kuparuk River

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Hist. Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (8.6% discount)	**0.92**	8,291	4,336	0	**12,627**	6,436	4,094	0	10,530
High Price	**0.88**	9,318	4,482	0	**13,799**	6,699	4,138	0	10,837
Fixed Price	**0.92**	13,206	6,765	0	**19,971**	12,038	5,874	0	17,912
Low Discount (2%)	**0.72**	4,001	1,668	0	**5,670**	6,436	4,094	0	10,530
High Discount (15%)	**0.49**	9,579	4,394	0	**13,974**	6,436	4,094	0	10,530
Steeper Cost Func.	**0.91**	7,204	4,249	0	**11,453**	4,823	4,095	0	8,918
Shallower Cost Func.	**0.92**	7,969	4,303	0	**12,272**	6,484	4,074	0	10,558
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.87**	7,765	5,464	0	**13,229**	5,164	5,368	0	10,532
2) Tax on Gross w/ Credits	**0.93**	6,611	6,148	719	**12,759**	4,792	5,769	769	10,561
3) Low Tax on Gross w/ Credits	**0.92**	7,890	4,285	855	**12,175**	6,373	4,164	646	10,537
4) Tax on Gross w/ High Credits	**0.92**	6,828	5,304	1,658	**12,132**	5,079	5,482	1,072	10,561
5) Tax on Net w/ Credits, 2007 Policy	**0.93**	6,856	5,663	785	**12,518**	5,178	5,365	769	10,543
Reference (5% discount)	**-0.19**	21,302	0	0	**21,302**	23,227	0	0	23,227
High Price	**-0.34**	12,655	0	0	**12,655**	16,005	0	0	16,005
Reference Price	**-0.18**	13,759	0	0	**13,759**	15,737	0	0	15,737
Low Discount (2%)	**-0.07**	13,641	0	0	**13,641**	23,227	0	0	23,227
High Discount (10%)	**-0.32**	33,443	0	0	**33,443**	23,227	0	0	23,227
Steeper Cost Func.	**-0.16**	19,851	0	0	**19,851**	21,408	0	0	21,408
Shallower Cost Func.	**-0.18**	21,289	0	0	**21,289**	23,221	0	0	23,221
Taxes no ELF	**-0.20**	15,098	6,246	0	**21,344**	16,371	6,872	0	23,243
Taxes with ELF	**-0.19**	15,985	5,253	0	**21,238**	17,401	5,832	0	23,234

Table 18: Present discounted values and correlation coefficients for Kuparuk River model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars.
[i]including credits; [ii]net of credits.

Milne Point		Modeled Production, through 2175				Hist. Actual Prod., through 2006			
Scenario	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (9.5% discount)	**0.89**	968	761	0	**1,728**	-2,262	383	0	-1,878
High Price	**0.88**	2,485	1,385	0	**3,870**	-2,103	412	0	-1,691
Fixed Price	**0.83**	2,278	1,245	0	**3,523**	-1,766	514	0	-1,251
Low Discount (2%)	**-0.56**	681	391	0	**1,071**	-2,262	383	0	-1,878
High Discount (15%)	**0.85**	1,834	1,031	0	**2,865**	-2,262	383	0	-1,878
Steeper Cost Func.	**0.80**	1,056	1,045	0	**2,101**	-2,772	383	0	-2,389
Shallower Cost Func.	**0.83**	1,814	1,355	0	**3,169**	-2,168	507	0	-1,661
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.86**	980	996	0	**1,976**	-2,352	469	0	-1,882
2) Tax on Gross w/ Credits	**0.84**	1,418	1,643	100	**3,061**	-2,388	524	267	-1,864
3) Low Tax on Gross w/ Credits	**0.84**	1,501	1,187	253	**2,688**	-2,235	368	219	-1,867
4) Tax on Gross w/ High Credits	**0.86**	1,452	1,526	306	**2,978**	-2,350	487	313	-1,864
5) Tax on Net w/ Credits, 2007 Policy	**0.83**	1,423	1,402	210	**2,825**	-2,255	396	267	-1,859
Reference (5% discount)	**-0.82**	7,822	0	0	**7,822**	2,259	0	0	2,259
High Price	**-0.57**	4,132	0	0	**4,132**	1,778	0	0	1,778
Reference Price	**-0.74**	3,845	0	0	**3,845**	1,602	0	0	1,602
Low Discount (2%)	**-0.72**	4,996	0	0	**4,996**	2,259	0	0	2,259
High Discount (10%)	**-0.85**	9,605	0	0	**9,605**	2,259	0	0	2,259
Steeper Cost Func.	**-0.74**	7,990	0	0	**7,990**	1,625	0	0	1,625
Shallower Cost Func.	**-0.83**	7,798	0	0	**7,798**	2,411	0	0	2,411
Taxes no ELF	**-0.80**	5,435	2,521	0	**7,956**	1,513	746	0	2,259
Taxes with ELF	**-0.80**	6,198	1,752	0	**7,950**	1,717	528	0	2,245

Table 19: Present discounted values and correlation coefficients for Milne Point model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Colville River		**Modeled Production, through 2175**				**Hist. Actual Prod., through 2006**			
Scenario	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (20.5%)	**0.67**	2,204	1,302	0	**3,506**	1,294	837	0	2,132
High Price	**0.67**	3,282	1,740	0	**5,022**	1,807	1,002	0	2,809
Fixed Price	**0.60**	2,514	1,596	0	**4,110**	1,779	999	0	2,778
Low Discount (10%)	**0.39**	2,356	983	0	**3,340**	1,294	837	0	2,132
High Discount (30%)	**0.66**	1,917	1,392	0	**3,309**	1,294	837	0	2,132
Steeper Cost Func.	**0.70**	1,241	966	0	**2,207**	495	838	0	1,333
Shallower Cost Func.	**0.67**	2,318	1,208	0	**3,527**	1,353	774	0	2,127
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.67**	2,047	1,514	0	**3,561**	1,066	1,114	0	2,180
2) Tax on Gross w/ Credits	**0.54**	1,307	2,079	94	**3,387**	910	1,166	90	2,075
3) Low Tax on Gross w/ Credits	**0.66**	1,937	1,523	68	**3,460**	1,235	834	90	2,069
4) Tax on Gross w/ High Credits	**0.66**	1,496	2,006	126	**3,502**	996	1,080	181	2,075
5) Tax on Net w/ Credits, 2007 Policy	**0.67**	1,808	1,602	62	**3,409**	1,088	962	90	2,050
Reference (5%)	**-0.69**	3,418	0	0	**3,418**	3,328	0	0	3,328
High Price	**-0.65**	3,677	0	0	**3,677**	3,304	0	0	3,304
Reference Price	**-0.68**	2,593	0	0	**2,593**	2,637	0	0	2,637
Low Discount (2%)	**-0.68**	2,204	0	0	**2,204**	3,328	0	0	3,328
High Discount (10%)	**-0.70**	4,627	0	0	**4,627**	3,328	0	0	3,328
Steeper Cost Func.	**-0.68**	2,475	0	0	**2,475**	2,444	0	0	2,444
Shallower Cost Func.	**-0.69**	3,418	0	0	**3,418**	3,328	0	0	3,328
Taxes no ELF	**-0.68**	2,196	1,233	0	**3,430**	2,169	1,159	0	3,328
Taxes with ELF	**-0.67**	2,559	765	0	**3,324**	2,317	1,042	0	3,359

Table 20: Present discounted values and correlation coefficients for Colville River model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Endicott		**Modeled Production, through 2175**				**Hist. Actual Prod., through 2006**			
Scenario	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (12%)	**0.93**	2,336	1,077	0	**3,413**	2,073	996	0	3,070
High Price	**0.92**	2,558	1,043	0	**3,601**	2,096	987	0	3,083
Fixed Price	**0.92**	3,830	1,665	0	**5,495**	3,528	1,445	0	4,972
Low Discount (5%)	**0.88**	1,758	408	0	**2,166**	2,073	996	0	3,070
High Discount (20%)	**0.91**	2,278	1,320	0	**3,598**	2,073	996	0	3,070
Steeper Cost Func.	**0.73**	1,769	1,271	0	**3,039**	1,174	1,001	0	2,174
Shallower Cost Func.	**0.91**	2,426	1,026	0	**3,452**	2,173	947	0	3,120
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.83**	2,049	1,274	0	**3,323**	1,787	1,288	0	3,075
2) Tax on Gross w/ Credits	**0.91**	1,730	1,871	64	**3,601**	1,497	1,579	82	3,076
3) Low Tax on Gross w/ Credits	**0.92**	2,071	1,304	62	**3,375**	1,930	1,140	82	3,070
4) Tax on Gross w/ High Credits	**0.93**	1,661	1,723	132	**3,384**	1,575	1,501	164	3,076
5) Tax on Net w/ Credits, 2007 Policy	**0.92**	1,888	1,489	60	**3,377**	1,715	1,347	82	3,062
Reference (5%)	**0.58**	4,422	0	0	**4,422**	5,429	0	0	5,429
High Price	**0.52**	3,096	0	0	**3,096**	3,523	0	0	3,523
Reference Price	**0.60**	2,347	0	0	**2,347**	3,518	0	0	3,518
Low Discount (2%)	**0.64**	2,777	0	0	**2,777**	5,429	0	0	5,429
High Discount (10%)	**0.50**	6,399	0	0	**6,399**	5,429	0	0	5,429
Steeper Cost Func.	**0.58**	3,676	0	0	**3,676**	4,446	0	0	4,446
Shallower Cost Func.	**0.58**	4,405	0	0	**4,405**	5,417	0	0	5,417
Taxes no ELF	**0.59**	3,027	1,454	0	**4,481**	3,681	1,748	0	5,429
Taxes with ELF	**0.60**	3,462	960	0	**4,422**	4,003	1,441	0	5,444

Table 21: Present discounted values and correlation coefficients for Endicott model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Northstar		**Modeled Production, through 2175**				**Hist. Actual Prod., through 2006**			
Scenario	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit (46%)	**0.68**	204	275	0	**480**	-90	394	0	304
High Price	**0.64**	450	388	0	**838**	158	474	0	632
Fixed Price	**0.61**	355	345	0	**701**	118	468	0	585
Low Discount (30%)	**0.55**	230	268	0	**499**	-90	394	0	304
High Discount (60%)	**0.61**	212	269	0	**481**	-90	394	0	304
Steeper Cost Func.	**0.69**	234	105	0	**338**	-1,297	394	0	-903
Shallower Cost Func.	**0.80**	267	230	0	**497**	-24	361	0	338
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.49**	162	308	0	**469**	-226	524	0	298
2) Tax on Gross w/ Credits	**0.50**	72	363	13	**435**	-264	585	9	321
3) Low Tax on Gross w/ Credits	**0.59**	175	367	12	**542**	-110	428	9	318
4) Tax on Gross w/ High Credits	**0.50**	89	445	25	**534**	-256	576	18	321
5) Tax on Net w/ Credits, 2007 Policy	**0.66**	185	313	11	**497**	-46	280	9	234
Reference (5%)	**0.13**	602	0	0	**602**	578	0	0	578
High Price	**0.27**	878	0	0	**878**	593	0	0	593
Reference Price	**0.17**	504	0	0	**504**	258	0	0	258
Low Discount (2%)	**0.16**	544	0	0	**544**	578	0	0	578
High Discount (10%)	**-0.34**	1,048	0	0	**1,048**	578	0	0	578
Steeper Cost Func.	**-0.14**	381	0	0	**381**	-1,041	0	0	-1,041
Shallower Cost Func.	**0.10**	612	0	0	**612**	587	0	0	587
Taxes no ELF	**-0.19**	403	376	0	**779**	22	556	0	578
Taxes with ELF	**0.12**	441	148	0	**589**	113	486	0	599

Table 22: Present discounted values and correlation coefficients for Northstar model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

North Slope Total		**Modeled Production, through 2175**				**Hist. Actual Prod., through 2006**			
Scenario	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.92**	63,783	32,790	0	**96,573**	33,138	31,118	0	64,256
High Price	**0.86**	69,175	33,560	0	**102,734**	33,203	30,977	0	64,180
Fixed Price	**0.93**	102,216	54,111	0	**156,327**	76,592	46,765	0	123,357
Low Discount	**0.03**	34,771	13,024	0	**47,795**	33,138	31,118	0	64,256
High Discount	**0.71**	67,418	36,912	0	**104,330**	33,138	31,118	0	64,256
Steeper Cost Func.	**0.93**	57,454	32,455	0	**89,910**	24,865	31,130	0	55,995
Shallower Cost Func.	**0.91**	63,000	33,234	0	**96,234**	34,004	30,944	0	64,949
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.91**	53,779	43,967	0	**97,746**	21,598	42,631	0	64,228
2) Tax on Gross w/ Credits	**0.92**	53,567	44,582	3,963	**98,149**	23,798	40,444	4,685	64,242
3) Low Tax on Gross w/ Credits	**0.91**	62,207	31,148	5,070	**93,355**	34,871	29,245	3,961	64,116
4) Tax on Gross w/ High Credits	**0.92**	53,631	38,837	10,956	**92,468**	25,583	38,658	6,600	64,242
5) Tax on Net w/ Credits, 2007 Policy	**0.91**	55,333	41,867	4,190	**97,200**	25,888	38,193	4,685	64,081
Reference (5%)	**0.74**	174,958	0	0	**174,958**	169,904	0	0	169,904
High Price	**0.56**	94,565	0	0	**94,565**	108,960	0	0	108,960
Reference Price	**0.78**	88,021	0	0	**88,021**	109,045	0	0	109,045
Low Discount (2%)	**0.92**	93,446	0	0	**93,446**	169,904	0	0	169,904
High Discount (10%)	**0.60**	240,822	0	0	**240,822**	169,904	0	0	169,904
Steeper Cost Func.	**0.77**	167,997	0	0	**167,997**	155,031	0	0	155,031
Shallower Cost Func.	**0.74**	174,936	0	0	**174,936**	171,691	0	0	171,691
Taxes no ELF	**0.73**	124,655	49,470	0	**174,125**	120,528	49,622	0	170,150
Taxes with ELF	**0.73**	127,554	46,153	0	**173,707**	122,469	47,680	0	170,149

Table 23: Present discounted values and correlation coefficients for the North Slope model results and historical production, with fixed 5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Appendix L: Present Discounted Values and Correlation Coefficients for Model Results and Historical Production by Unit and North Slope Total, variable discount rate as defined in each scenario

This page intentionally left blank.

Prudhoe Bay

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Historical Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.92**	37,234	19,309	0	**56,543**	16,346	19,904	0	36,250
High Price	**0.90**	35,015	17,812	0	**52,827**	15,134	19,449	0	34,583
Fixed Price	**0.92**	63,080	34,268	0	**97,348**	46,068	30,866	0	76,935
Low Discount	**0.52**	95,468	33,398	0	**128,867**	43,337	32,811	0	76,148
High Discount	**0.70**	22,491	11,799	0	**34,290**	637	12,037	0	12,674
Steeper Cost Func.	**0.91**	34,451	19,170	0	**53,621**	13,837	19,908	0	33,745
Shallower Cost Func.	**0.91**	36,070	19,334	0	**55,403**	16,701	19,818	0	36,519
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.90**	30,350	26,543	0	**56,893**	8,455	27,722	0	36,176
2) Tax on Gross w/ Credits	**0.91**	31,738	24,848	2,544	**56,585**	11,267	24,934	3,166	36,201
3) Low Tax on Gross w/ Credits	**0.91**	36,446	17,200	3,296	**53,645**	18,087	18,051	2,626	36,138
4) Tax on Gross w/ High Credits	**0.92**	31,494	21,080	7,633	**52,574**	12,303	23,898	4,294	36,201
5) Tax on Net w/ Credits, 2007 Policy	**0.91**	32,272	24,067	2,625	**56,340**	12,039	24,148	3,166	36,187
Reference	**0.82**	137,388	0	0	**137,388**	138,513	0	0	138,513
High Price	**0.68**	70,106	0	0	**70,106**	87,200	0	0	87,200
Reference Price	**0.81**	64,967	0	0	**64,967**	88,747	0	0	88,747
Low Discount	**0.87**	120,230	0	0	**120,230**	181,482	0	0	181,482
High Discount	**0.72**	143,513	0	0	**143,513**	96,583	0	0	96,583
Steeper Cost Func.	**0.82**	133,622	0	0	**133,622**	134,776	0	0	134,776
Shallower Cost Func.	**0.82**	137,409	0	0	**137,409**	138,523	0	0	138,523
Taxes no ELF	**0.80**	98,495	37,638	0	**136,133**	100,232	38,512	0	138,743
Taxes with ELF	**0.80**	98,909	37,274	0	**136,183**	100,378	38,321	0	138,699

Table 24: Present discounted values and correlation coefficients for Prudhoe Bay model results and historical production, with 7.4% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Kuparuk River

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Hist. Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.92**	4,928	2,755	0	**7,683**	3,634	2,835	0	6,469
High Price	**0.88**	4,724	2,598	0	**7,321**	3,658	2,825	0	6,483
Fixed Price	**0.92**	8,760	4,695	0	**13,455**	7,766	4,135	0	11,901
Low Discount	**0.72**	18,707	7,662	0	**26,369**	10,444	5,797	0	16,241
High Discount	**0.49**	2,978	1,346	0	**4,323**	1,243	1,650	0	2,893
Steeper Cost Func.	**0.91**	4,371	2,649	0	**7,020**	2,570	2,836	0	5,406
Shallower Cost Func.	**0.92**	4,725	2,727	0	**7,452**	3,658	2,812	0	6,469
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.87**	4,647	3,358	0	**8,005**	2,753	3,721	0	6,474
2) Tax on Gross w/ Credits	**0.93**	3,897	3,880	584	**7,778**	2,583	3,894	683	6,477
3) Low Tax on Gross w/ Credits	**0.92**	4,674	2,699	700	**7,372**	3,656	2,805	564	6,462
4) Tax on Gross w/ High Credits	**0.92**	4,087	3,257	1,324	**7,344**	2,797	3,680	916	6,477
5) Tax on Net w/ Credits, 2007 Policy	**0.93**	4,011	3,612	639	**7,623**	2,841	3,628	683	6,468
Reference	**-0.19**	21,302	0	0	**21,302**	23,227	0	0	23,227
High Price	**-0.34**	12,655	0	0	**12,655**	16,005	0	0	16,005
Reference Price	**-0.18**	13,759	0	0	**13,759**	15,737	0	0	15,737
Low Discount	**-0.07**	23,019	0	0	**23,019**	32,269	0	0	32,269
High Discount	**-0.32**	26,173	0	0	**26,173**	14,631	0	0	14,631
Steeper Cost Func.	**-0.16**	19,851	0	0	**19,851**	21,408	0	0	21,408
Shallower Cost Func.	**-0.18**	21,289	0	0	**21,289**	23,221	0	0	23,221
Taxes no ELF	**-0.20**	15,098	6,246	0	**21,344**	16,371	6,872	0	23,243
Taxes with ELF	**-0.19**	15,985	5,253	0	**21,238**	17,401	5,832	0	23,234

Table 25: Present discounted values and correlation coefficients for Kuparuk River model results and historical production, with 8.6% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Milne Point

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Hist. Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.89**	383	403	0	**786**	-1,843	239	0	-1,604
High Price	**0.88**	705	564	0	**1,269**	-1,768	251	0	-1,517
Fixed Price	**0.83**	959	651	0	**1,610**	-1,530	327	0	-1,202
Low Discount	**-0.56**	5,807	2,637	0	**8,444**	-2,626	550	0	-2,076
High Discount	**0.85**	207	240	0	**447**	-1,478	148	0	-1,330
Steeper Cost Func.	**0.80**	171	399	0	**570**	-2,168	239	0	-1,929
Shallower Cost Func.	**0.83**	718	587	0	**1,305**	-1,759	301	0	-1,458
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.86**	353	508	0	**861**	-1,906	297	0	-1,609
2) Tax on Gross w/ Credits	**0.84**	468	667	61	**1,135**	-1,894	301	171	-1,593
3) Low Tax on Gross w/ Credits	**0.84**	547	523	145	**1,070**	-1,807	212	138	-1,595
4) Tax on Gross w/ High Credits	**0.86**	514	634	192	**1,148**	-1,874	280	197	-1,593
5) Tax on Net w/ Credits, 2007 Policy	**0.83**	519	603	118	**1,122**	-1,809	220	171	-1,589
Reference	**-0.82**	7,822	0	0	**7,822**	2,259	0	0	2,259
High Price	**-0.57**	4,132	0	0	**4,132**	1,778	0	0	1,778
Reference Price	**-0.74**	3,845	0	0	**3,845**	1,602	0	0	1,602
Low Discount	**-0.72**	7,773	0	0	**7,773**	3,288	0	0	3,288
High Discount	**-0.85**	7,554	0	0	**7,554**	1,300	0	0	1,300
Steeper Cost Func.	**-0.74**	7,990	0	0	**7,990**	1,625	0	0	1,625
Shallower Cost Func.	**-0.83**	7,798	0	0	**7,798**	2,411	0	0	2,411
Taxes no ELF	**-0.80**	5,435	2,521	0	**7,956**	1,513	746	0	2,259
Taxes with ELF	**-0.80**	6,198	1,752	0	**7,950**	1,717	528	0	2,245

Table 26: Present discounted values and correlation coefficients for Milne Point model results and historical production, with 9.5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Colville River		**Modeled Production, through 2175**				**Hist. Actual Prod., through 2006**			
Scenario	Corr.	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.67**	1,118	622	0	**1,740**	780	530	0	1,310
High Price	**0.67**	1,602	819	0	**2,421**	1,092	628	0	1,721
Fixed Price	**0.60**	1,435	853	0	**2,288**	1,111	637	0	1,749
Low Discount	**0.39**	1,581	638	0	**2,219**	1,085	713	0	1,798
High Discount	**0.66**	814	525	0	**1,338**	603	422	0	1,025
Steeper Cost Func.	**0.70**	569	367	0	**936**	303	530	0	833
Shallower Cost Func.	**0.67**	1,173	561	0	**1,734**	814	490	0	1,305
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.67**	964	706	0	**1,670**	647	703	0	1,350
2) Tax on Gross w/ Credits	**0.54**	716	1,022	67	**1,738**	524	735	77	1,259
3) Low Tax on Gross w/ Credits	**0.66**	1,024	720	45	**1,744**	732	523	77	1,255
4) Tax on Gross w/ High Credits	**0.66**	796	931	86	**1,728**	588	671	153	1,259
5) Tax on Net w/ Credits, 2007 Policy	**0.67**	927	792	41	**1,719**	631	613	77	1,244
Reference	**-0.69**	3,418	0	0	**3,418**	3,328	0	0	3,328
High Price	**-0.65**	3,677	0	0	**3,677**	3,304	0	0	3,304
Reference Price	**-0.68**	2,593	0	0	**2,593**	2,637	0	0	2,637
Low Discount	**-0.68**	3,529	0	0	**3,529**	3,647	0	0	3,647
High Discount	**-0.70**	3,810	0	0	**3,810**	2,892	0	0	2,892
Steeper Cost Func.	**-0.68**	2,475	0	0	**2,475**	2,444	0	0	2,444
Shallower Cost Func.	**-0.69**	3,418	0	0	**3,418**	3,328	0	0	3,328
Taxes no ELF	**-0.68**	2,196	1,233	0	**3,430**	2,169	1,159	0	3,328
Taxes with ELF	**-0.67**	2,559	765	0	**3,324**	2,317	1,042	0	3,359

Table 27: Present discounted values and correlation coefficients for Colville River model results and historical production, with 20.5% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Endicott

Scenario	Corr.	Modeled Production, through 2175 PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Hist. Actual Prod., through 2006 PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.93**	1,428	667	0	**2,095**	1,302	674	0	1,977
High Price	**0.92**	1,347	581	0	**1,928**	1,280	660	0	1,940
Fixed Price	**0.92**	2,508	1,124	0	**3,632**	2,303	989	0	3,292
Low Discount	**0.88**	1,758	408	0	**2,166**	2,073	996	0	3,070
High Discount	**0.91**	1,026	591	0	**1,617**	840	468	0	1,307
Steeper Cost Func.	**0.73**	1,102	777	0	**1,880**	752	678	0	1,430
Shallower Cost Func.	**0.91**	1,500	612	0	**2,111**	1,379	628	0	2,008
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.83**	1,195	757	0	**1,952**	1,106	880	0	1,986
2) Tax on Gross w/ Credits	**0.91**	1,147	1,196	51	**2,344**	949	1,023	69	1,972
3) Low Tax on Gross w/ Credits	**0.92**	1,299	769	52	**2,069**	1,233	735	69	1,968
4) Tax on Gross w/ High Credits	**0.93**	1,072	1,032	114	**2,104**	1,011	961	139	1,972
5) Tax on Net w/ Credits, 2007 Policy	**0.92**	1,167	914	51	**2,081**	1,084	880	69	1,964
Reference	**0.58**	4,422	0	0	**4,422**	5,429	0	0	5,429
High Price	**0.52**	3,096	0	0	**3,096**	3,523	0	0	3,523
Reference Price	**0.60**	2,347	0	0	**2,347**	3,518	0	0	3,518
Low Discount	**0.64**	4,452	0	0	**4,452**	6,617	0	0	6,617
High Discount	**0.50**	5,161	0	0	**5,161**	4,063	0	0	4,063
Steeper Cost Func.	**0.58**	3,676	0	0	**3,676**	4,446	0	0	4,446
Shallower Cost Func.	**0.58**	4,405	0	0	**4,405**	5,417	0	0	5,417
Taxes no ELF	**0.59**	3,027	1,454	0	**4,481**	3,681	1,748	0	5,429
Taxes with ELF	**0.60**	3,462	960	0	**4,422**	4,003	1,441	0	5,444

Table 28: Present discounted values and correlation coefficients for Endicott model results and historical production, with 12% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Northstar

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Historical Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.68**	95	118	0	**213**	35	167	0	202
High Price	**0.64**	181	157	0	**338**	132	198	0	330
Fixed Price	**0.61**	179	168	0	**347**	128	200	0	328
Low Discount	**0.55**	127	151	0	**279**	16	221	0	238
High Discount	**0.61**	84	105	0	**189**	41	135	0	177
Steeper Cost Func.	**0.69**	5	4	0	**9**	-380	167	0	-213
Shallower Cost Func.	**0.80**	112	90	0	**202**	58	154	0	212
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.49**	58	148	0	**206**	-23	222	0	199
2) Tax on Gross w/ Credits	**0.50**	45	177	8	**222**	-34	243	6	209
3) Low Tax on Gross w/ Credits	**0.59**	92	138	7	**230**	30	178	6	208
4) Tax on Gross w/ High Credits	**0.50**	50	171	15	**221**	-30	239	12	209
5) Tax on Net w/ Credits, 2007 Policy	**0.66**	92	129	7	**220**	43	138	6	181
Reference	**0.13**	602	0	0	**602**	578	0	0	578
High Price	**0.27**	878	0	0	**878**	593	0	0	593
Reference Price	**0.17**	504	0	0	**504**	258	0	0	258
Low Discount	**0.16**	710	0	0	**710**	588	0	0	588
High Discount	**-0.34**	1,019	0	0	**1,019**	559	0	0	559
Steeper Cost Func.	**-0.14**	381	0	0	**381**	-1,041	0	0	-1,041
Shallower Cost Func.	**0.10**	612	0	0	**612**	587	0	0	587
Taxes no ELF	**-0.19**	403	376	0	**779**	22	556	0	578
Taxes with ELF	**0.12**	441	148	0	**589**	113	486	0	599

Table 29: Present discounted values and correlation coefficients for Northstar model results and historical production, with 46% discount rate used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

North Slope Total

Scenario	Corr.	Modeled Production, through 2175: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes	Hist. Actual Prod., through 2006: PDV Profits[i]	PDV Taxes[ii]	PDV Credits	Sum of Profits + Taxes
Best Fit	**0.92**	45,187	23,873	0	**69,060**	20,255	24,348	0	44,603
High Price	**0.86**	43,572	22,532	0	**66,104**	19,528	24,011	0	43,539
Fixed Price	**0.93**	76,921	41,758	0	**118,680**	55,847	37,155	0	93,002
Low Discount	**0.03**	123,450	44,894	0	**168,343**	54,330	41,088	0	95,419
High Discount	**0.71**	27,600	14,605	0	**42,205**	1,886	14,859	0	16,745
Steeper Cost Func.	**0.93**	40,669	23,366	0	**64,035**	14,915	24,357	0	39,272
Shallower Cost Func.	**0.91**	44,298	23,911	0	**68,208**	20,851	24,204	0	45,055
Hypothetical Tax Scenarios									
1) High Tax on Gross w/ ELF	**0.91**	37,567	32,020	0	**69,587**	11,033	33,545	0	44,577
2) Tax on Gross w/ Credits	**0.92**	38,011	31,791	3,315	**69,802**	13,395	31,130	4,172	44,525
3) Low Tax on Gross w/ Credits	**0.91**	44,082	22,048	4,246	**66,130**	21,931	22,505	3,480	44,436
4) Tax on Gross w/ High Credits	**0.92**	38,013	27,105	9,364	**65,118**	14,795	29,730	5,712	44,525
5) Tax on Net w/ Credits, 2007 Policy	**0.91**	38,988	30,118	3,481	**69,105**	14,829	29,626	4,172	44,455
Reference	**0.74**	174,958	0	0	**174,958**	169,904	0	0	169,904
High Price	**0.56**	94,565	0	0	**94,565**	108,960	0	0	108,960
Reference Price	**0.78**	88,021	0	0	**88,021**	109,045	0	0	109,045
Low Discount	**0.92**	159,721	0	0	**159,721**	224,320	0	0	224,320
High Discount	**0.60**	187,232	0	0	**187,232**	116,792	0	0	116,792
Steeper Cost Func.	**0.77**	167,997	0	0	**167,997**	155,031	0	0	155,031
Shallower Cost Func.	**0.74**	174,936	0	0	**174,936**	171,691	0	0	171,691
Taxes no ELF	**0.73**	124,655	49,470	0	**174,125**	120,528	49,622	0	170,150
Taxes with ELF	**0.73**	127,554	46,153	0	**173,707**	122,469	47,680	0	170,149

Table 30: Present discounted values and correlation coefficients for North Slope model results and historical production, with field-specific discount rates used for calculation of PDV. Note, all profits, taxes, and credits are in millions of 1982-84 dollars. [i]including credits; [ii]net of credits.

Appendix M: Derivation of Model Structure for Variable Discount Rate

In this appendix, we derive the model structure for incorporating a variable discount rate to account for recovery of capital investments as a primary business consideration (see section 5.5 for further explanation).

The producer's optimal control problem is to choose the production profile {Q(t)} to maximize the present discounted value of the entire stream of profits, given the initial stock X(0) and given the relationship between production Q(t) and the cumulative stock produced X(t), and subject to the constraints that both production and stock are nonnegative. Mathematically,

$$\begin{aligned} H &= [P(t)Q_{it}(1-LR_i-ST_iELF(Q_i(t))) - C(Q_{it},S_{it})]\,\beta\delta^t - p(t)\,\beta\delta^tQ_{it} \\ &= P(t)Q_{it}\beta\delta^t(1-LR_i-ST_iELF(Q_i(t))) - C(Q_{it},S_{it})\beta\delta^t - p(t)\,\beta\delta^tQ_{it} \\ &= P(t)Q_{it}\beta\delta^t - P(t)Q_{it}\beta\delta^tLR_i - P(t)Q_{it}\beta\delta^tST_iELF(Q_i(t)) - C(Q_{it},S_{it})\beta\delta^t - p(t)\,\beta\delta^tQ_{it} \\ &= \beta\delta^t[P(t)Q_{it}(1-LR_i-ST_iELF(Q_i(t))) - C(Q_{it},S_{it}) - p(t)Q_{it}] \end{aligned}$$

Then the three first-order conditions for dynamic optimality (from the Maximum Principle) are the following.

1.) Static optimality in the current period: Price = Marginal Cost for perfect competition (i.e., price takers). Or, the shadow price p(t) must equal price minus marginal cost of production. More formally,

$$\frac{\partial H}{\partial Q} = \beta\delta^tP(t) - \beta\delta^tLR_iP(t) - [\beta\delta^tST_iELF(Q_i(t))P(t) + \beta\delta^tST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q}] - \beta\delta^t\frac{\partial C}{\partial Q} - \beta\delta^tp(t) = 0$$

Cancelling the $\beta\delta^t$ terms yields,

$$\frac{\partial H}{\partial Q} = P(t)(1 - LR_i - ST_iELF(Q_i(t))) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q} - p(t) = 0$$

$$p(t) = P(t)(1 - LR_i - ST_iELF(Q_i(t))) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}$$

The term $(LR_i + ST_iELF(Q_i(t)))$ is an approximation of "total government take" (%), although in this intermediate-complexity model it does not include state property tax, state income tax, or federal income tax.

2.) Evolution of the shadow price over time, to ensure inter-period optimality over all finite sub-periods.

In general, where $u(Q,S) = P(t)Q_{it} - C(Q_{it},S_{it})$,

$$\frac{dH}{dS} = \frac{-d\mu(t)}{dt} \qquad \text{Where } \mu(t) = p(t)e^{-r(t)}$$

$$H = P(t)Q_{it}\beta\delta^t - C(Q_{it},S_{it})\beta\delta^t - p(t)\,\beta\delta^tQ_{it}$$

Thus, $\frac{dH}{dS} = -\beta\delta^t \frac{\partial C}{\partial S}$

$\mu(t) = p(t)e^{-r(t)t}$

Thus, $\frac{d\mu(t)}{dt} = \frac{dp(t)}{dt} e^{-r(t)t} + p(t)(-\frac{dr(t)}{dt}t - r(t))(e^{-r(t)t})$

$e^{-r(t)t} = \beta\delta^t$

Thus, $\frac{d\mu(t)}{dt} = \frac{dp(t)}{dt} \beta\delta^t + p(t)(-\frac{dr(t)}{dt}t - r(t))\beta\delta^t$

Thus,

$$-\frac{dp(t)}{dt}\beta\delta^t - p(t)(-\frac{dr(t)}{dt}t - r(t))\beta\delta^t = -\beta\delta^t \frac{\partial C}{\partial S}$$

$$-\frac{dp(t)}{dt} - p(t)(-\frac{dr(t)}{dt}t - r(t)) = -\frac{\partial C}{\partial S}$$

$$\frac{dp(t)}{dt} = \frac{\partial C}{\partial S} - p(t)(-\frac{dr(t)}{dt}t - r(t))$$

Substitute the following,

$e^{r(t)t} = \frac{1}{\beta\delta^t}$ since $e^{-r(t)t} = \beta\delta^t$

p(t) = P(t) - $\frac{\partial C}{\partial Q}$

To get the following results

$$\frac{dp(t)}{dt} = \frac{\partial C}{\partial S} + (P(t) - \frac{\partial C}{\partial Q})(\frac{dr(t)}{dt}t + r(t))$$

or

$$\frac{dp(t)}{dt} = -\frac{\partial C}{\partial X} + (P(t) - \frac{\partial C}{\partial Q})(\frac{dr(t)}{dt}t + r(t))$$

These results are the same as the simple specification and nearly the same result as fixed-discount-rate models except the fixed discount rate has been replaced by $\frac{dr}{dt}$.

Note, r(t) = $\frac{-\ln(\beta\delta^t)}{t} = \ln(\beta\delta^t)(\frac{1}{t})$

Thus, $\frac{dr(t)}{dt} = (\frac{1}{\beta\delta^t})(\beta\delta^t \ln\beta\delta)(\frac{1}{t}) + (\frac{-1}{t^2})(\ln(\beta\delta^t)) = \frac{1}{t\ln(\beta\delta)} - \frac{\ln(\beta\delta^t)}{t^2}$

3.) The transversality condition, required for optimality over an infinite time horizon.

$\lim_{t\to\infty} \mu(t)S(t) = 0$

Thus,
$\lim_{t\to\infty} p(t)S(t)\beta\delta^t = 0$ or $\lim_{t\to\infty} p(t)X(t)\beta\delta^t = 0$

Rewriting the problem as a boundary value problem with differential equations for Q_{it} (instead of P_{it}) and X_{it} and boundary conditions proceeds as follows. Again following the methodology from Lin (2007), the Hotelling problem can be reformulated into the following ordinary differential equation boundary value problem.

Step 1: combine FOC 1 and 2

$$p(t) = P(t)(1-LR_i-ST_iELF(Q_i(t))) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}$$

$$= P(t) - P(t)LR_i - P(t)ST_iELF(Q_i(t)) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}$$

$$\frac{dp(t)}{dt} = \frac{d}{dt}P(t) - LR_i\frac{d}{dt}P(t) - \frac{d}{dt}[P(t)ST_iELF(Q_i(t))] - \frac{d}{dt}[ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q}] - \frac{d}{dt}\frac{\partial C}{\partial Q}$$

Where $\frac{d}{dt}[P(t)ST_iELF(Q_i(t))] = [ST_iP(t)\frac{d}{dt}(ELF(Q_i(t))) + ST_iELF(Q_i(t))\frac{d}{dt}P(t)]$

Where $\frac{d}{dt}[ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q}] = [ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q} + ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} + ST_iQ_{it}P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q}]$

Where $\frac{d}{dt}(ELF(Q_i(t))) = \frac{\partial ELF}{\partial Q}\frac{dQ}{dt}$

Where $\frac{d}{dt}\frac{\partial C}{\partial Q} = \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} + \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t}$ or $-\frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dXdQ}\frac{\partial X}{\partial t}$

Where $\frac{d}{dt}\frac{\partial ELF}{\partial Q} =$

$$\{\frac{d}{dt}[(vu^{v-1})(\frac{du}{dQ})] + \frac{d}{dt}[(\ln u)(u^v)(\frac{dv}{dQ})]\}[\frac{\partial f(Q)}{\partial Q}] + \{\frac{d}{dt}[\frac{\partial f(Q)}{\partial Q}]\}[(vu^{v-1})(\frac{du}{dQ}) + (\ln u)(u^v)(\frac{dv}{dQ})]$$

(see appendix G)

Then,

$$\frac{dp(t)}{dt} = (1 - LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{dQ}{dt} - ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q}$$
$$- ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_{it}P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t}$$

or

$$\frac{dp(t)}{dt} = (1 - LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{dQ}{dt} - ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q}$$
$$- ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_{it}P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dXdQ}\frac{\partial X}{\partial t}$$

Now, substitute this into the left side of FOC (e)

$$(1 - LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{dQ}{dt} - ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_{it}\frac{d}{dt}P(t)$$

$$ST_iQ_{it}P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t} = \frac{\partial C}{\partial S} + (P(t) - \frac{\partial C}{\partial Q})(\frac{dr(t)}{dt}t + r(t))$$

or

$$(1 - LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iP(t)\frac{\partial ELF}{\partial Q}\frac{dQ}{dt} - ST_i\frac{dQ}{dt}P(t)\frac{\partial ELF}{\partial Q} - ST_iQ_{it}\frac{d}{dt}P(t)$$

$$ST_iQ_{it}P(t)\frac{d}{dt}\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dQ^2}\frac{\partial Q}{\partial t} - \frac{d^2C}{dXdQ}\frac{\partial X}{\partial t} = -\frac{\partial C}{\partial X} + (P(t) - \frac{\partial C}{\partial Q})(\frac{dr(t)}{dt}t + r(t))$$

However, before using algebra to isolate $\frac{\partial Q}{\partial t}$ on the left side, it is necessary to write out the expression for $\frac{d}{dt}\frac{\partial ELF}{\partial Q}$ since it contains $\frac{\partial Q}{\partial t}$ terms (see appendix G). We then expand the expression above in appendix J to derive the following expression for $\frac{\partial Q}{\partial t}$.

$$\frac{\partial Q}{\partial t} = \frac{KA - LD + E}{H + IA + GB + FC + JD}$$

Where $E = (1 - LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dSdQ}\frac{\partial S}{\partial t} - \frac{\partial C}{\partial S} - (P(t) - \frac{\partial C}{\partial Q})(\frac{dr(t)}{dt}t + r(t))$

or

Where $E = (1 - LR_i - ST_iELF(Q_i(t)))\frac{d}{dt}P(t) - ST_iQ_{it}\frac{d}{dt}P(t)\frac{\partial ELF}{\partial Q} - \frac{d^2C}{dXdQ}\frac{\partial X}{\partial t} + \frac{\partial C}{\partial X} - (P(t) - \frac{\partial C}{\partial Q})(\frac{dr(t)}{dt}t + r(t))$

And all other letters in the equation are as given in appendix J, and remain consistent across cost specifications and constant/variable discount rates.

This is one differential equation in the boundary value problem; it contains the information from FOC 1 and 2.

Step 2: combine FOC 1 and 3, making sure the limit contains a Q term so the boundary conditions will pin it down.

FOC 1: $p(t) = P(t)(1 - LR_i - ST_iELF(Q_i(t))) - ST_iQ_{it}P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}$

FOC 3: $\lim_{t\to\infty} p(t)S(t)\beta\delta^t = 0$ or $\lim_{t\to\infty} p(t)X(t)\beta\delta^t = 0$

Thus,

$$\lim_{t\to\infty} \left(P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - ST_i Q_{it} P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}\right)S(t)\beta\delta^t = 0$$

or

$$\lim_{t\to\infty} \left(P(t)(1 - LR_i - ST_i ELF(Q_i(t))) - ST_i Q_{it} P(t)\frac{\partial ELF}{\partial Q} - \frac{\partial C}{\partial Q}\right)X(t)\beta\delta^t = 0$$

This is one boundary condition in our boundary value problem; it contains the information from FOC 2 and 3, and the term $\frac{\partial C}{\partial Q}$ does contain a Q term. This is the same result as fixed-discount-rate models except e^{-rt} has been replaced by $\beta\delta^t$.

Step 3: Define the second differential equation and second boundary condition from the maximization constraints.

The second differential equation comes from the fact that the rate of change in cumulative production (X(t)) is equal to the rate of production (Q(t)) and the rate of change in reserves remaining (S(t)) is equal to the negative rate of production (-Q(t)). Thus, we have,

$$\frac{d}{dt}S(t) = -Q(t) \quad \text{or} \quad \frac{d}{dt}X(t) = Q(t)$$

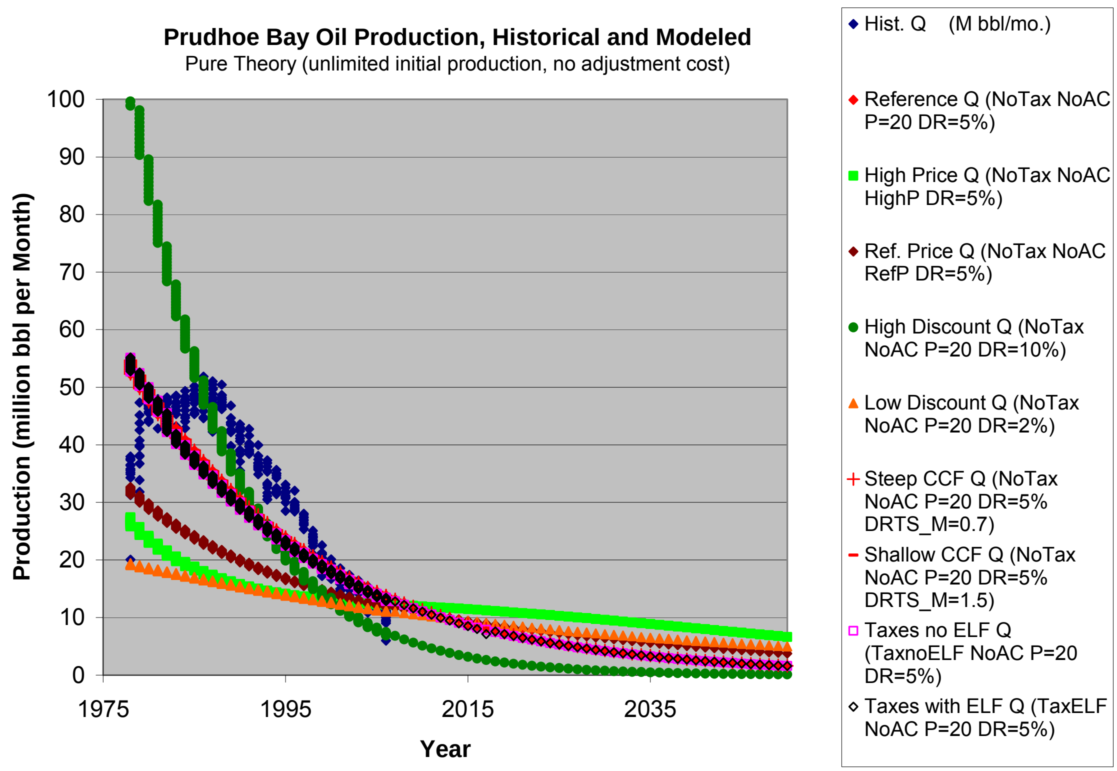
The second boundary condition comes directly from the constraints to which the producer's maximization problem is subject. That is,

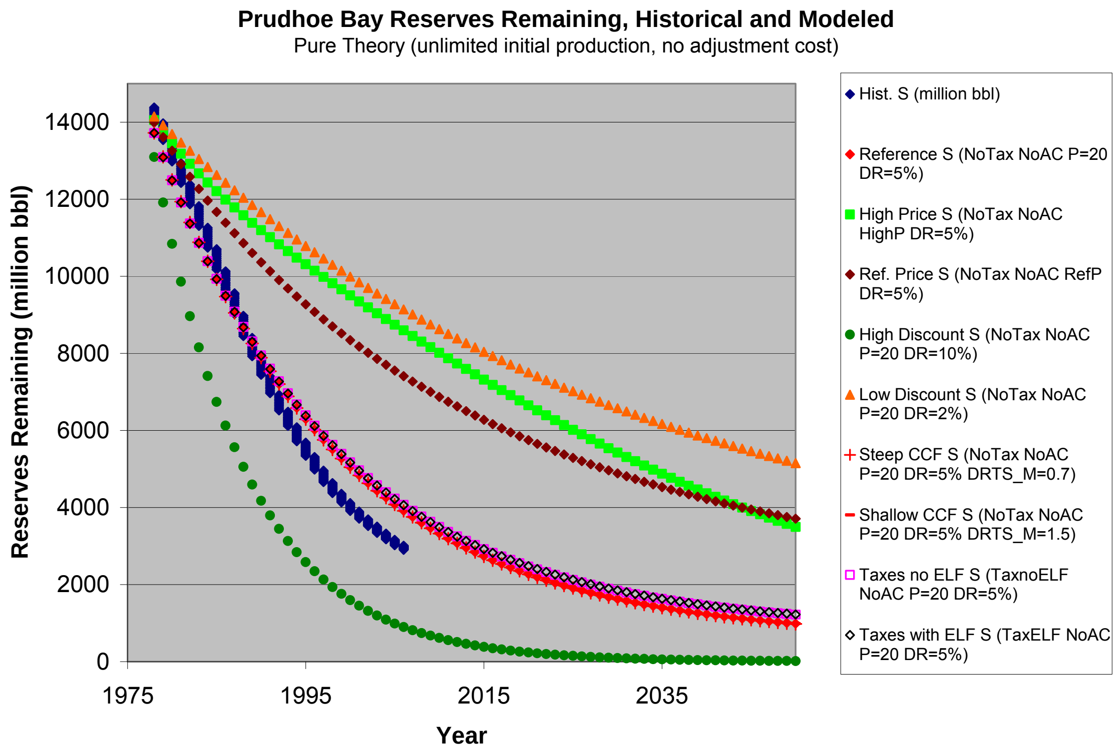
$S(0) = S_o$ or $X(0) = X_o$

Finally, the solution to the boundary value problem specified by the two differential equations and two boundary conditions is equivalent to the original producer's optimal control problem. The boundary value problem is solved with the software package Matlab.

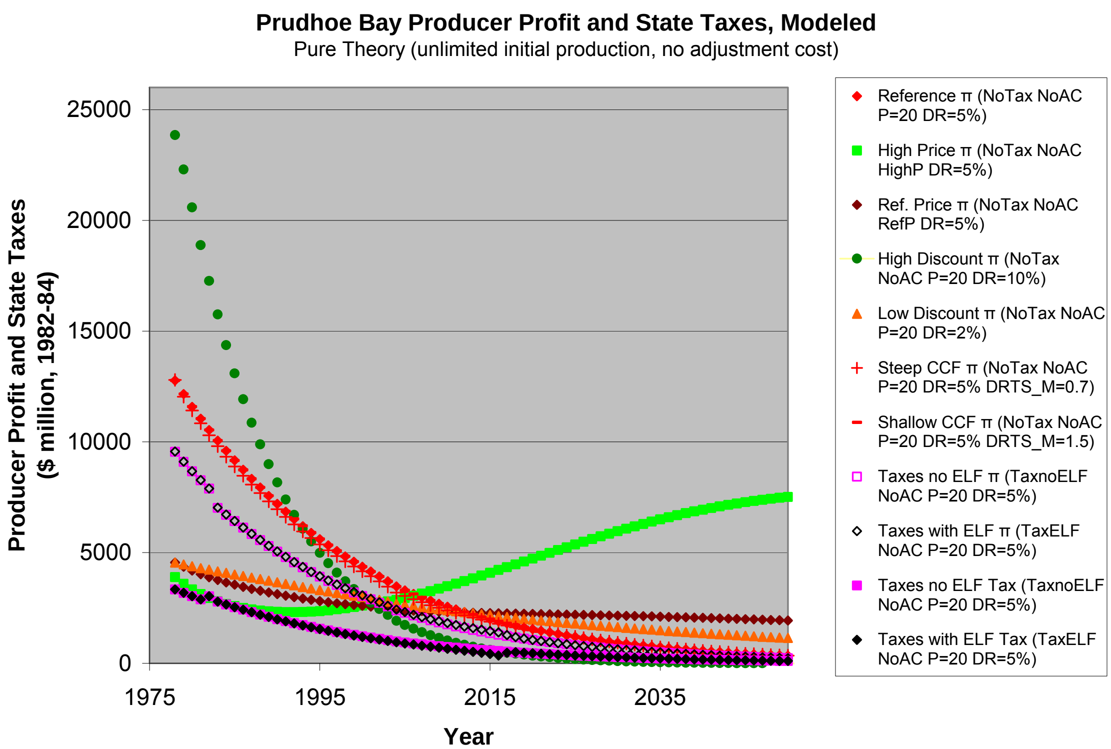
Appendix N: Model Results Plots by Field

Prudhoe Bay: *uncalibrated model results*

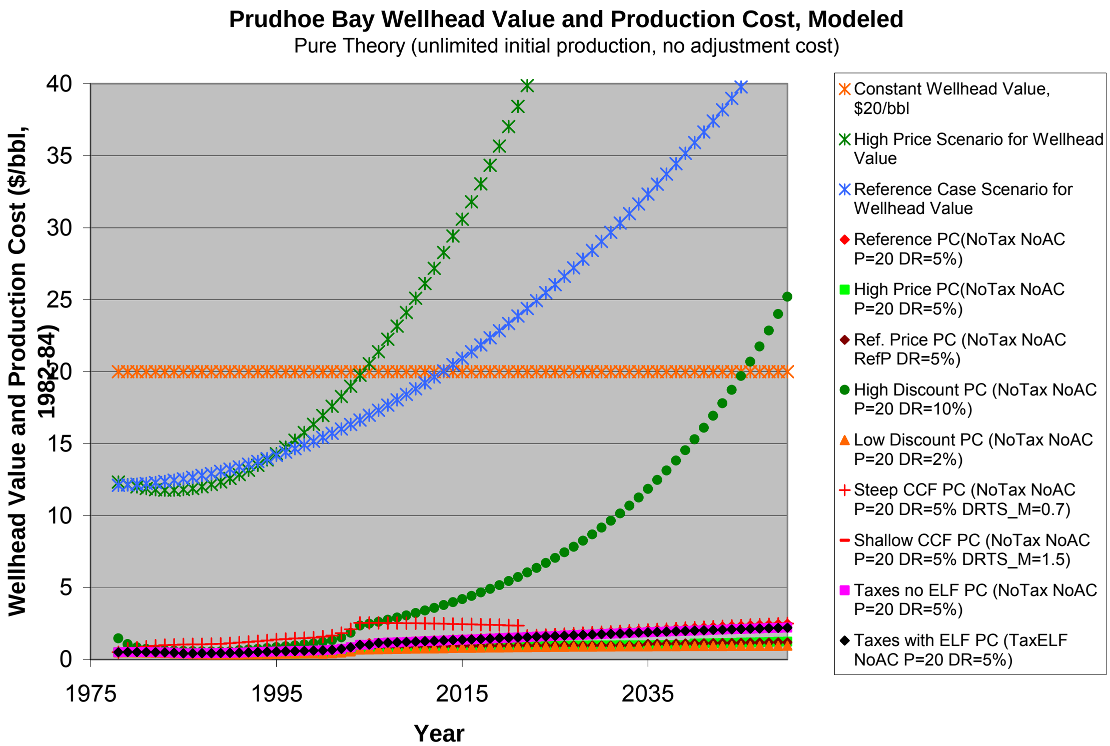




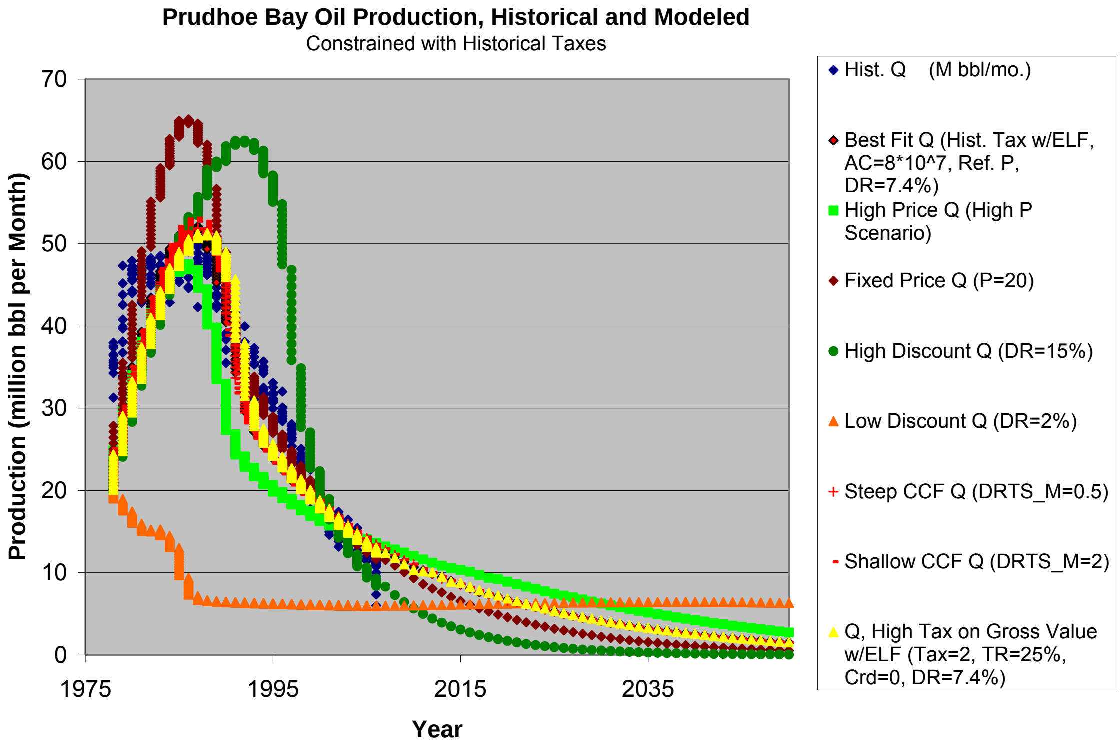
Prudhoe Bay Reserves Remaining, Historical and Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Reserves Remaining (million bbl)
14000
12000
10000
8000
6000
4000
2000
0
1975
1995
2015
2035
Year
Hist. S (million bbl)
Reference S (NoTax NoAC P=20 DR=5%)
High Price S (NoTax NoAC HighP DR=5%)
Ref. Price S (NoTax NoAC RefP DR=5%)
High Discount S (NoTax NoAC P=20 DR=10%)
Low Discount S (NoTax NoAC P=20 DR=2%)
Steep CCF S (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF S (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF S (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF S (TaxELF NoAC P=20 DR=5%)

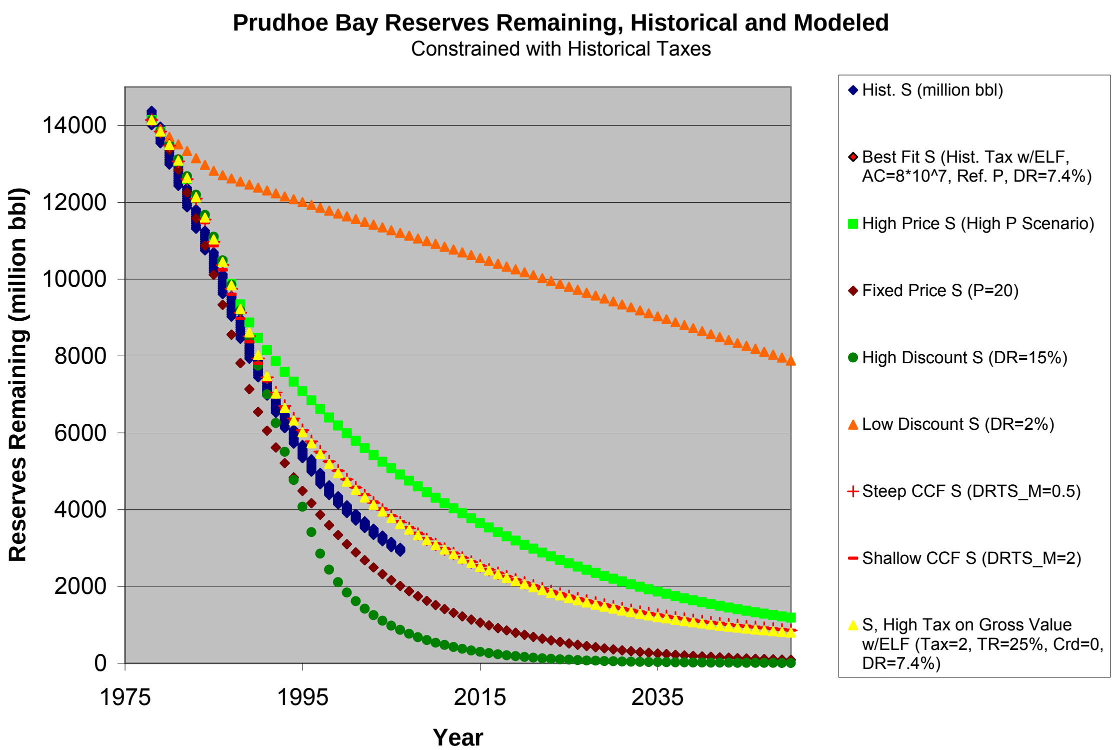


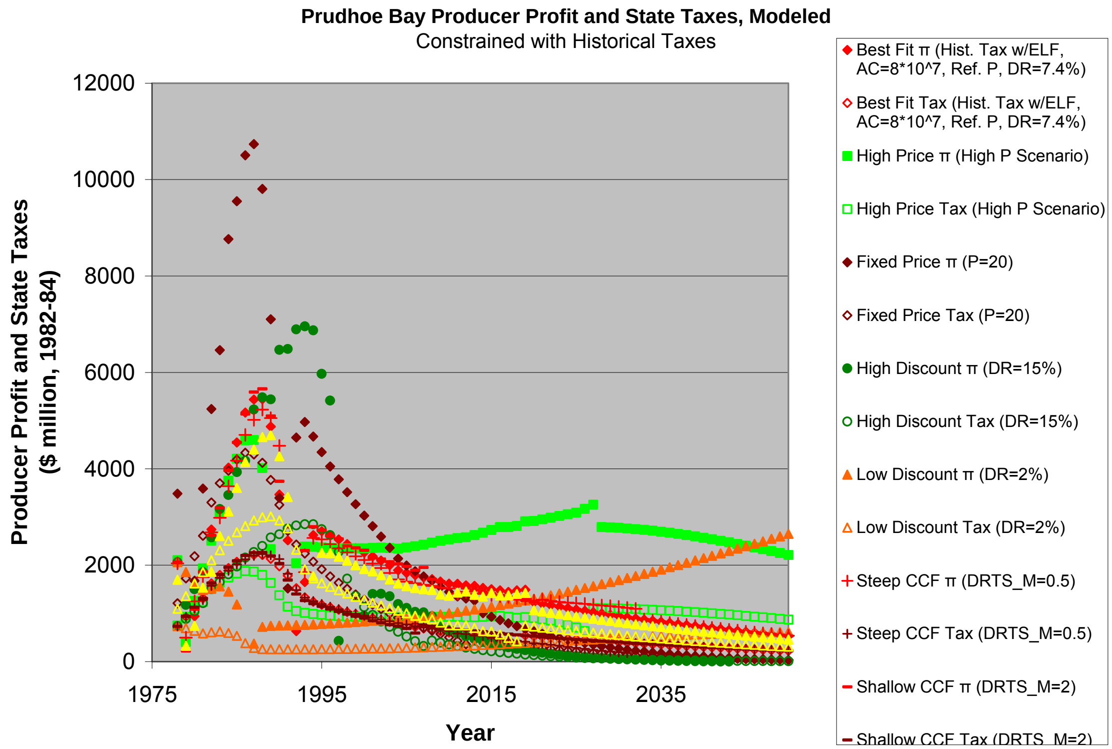
Prudhoe Bay Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
25000
20000
15000
10000
5000
0
1975
1995
2015
2035
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

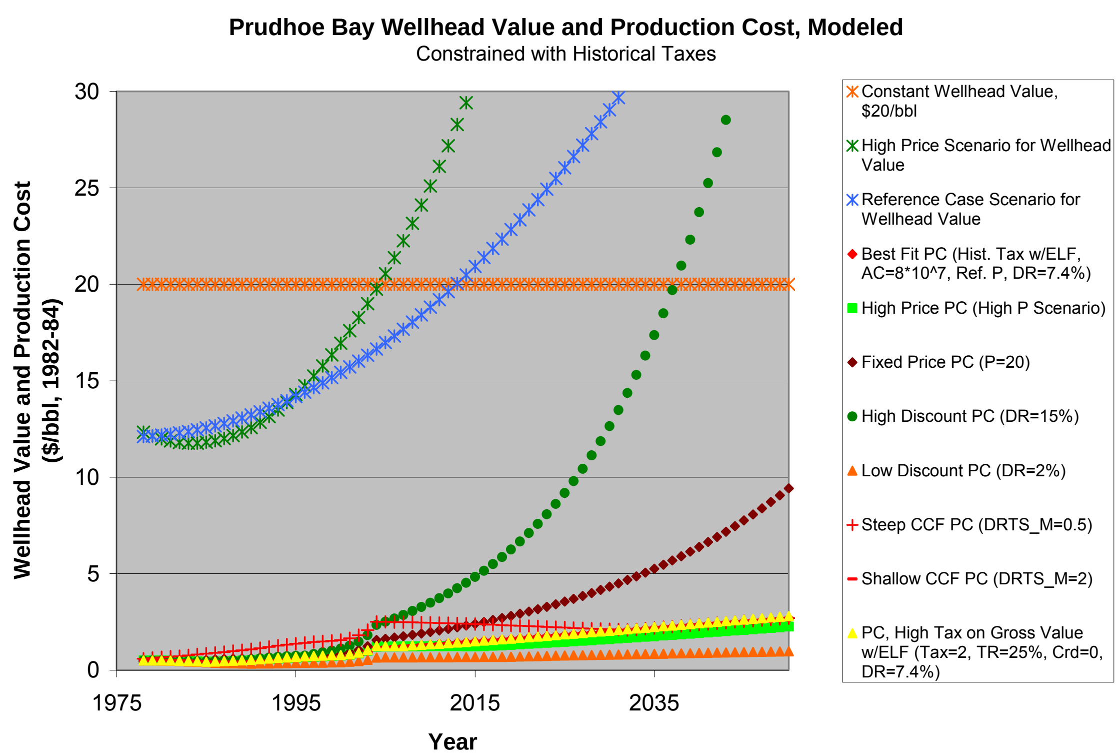

Prudhoe Bay Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
40
35
30
25
20
15
10
5
0
1975
1995
2015
2035
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

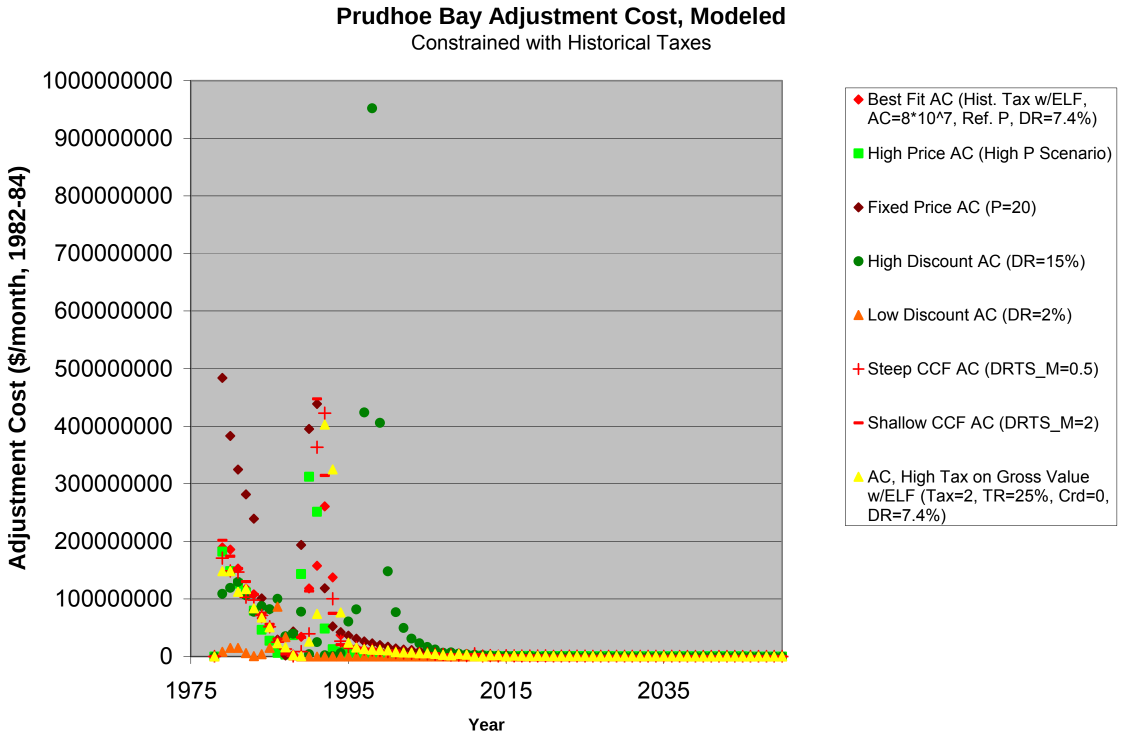
Prudhoe Bay: *calibrated model results*



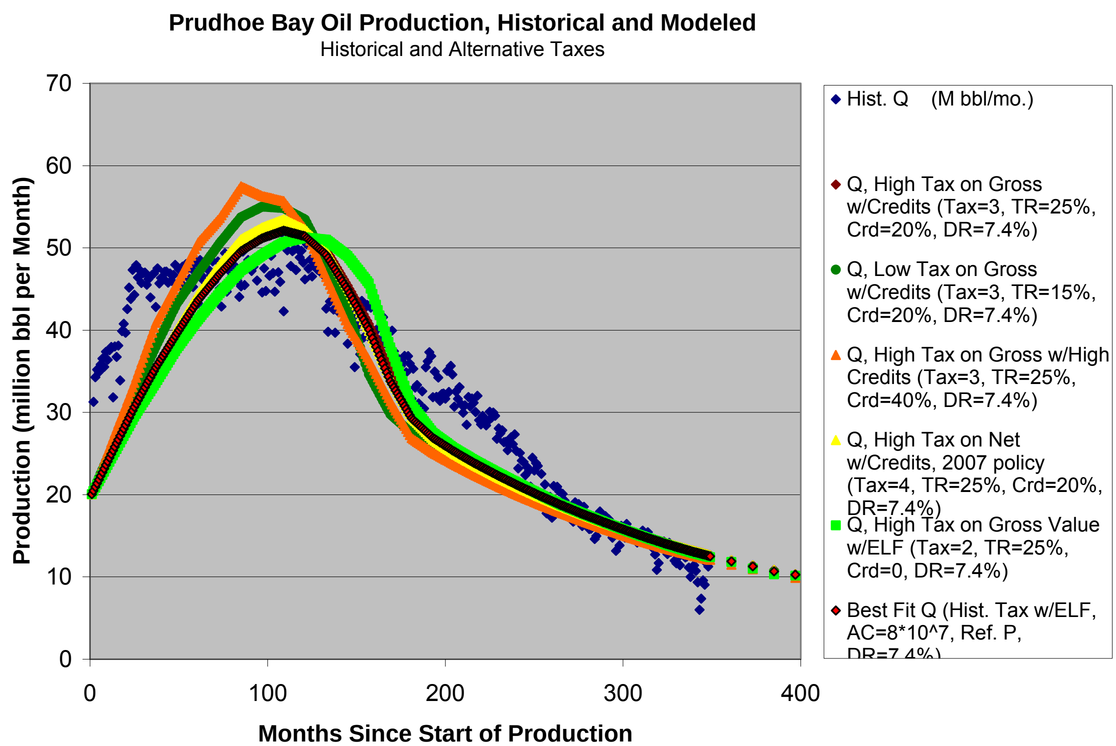


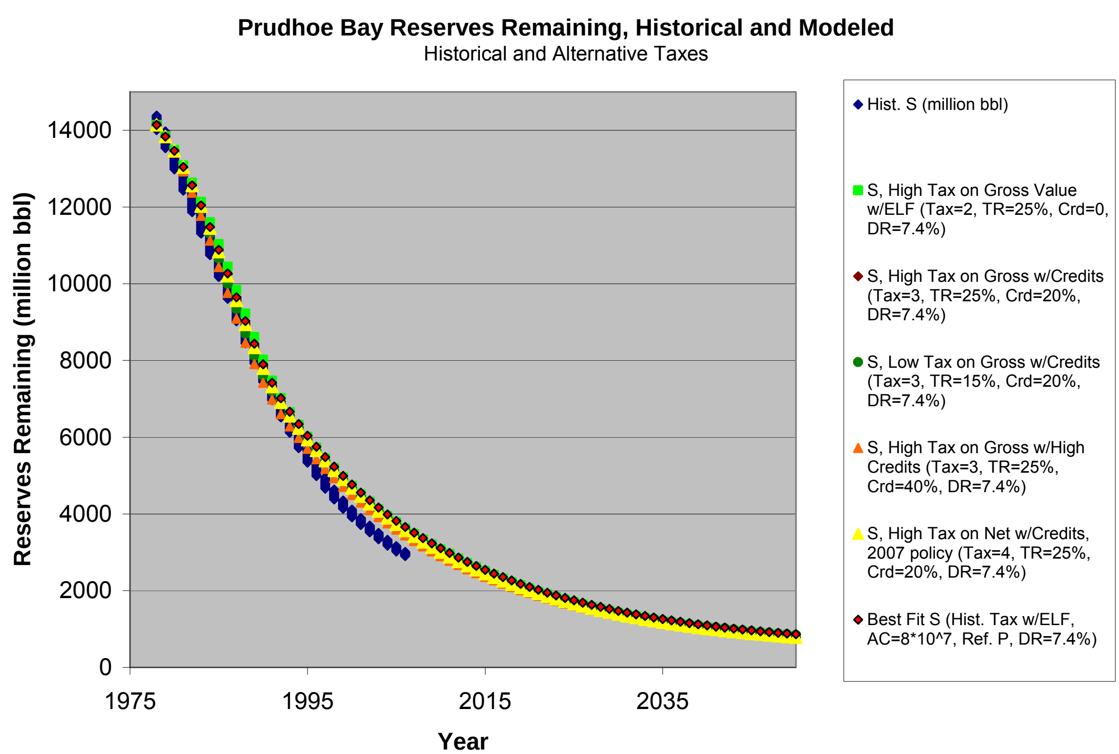

Prudhoe Bay Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
Best Fit π (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
Best Fit Tax (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π (DR=15%)
High Discount Tax (DR=15%)
Low Discount π (DR=2%)
Low Discount Tax (DR=2%)
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)

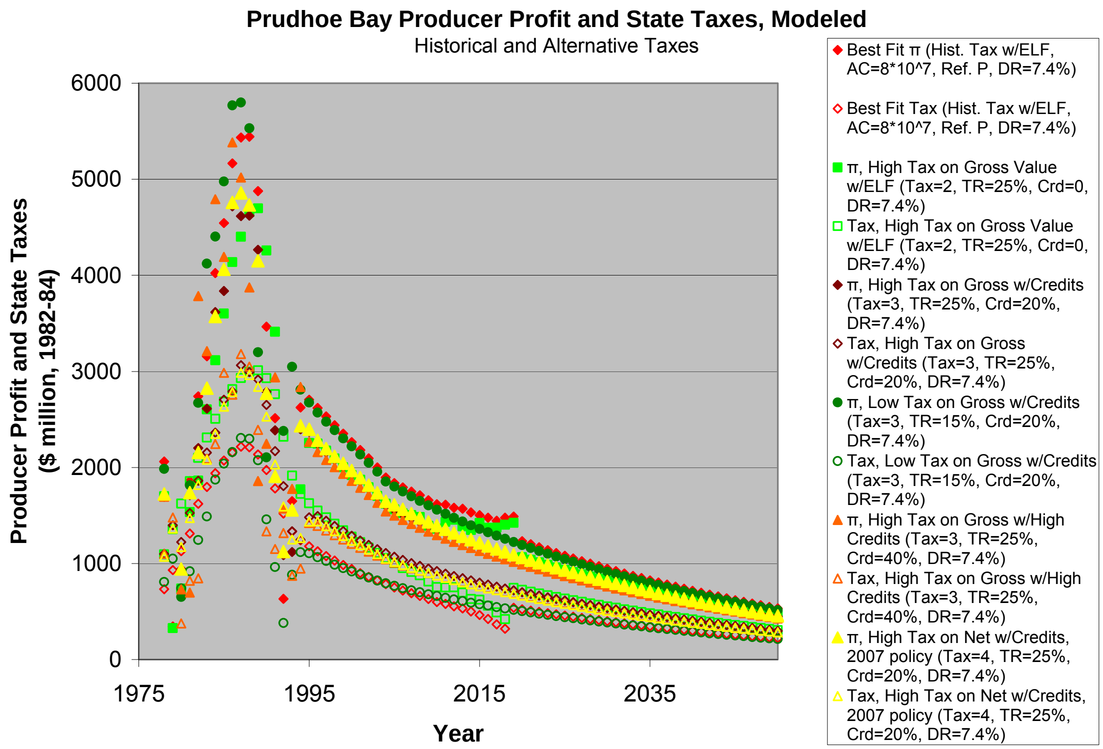

Prudhoe Bay Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC (DR=15%)
Low Discount PC (DR=2%)
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=7.4%)

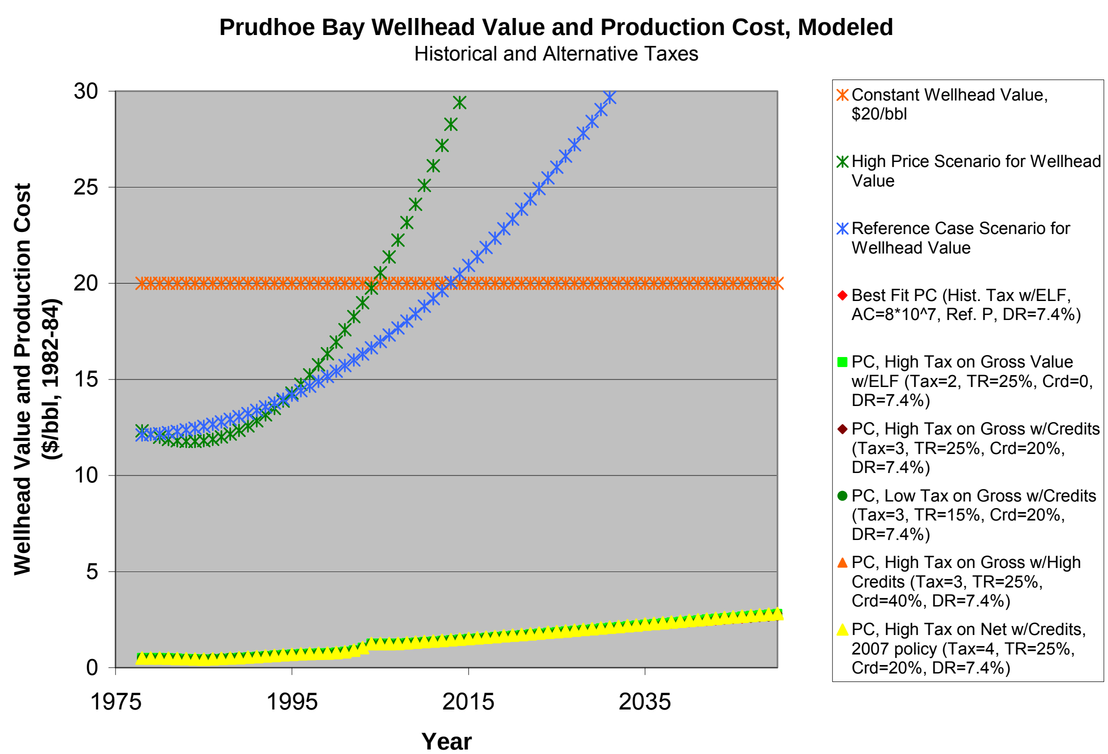

Prudhoe Bay Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
1000000000
900000000
800000000
700000000
600000000
500000000
400000000
300000000
200000000
100000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC (DR=15%)
Low Discount AC (DR=2%)
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=7.4%)

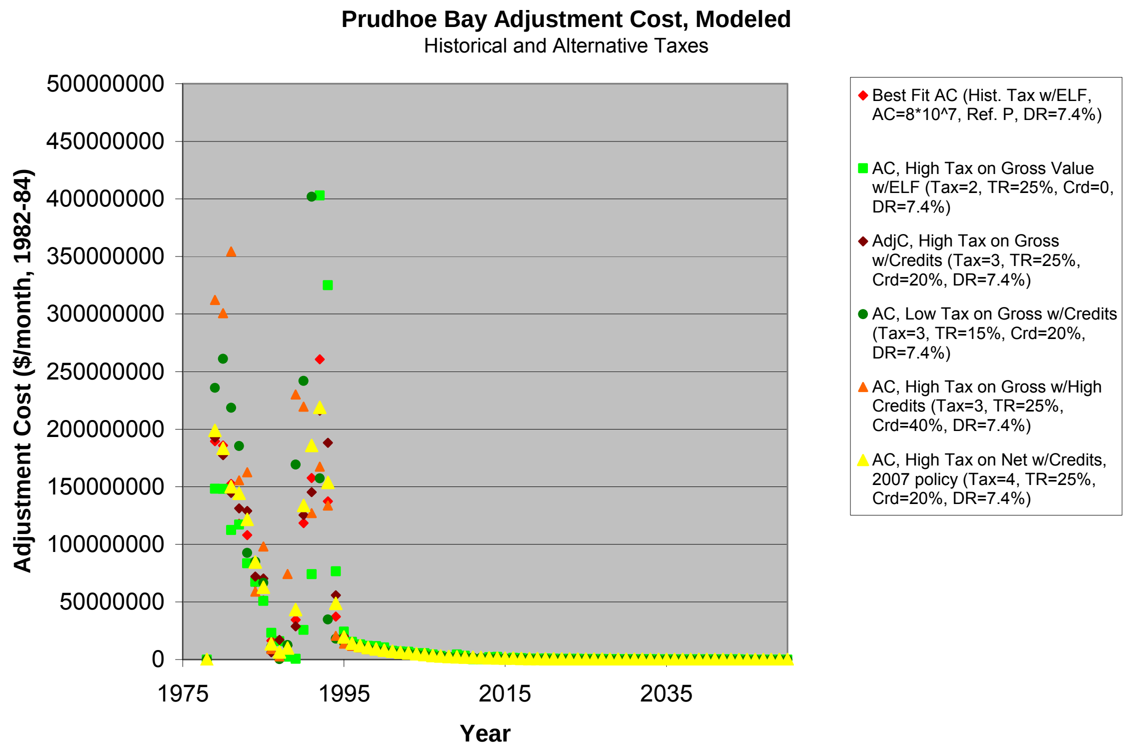
Prudhoe Bay: *tax scenario model results*





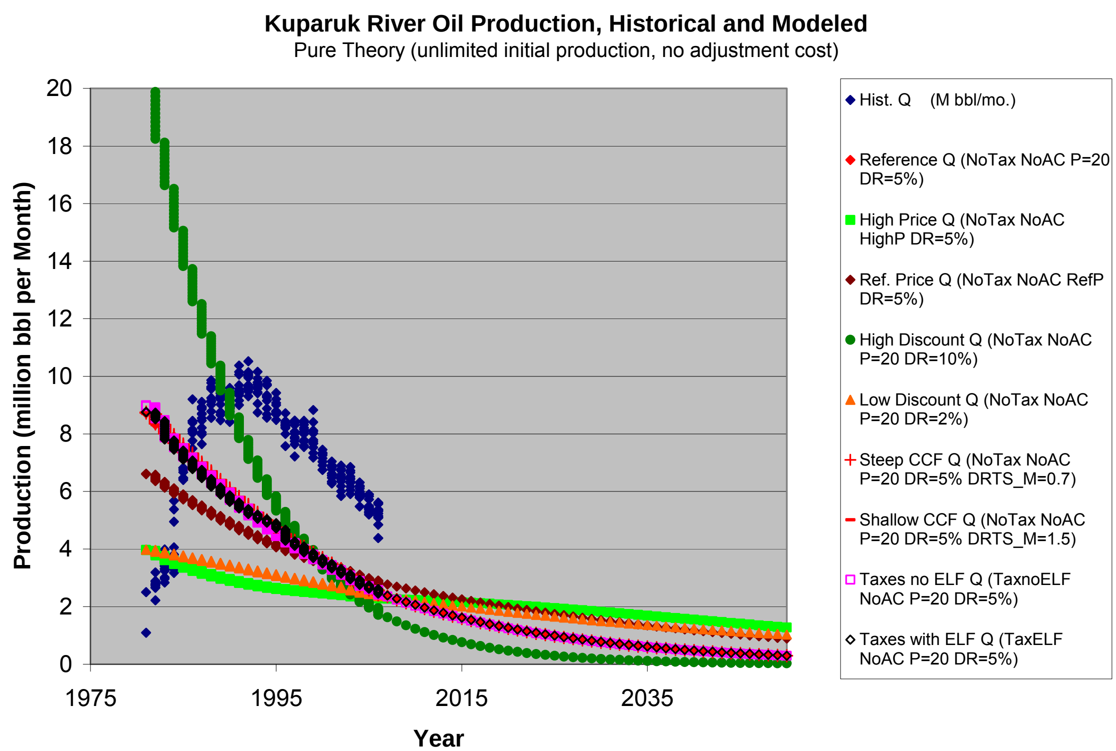

Prudhoe Bay Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes ($ million, 1982-84)
0
1000
2000
3000
4000
5000
6000
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
Best Fit Tax (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=7.4%)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=7.4%)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=7.4%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=7.4%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=7.4%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=7.4%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=7.4%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=7.4%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=7.4%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=7.4%)

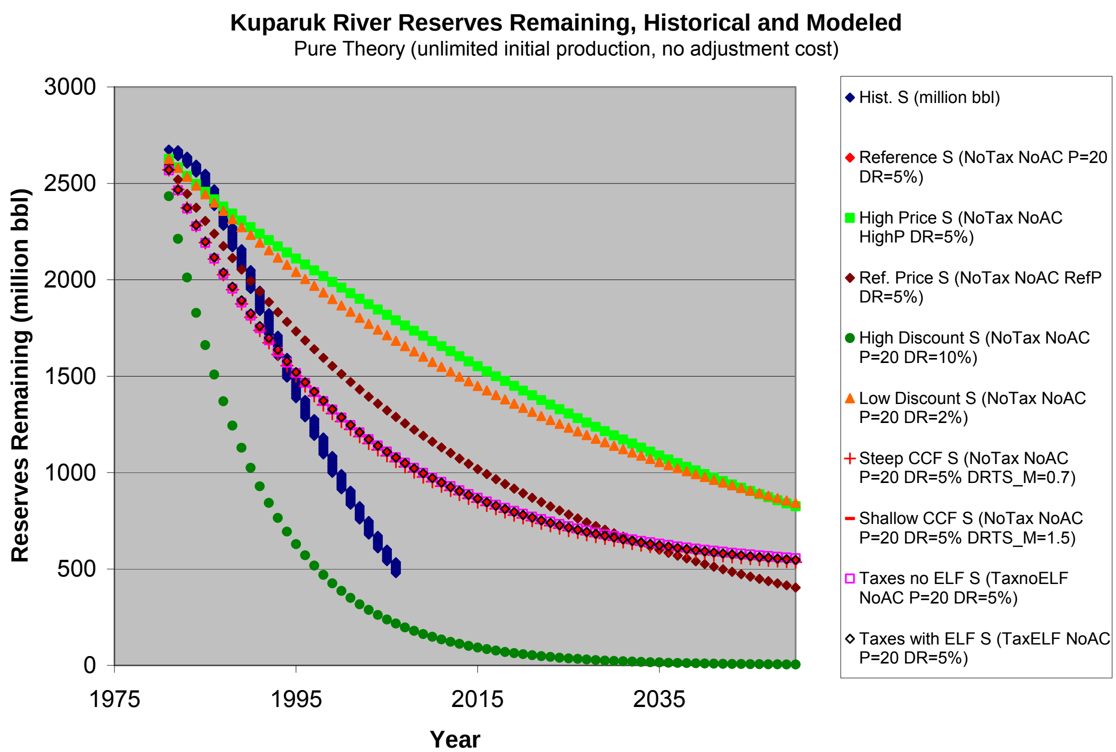

Prudhoe Bay Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
0
5
10
15
20
25
30
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=7.4%)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=7.4%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=7.4%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=7.4%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=7.4%)

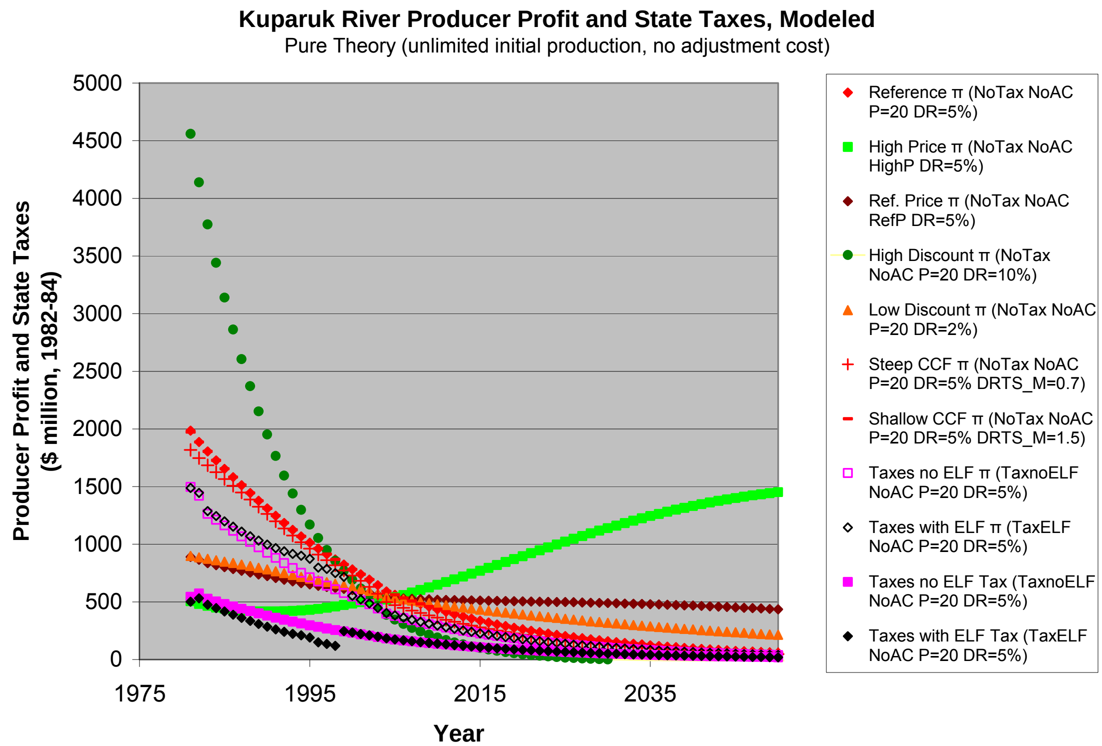


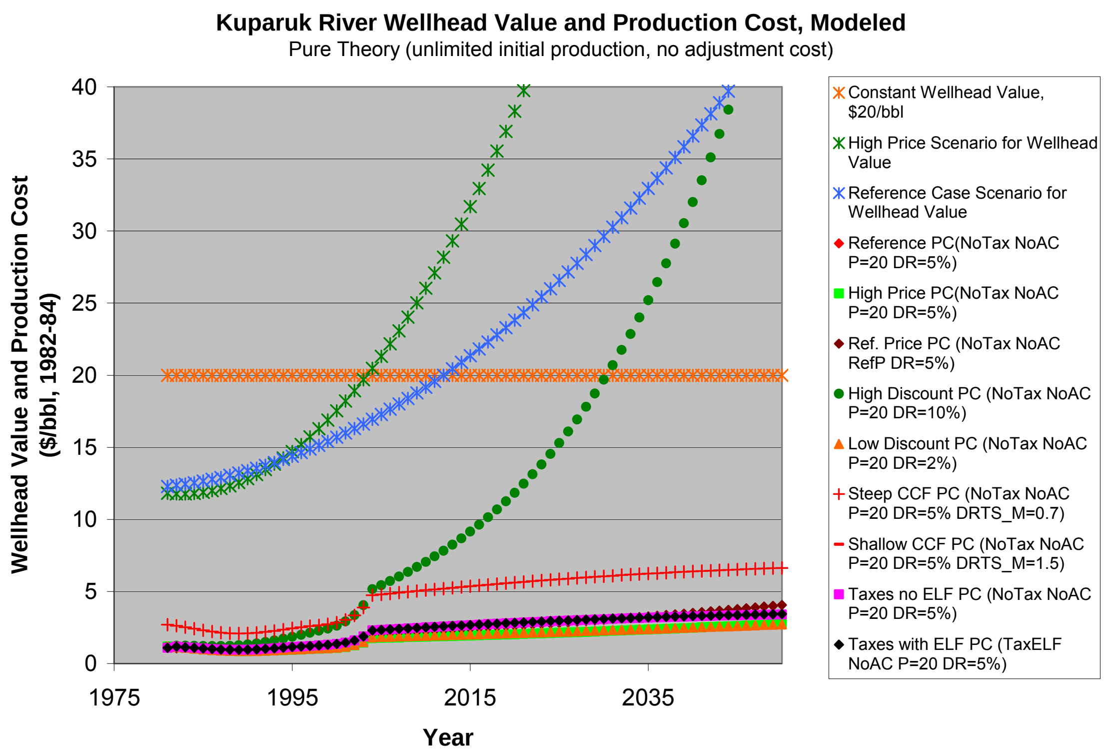
Prudhoe Bay Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
500000000
450000000
400000000
350000000
300000000
250000000
200000000
150000000
100000000
50000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=8*10^7, Ref. P, DR=7.4%)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=7.4%)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=7.4%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=7.4%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=7.4%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=7.4%)

Kuparuk River: *uncalibrated model results*

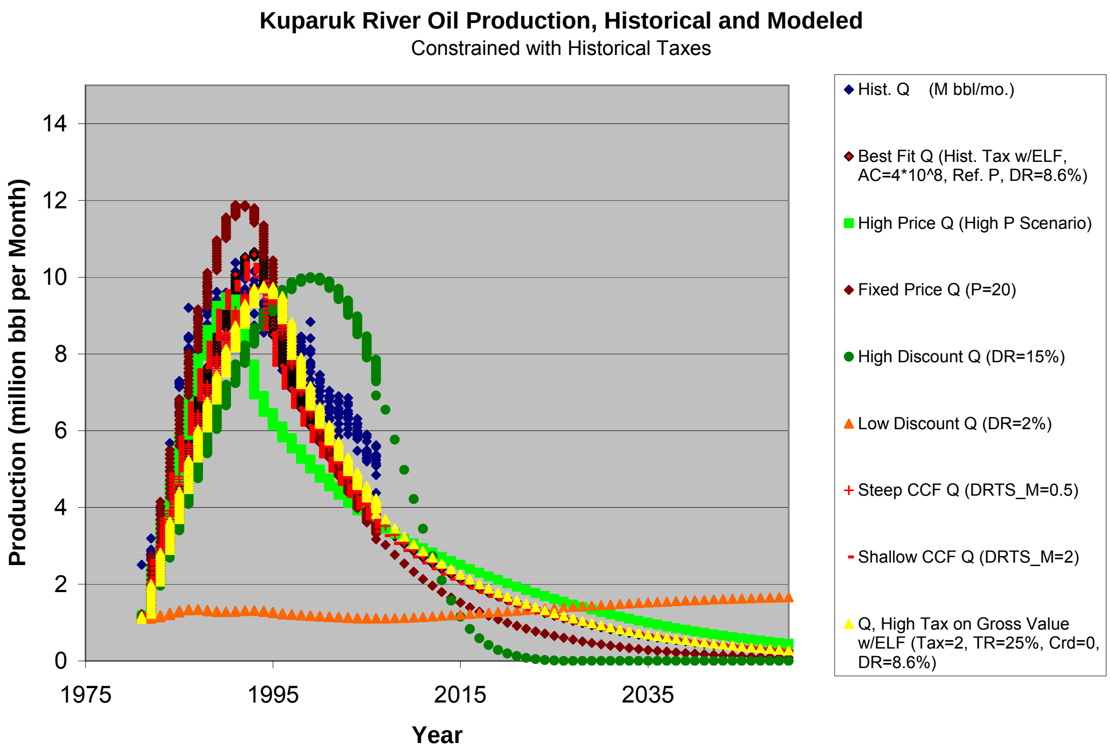


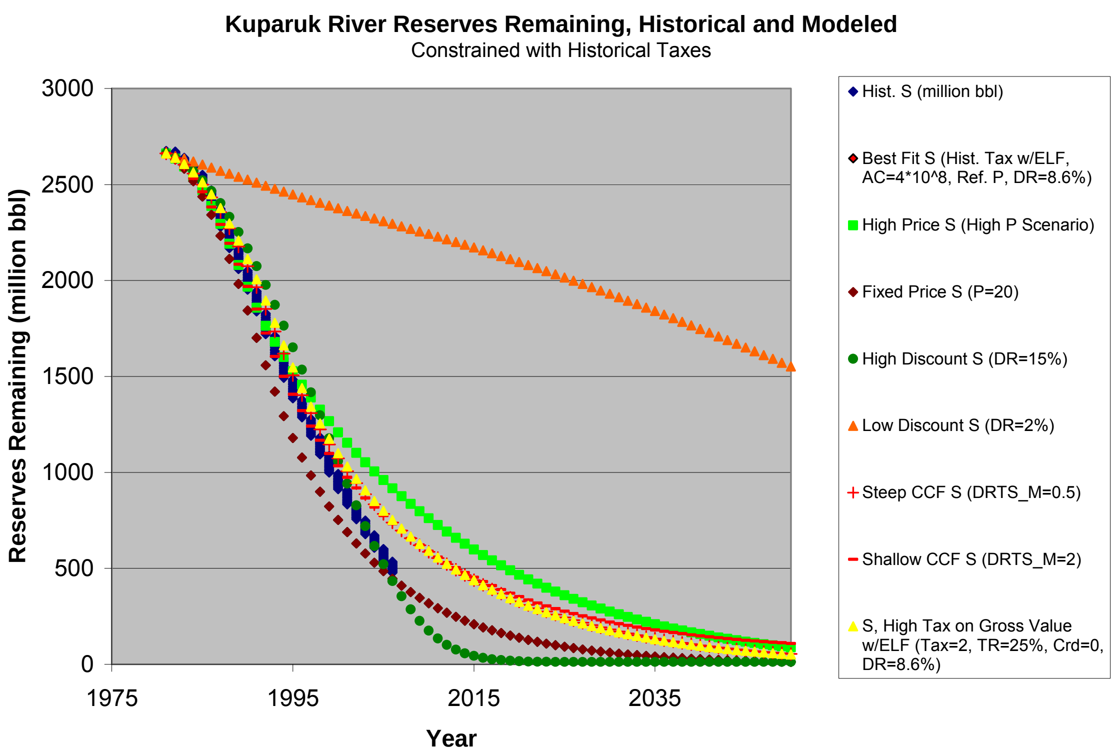


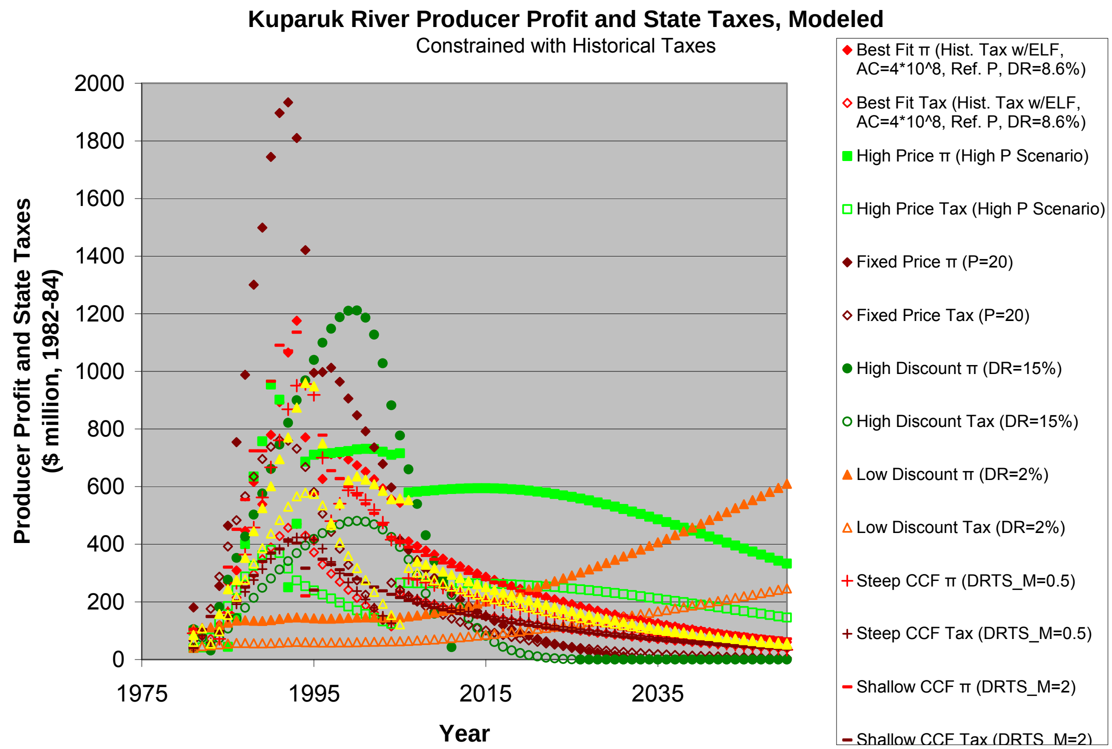

Kuparuk River Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

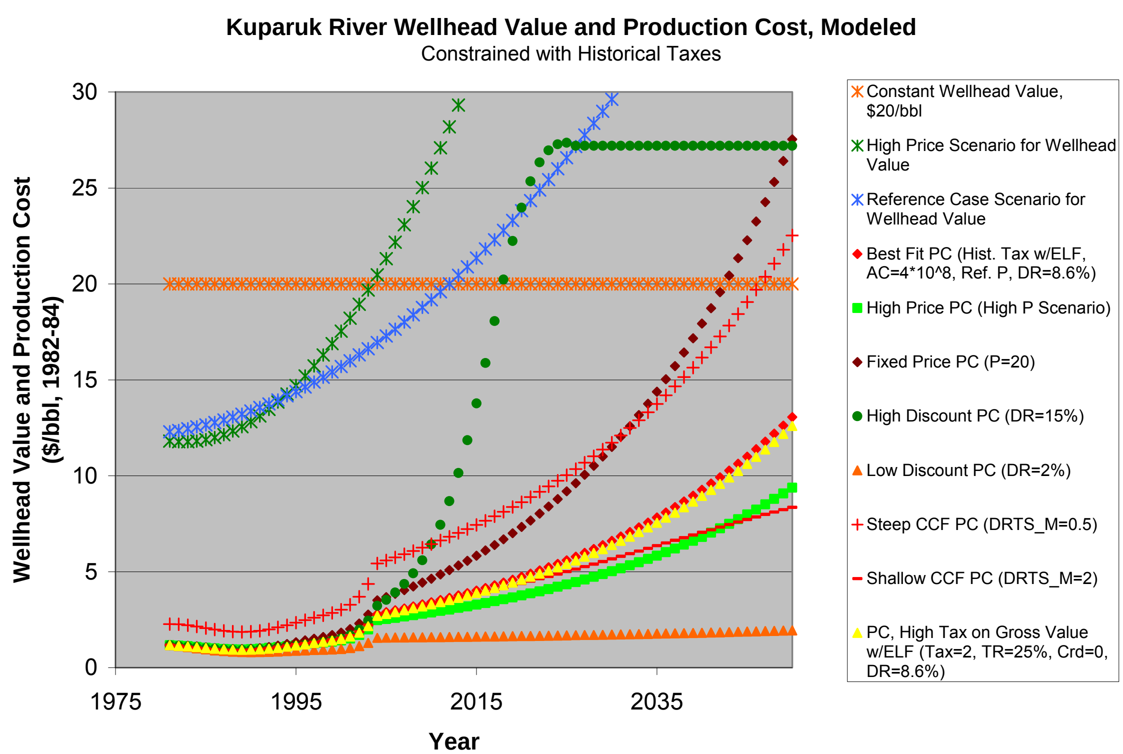

Kuparuk River Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

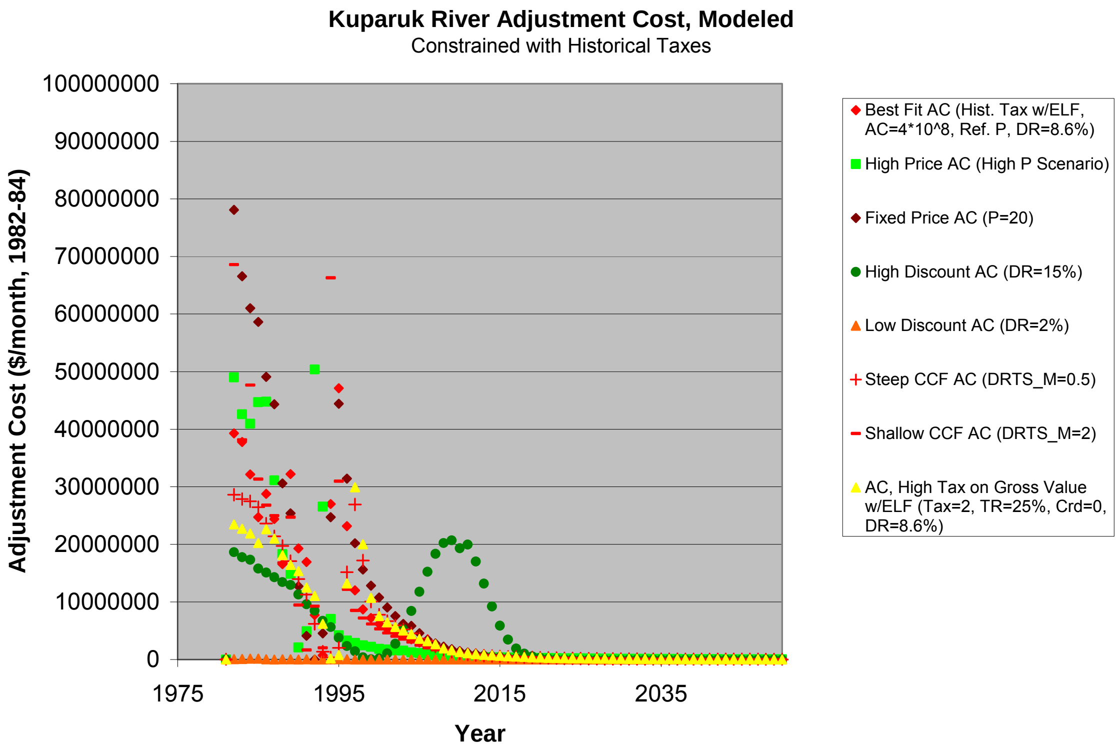
Kuparuk River: *calibrated model results*



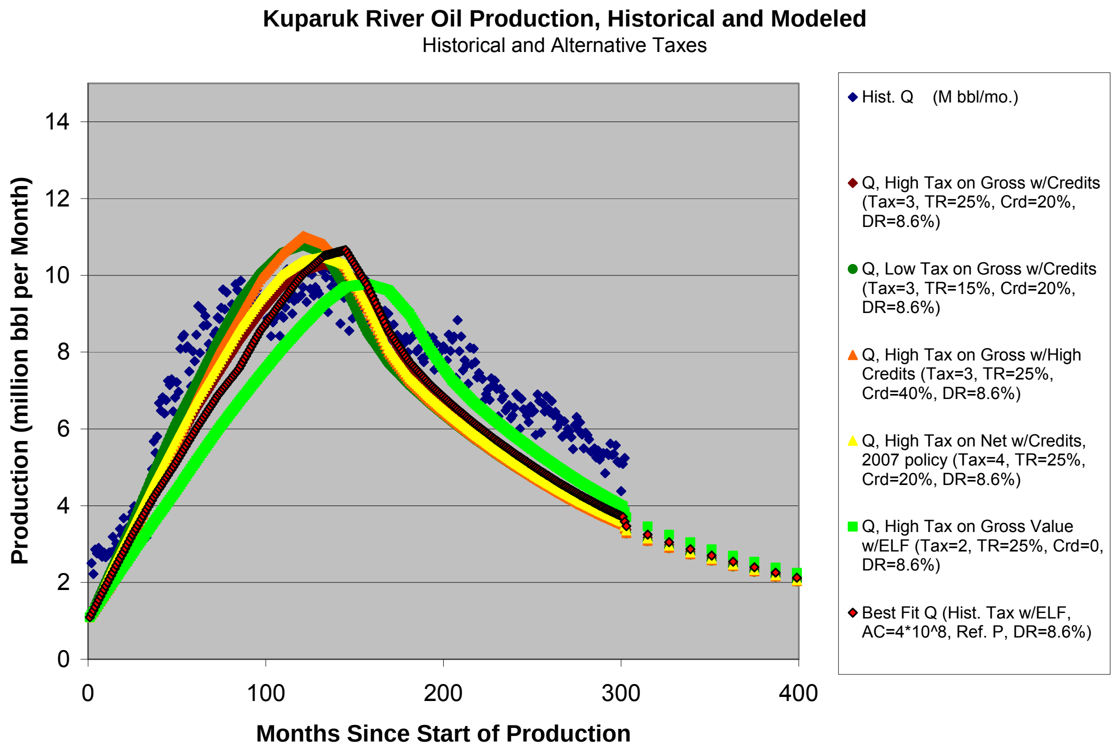


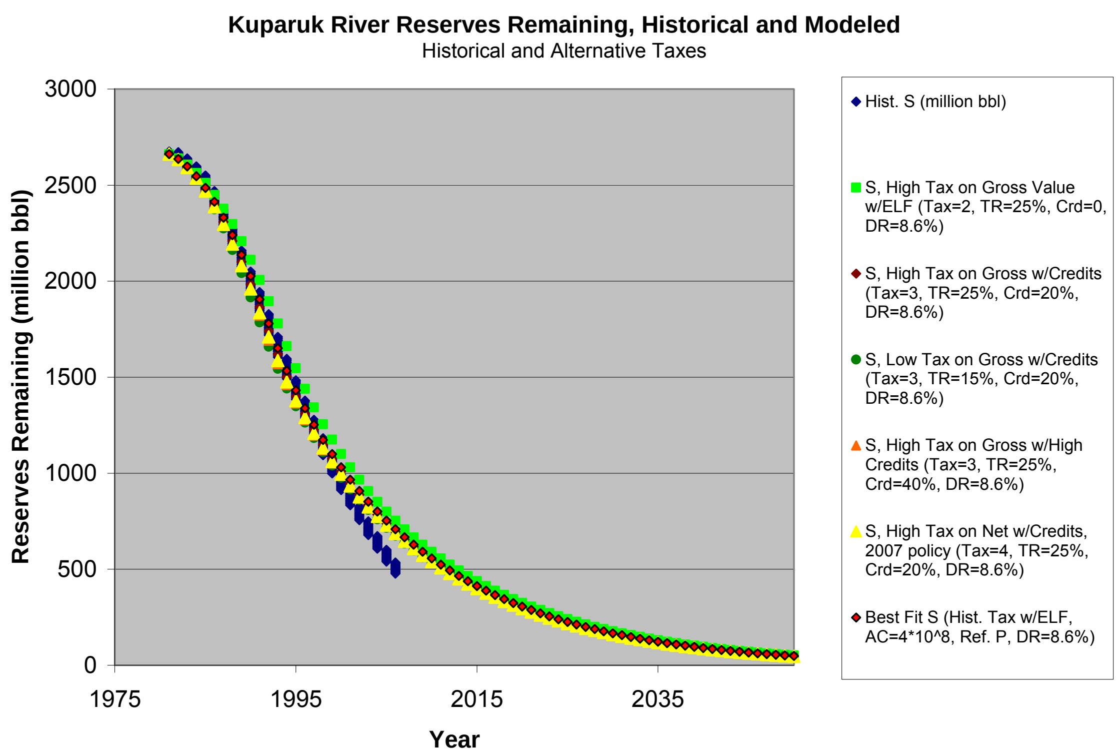

Kuparuk River Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
Best Fit π (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
Best Fit Tax (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π (DR=15%)
High Discount Tax (DR=15%)
Low Discount π (DR=2%)
Low Discount Tax (DR=2%)
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)
Shallow CCF Tax (DRTS_M=2)

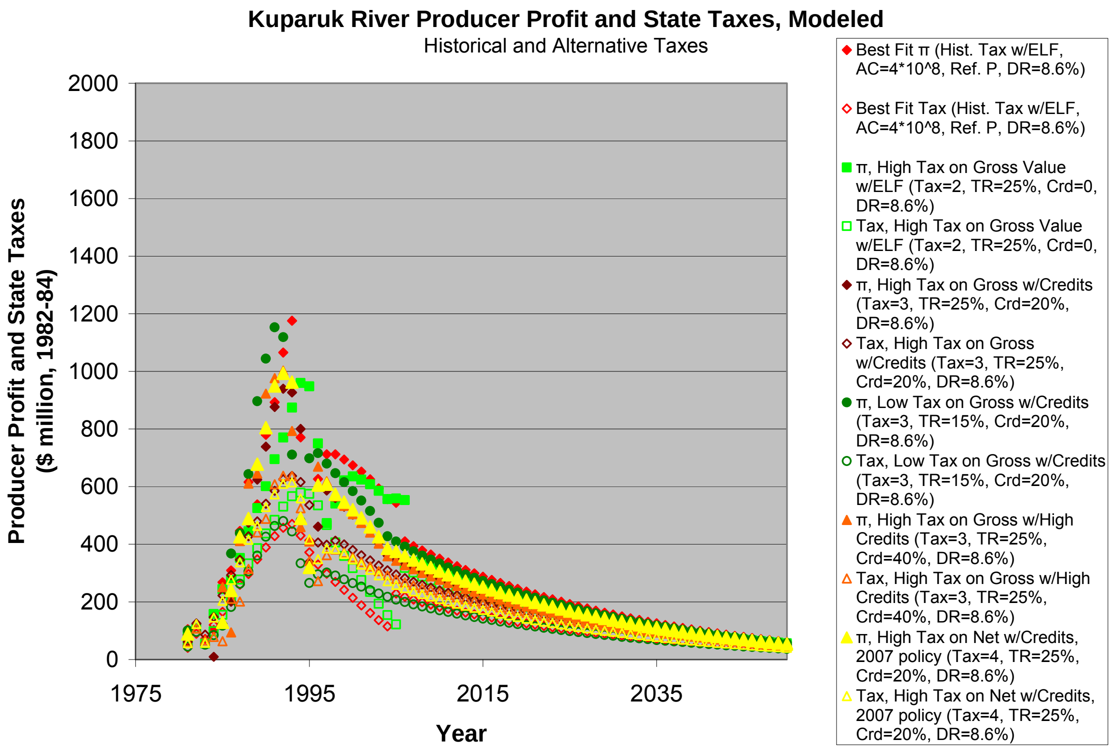

Kuparuk River Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC (DR=15%)
Low Discount PC (DR=2%)
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=8.6%)

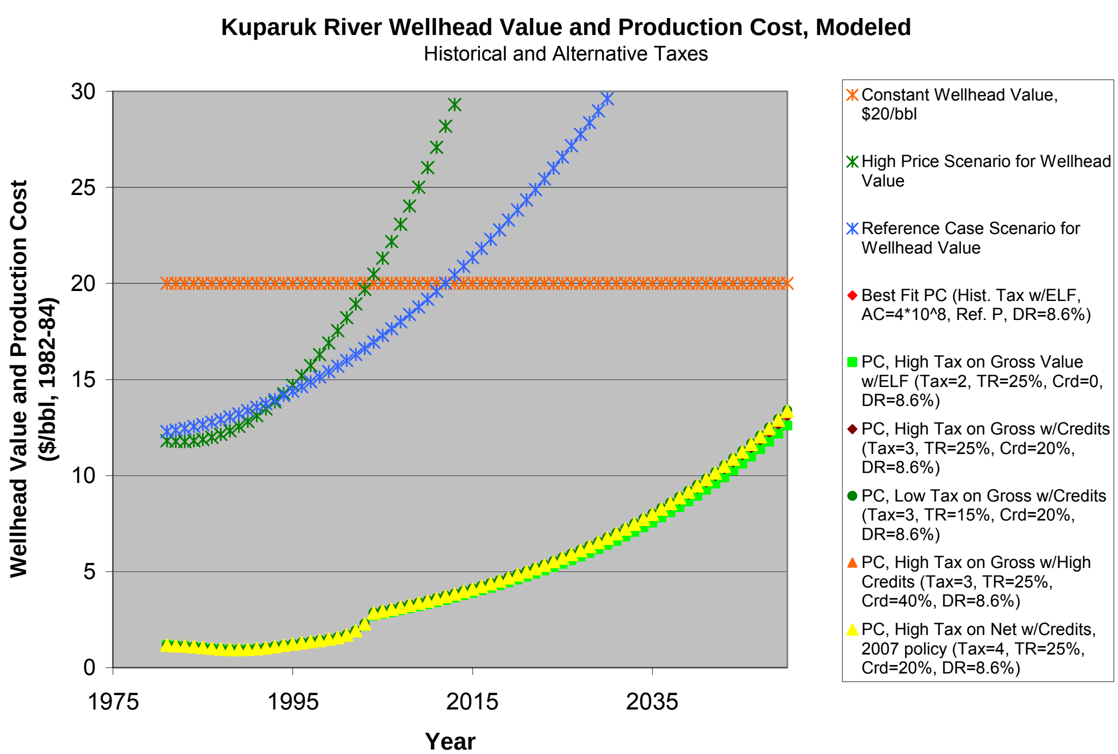

Kuparuk River Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
100000000
90000000
80000000
70000000
60000000
50000000
40000000
30000000
20000000
10000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC (DR=15%)
Low Discount AC (DR=2%)
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=8.6%)

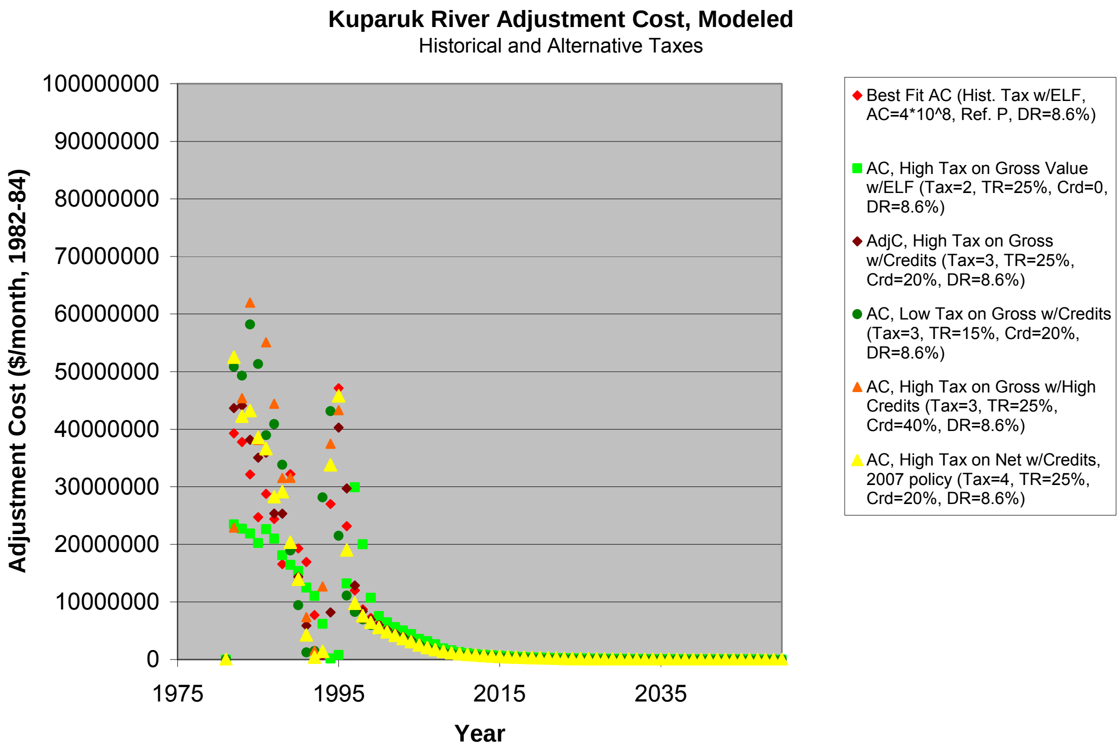
Kuparuk River: *tax scenario model results*



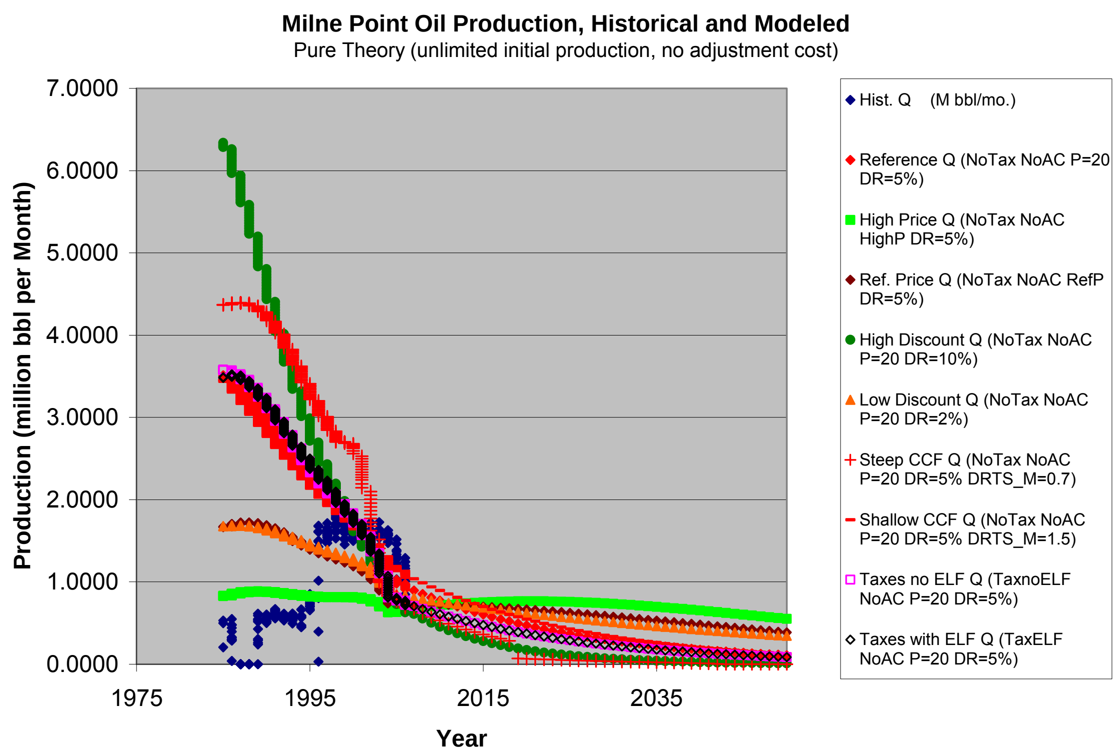


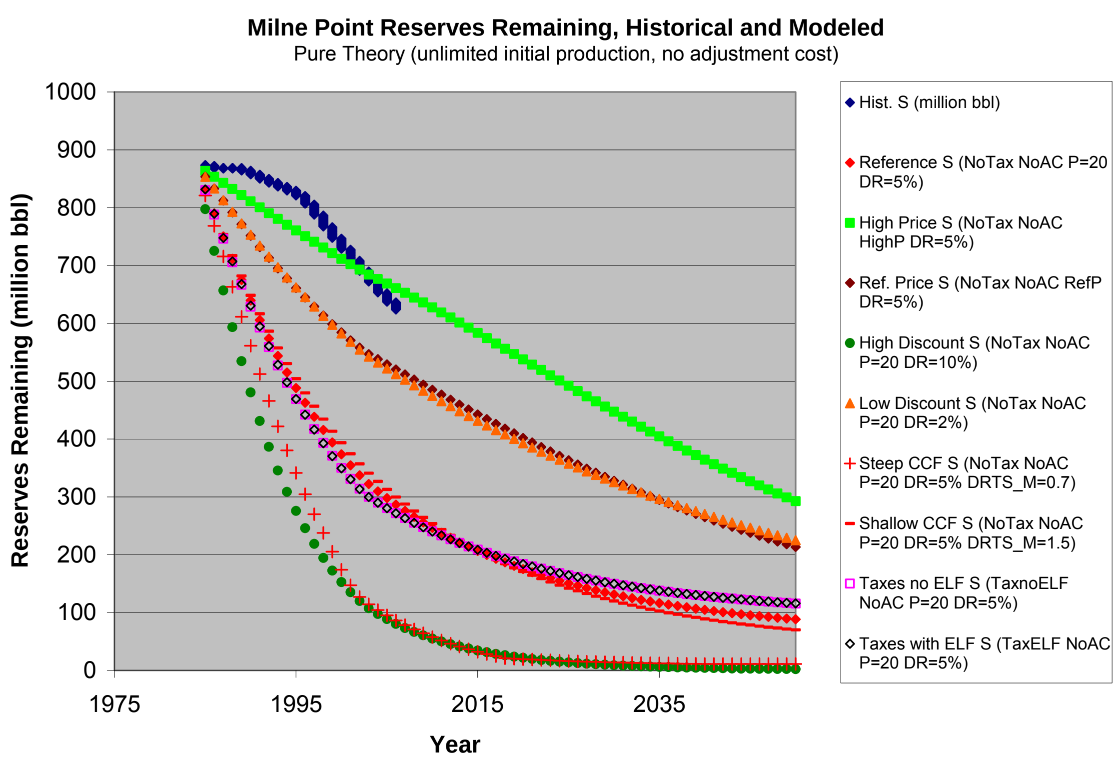

Kuparuk River Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
2000
1800
1600
1400
1200
1000
800
600
400
200
0
1975
1995
2015
2035
Best Fit π (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
Best Fit Tax (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=8.6%)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=8.6%)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=8.6%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=8.6%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=8.6%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=8.6%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=8.6%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=8.6%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=8.6%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=8.6%)

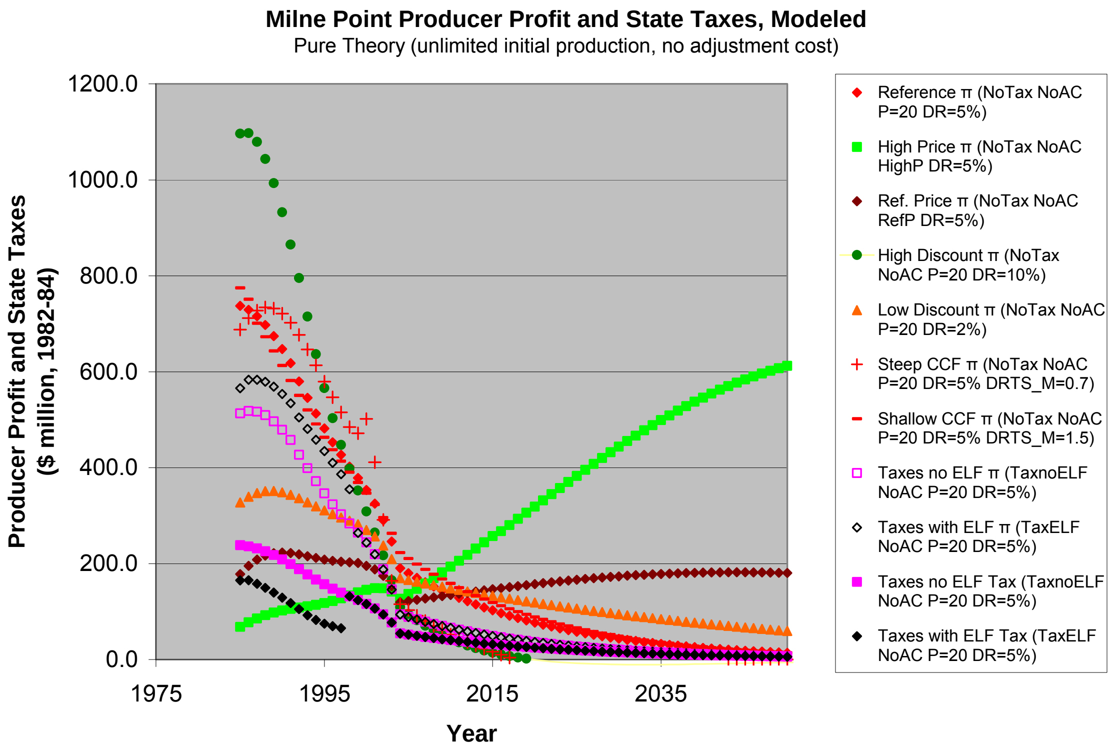

Kuparuk River Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
30
25
20
15
10
5
0
1975
1995
2015
2035
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=8.6%)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=8.6%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=8.6%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=8.6%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=8.6%)

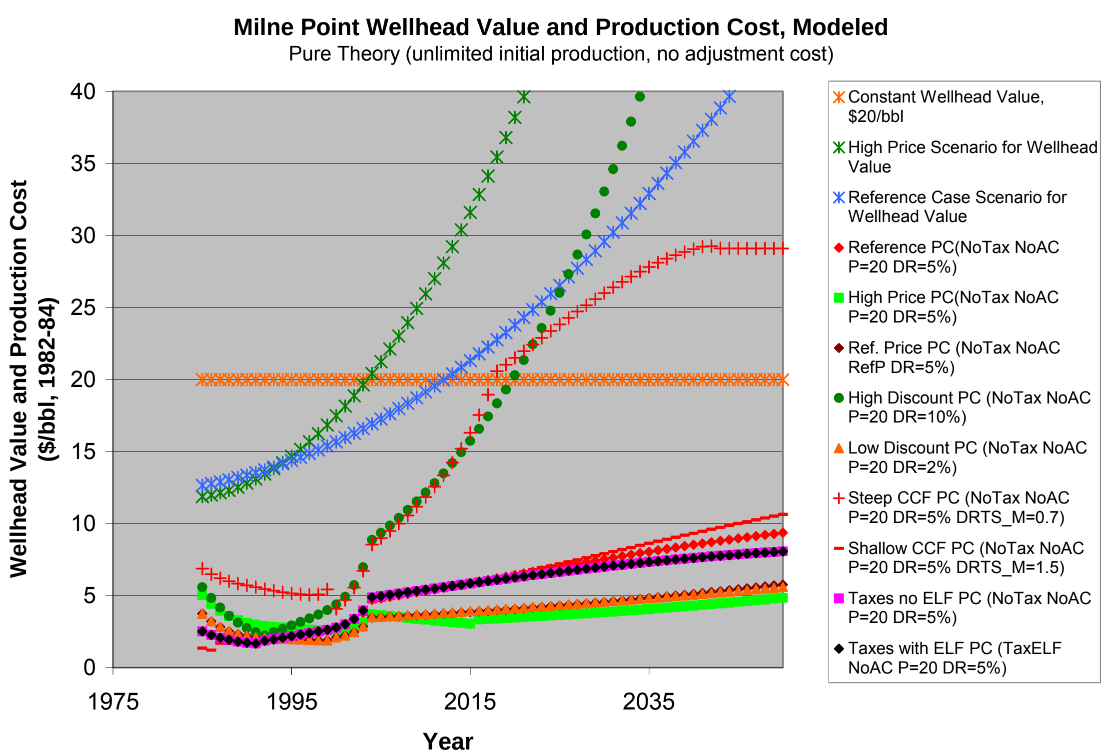

Kuparuk River Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
100000000
90000000
80000000
70000000
60000000
50000000
40000000
30000000
20000000
10000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=4*10^8, Ref. P, DR=8.6%)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=8.6%)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=8.6%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=8.6%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=8.6%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=8.6%)

Milne Point: *uncalibrated model results*

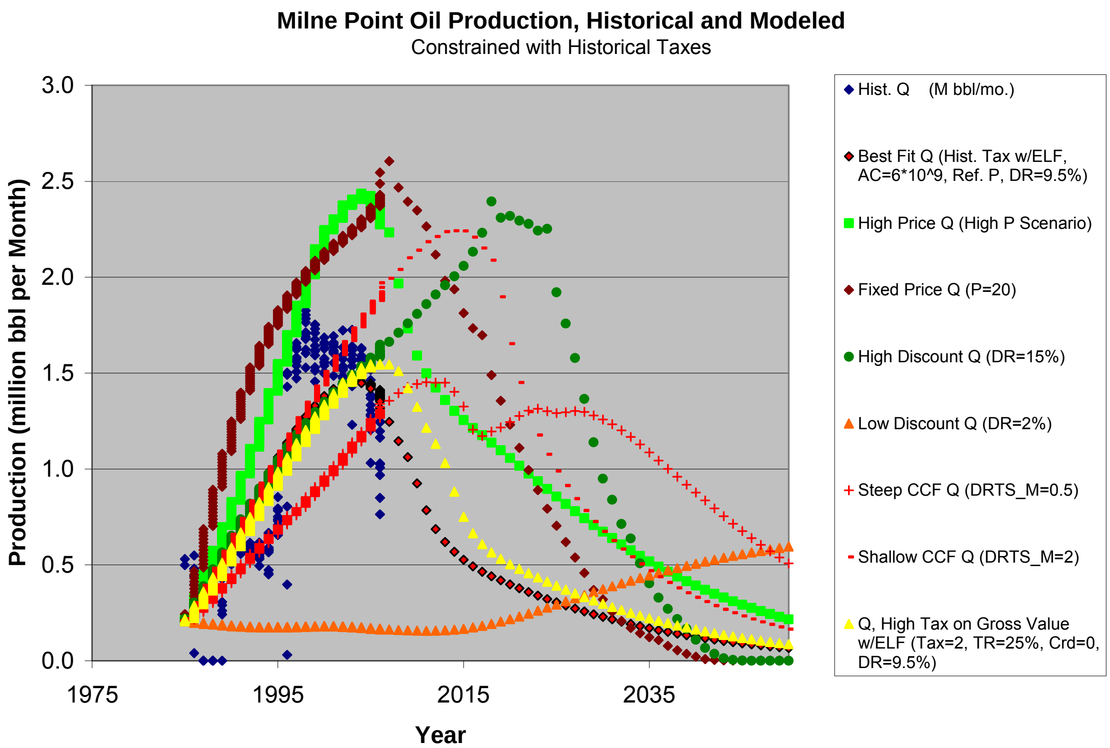


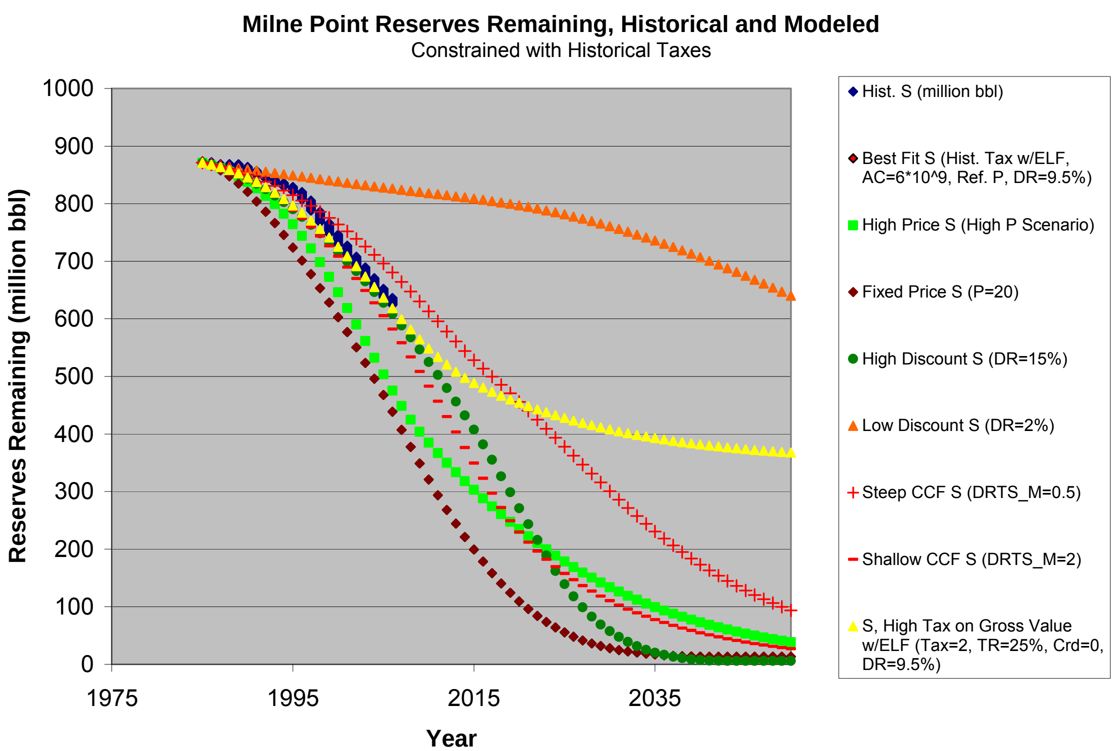


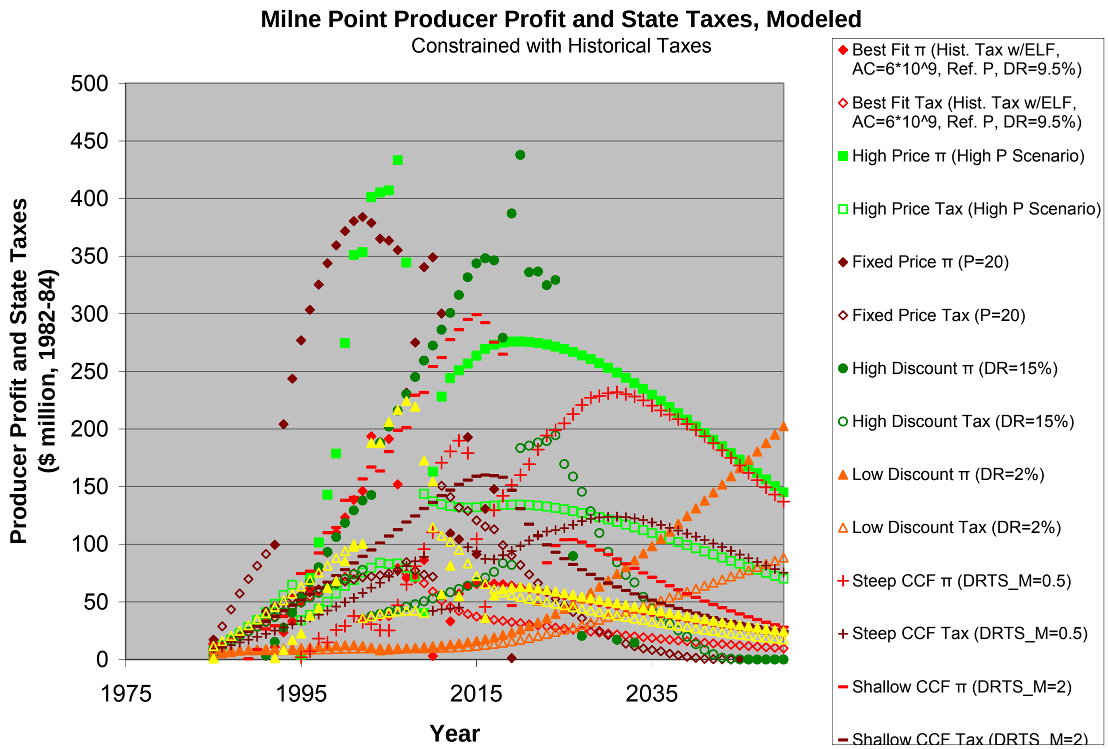

Milne Point Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
1200.0
1000.0
800.0
600.0
400.0
200.0
0.0
1975
1995
2015
2035
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

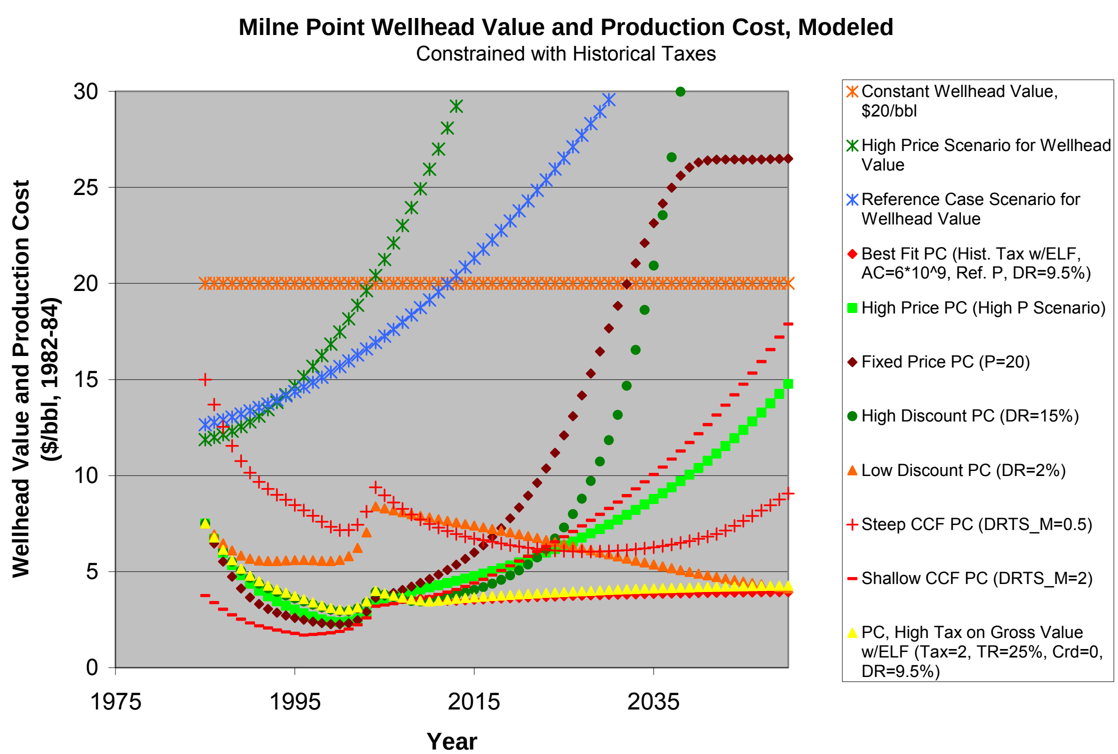

Milne Point Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
40
35
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

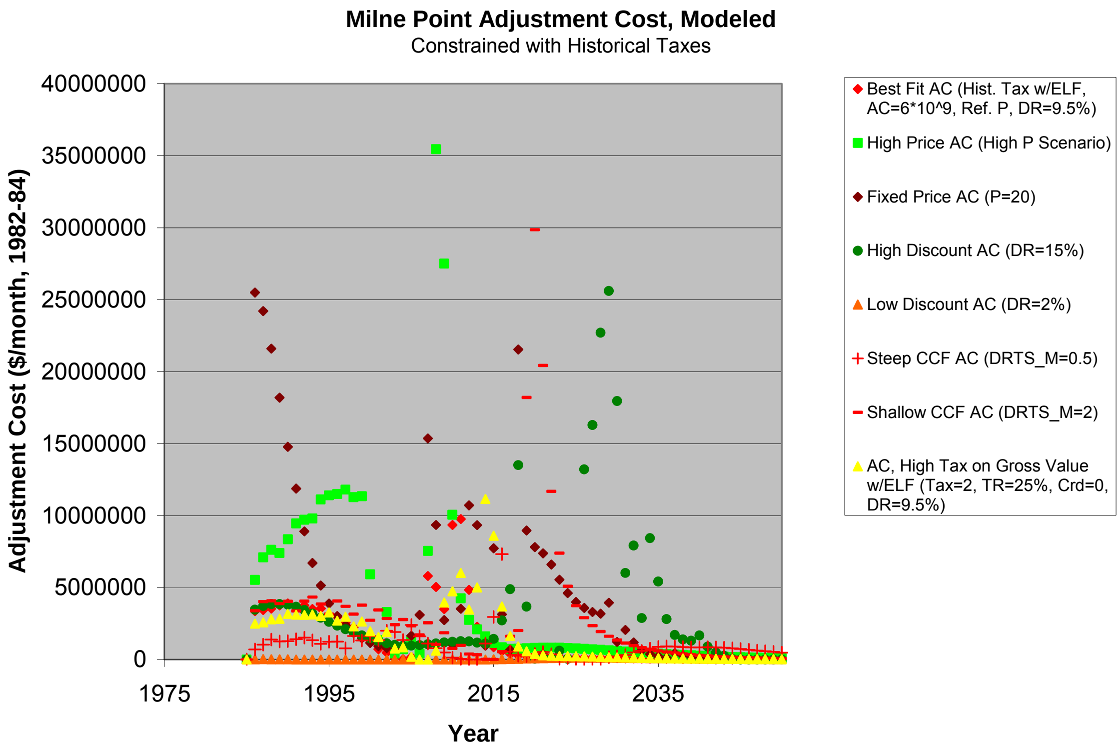
Milne Point: *calibrated model results*



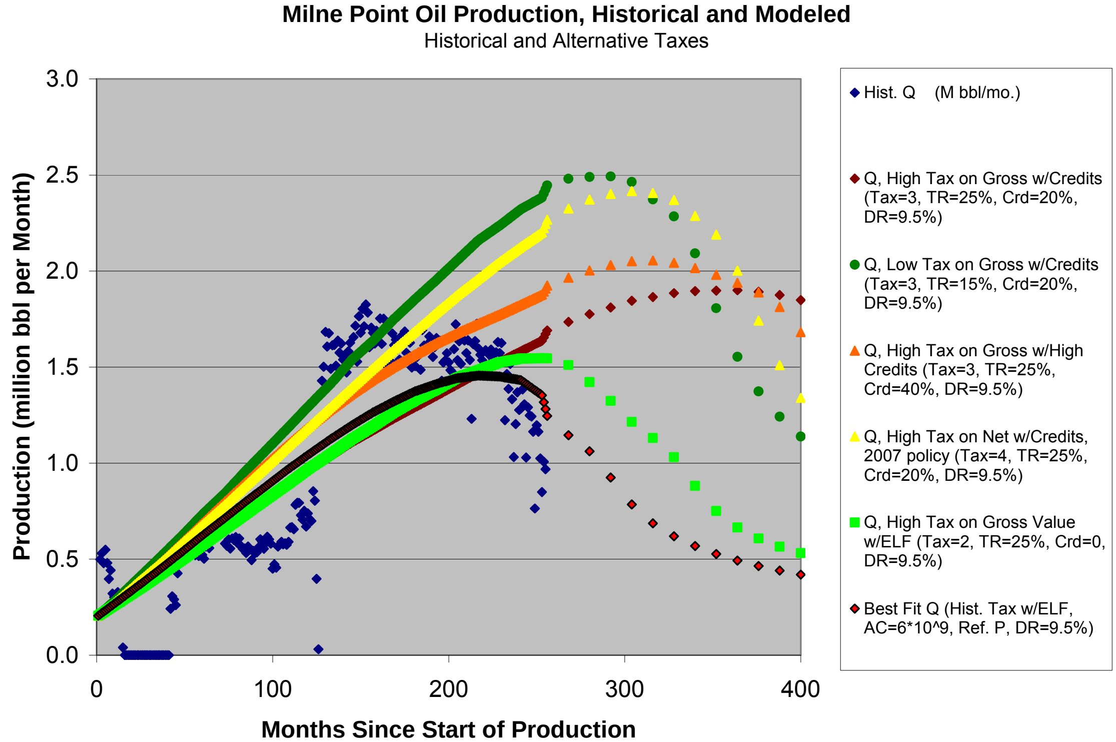


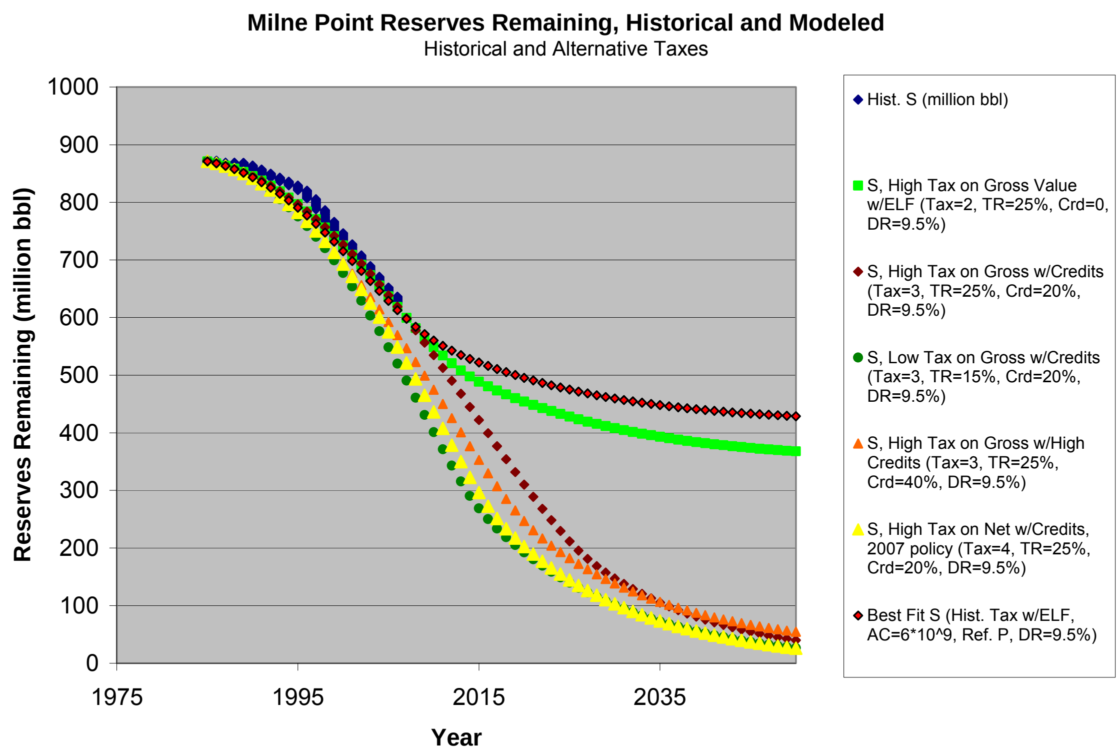

Milne Point Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
Best Fit π (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
Best Fit Tax (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π (DR=15%)
High Discount Tax (DR=15%)
Low Discount π (DR=2%)
Low Discount Tax (DR=2%)
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)
Shallow CCF Tax (DRTS_M=2)

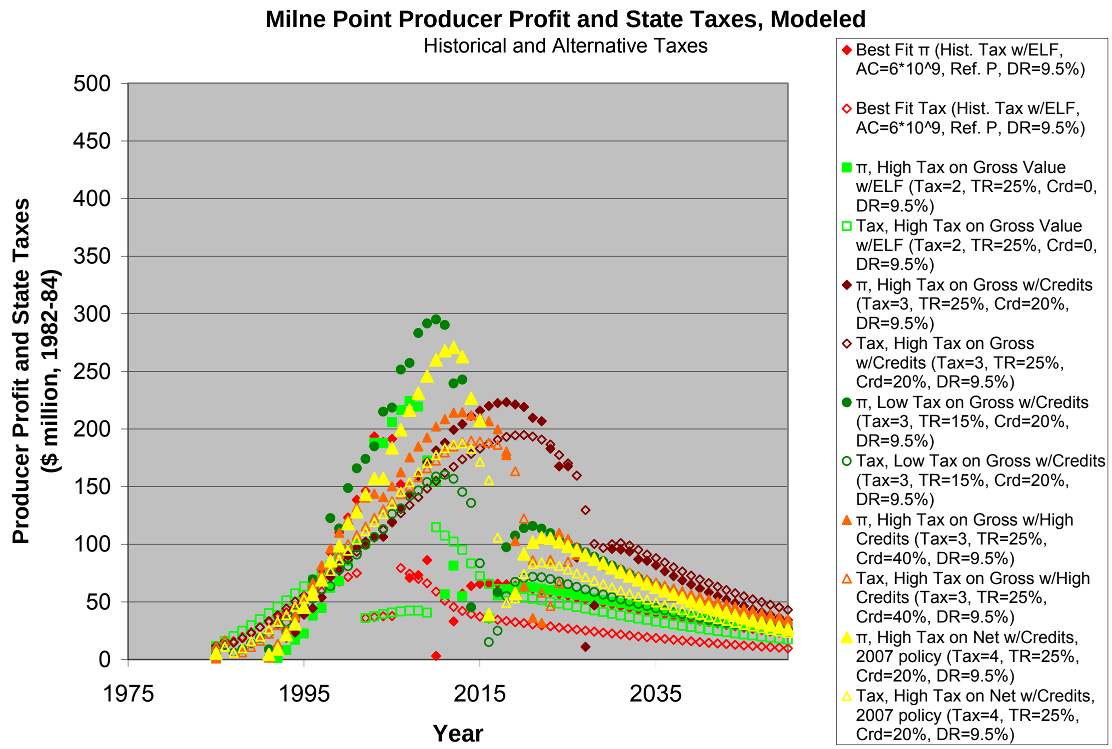

Milne Point Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC (DR=15%)
Low Discount PC (DR=2%)
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=9.5%)

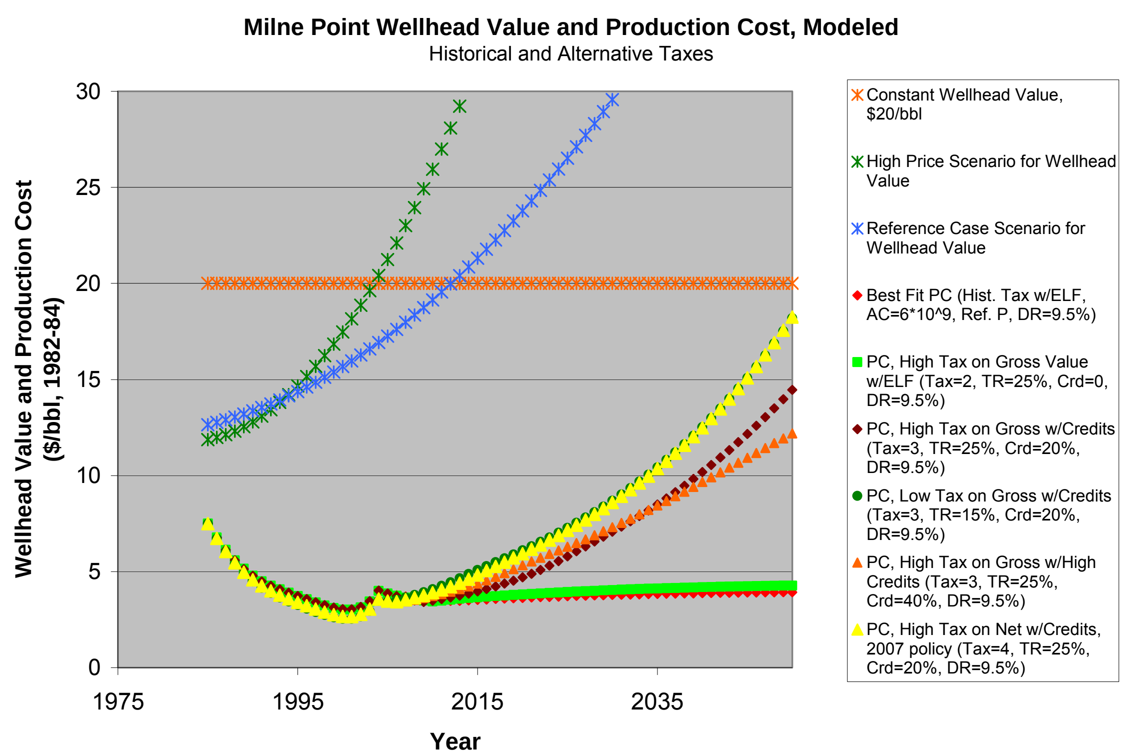

Milne Point Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
40000000
35000000
30000000
25000000
20000000
15000000
10000000
5000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC (DR=15%)
Low Discount AC (DR=2%)
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=9.5%)

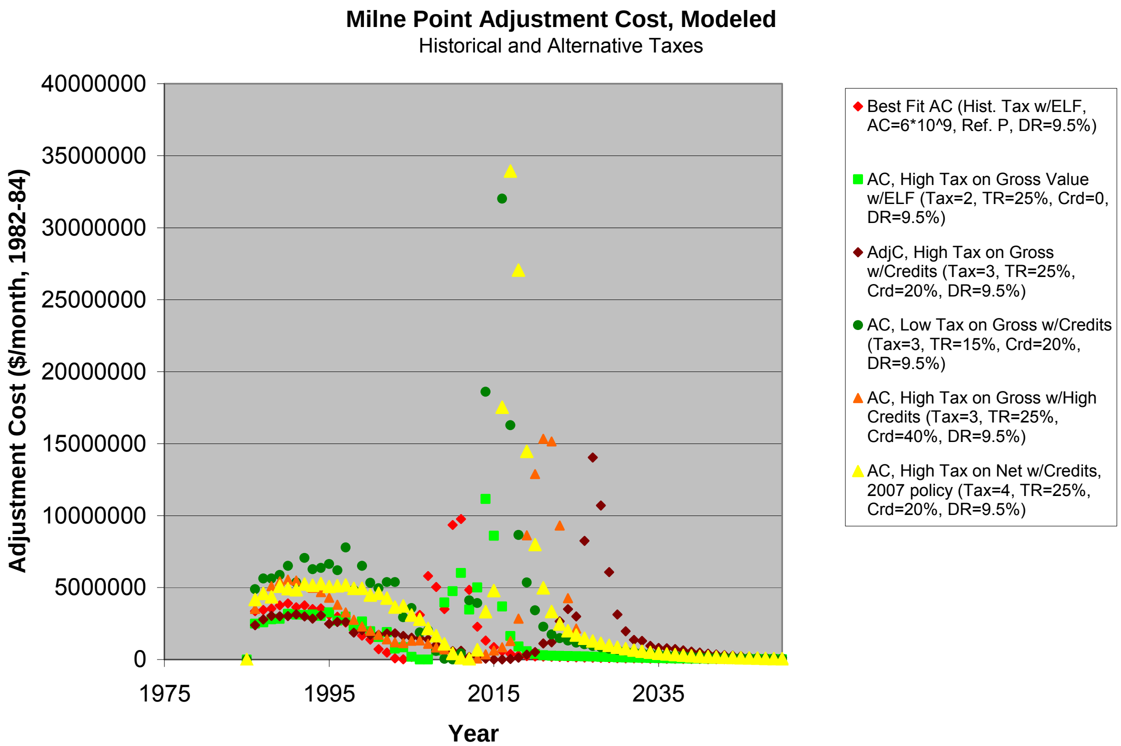
Milne Point: *tax scenario model results*



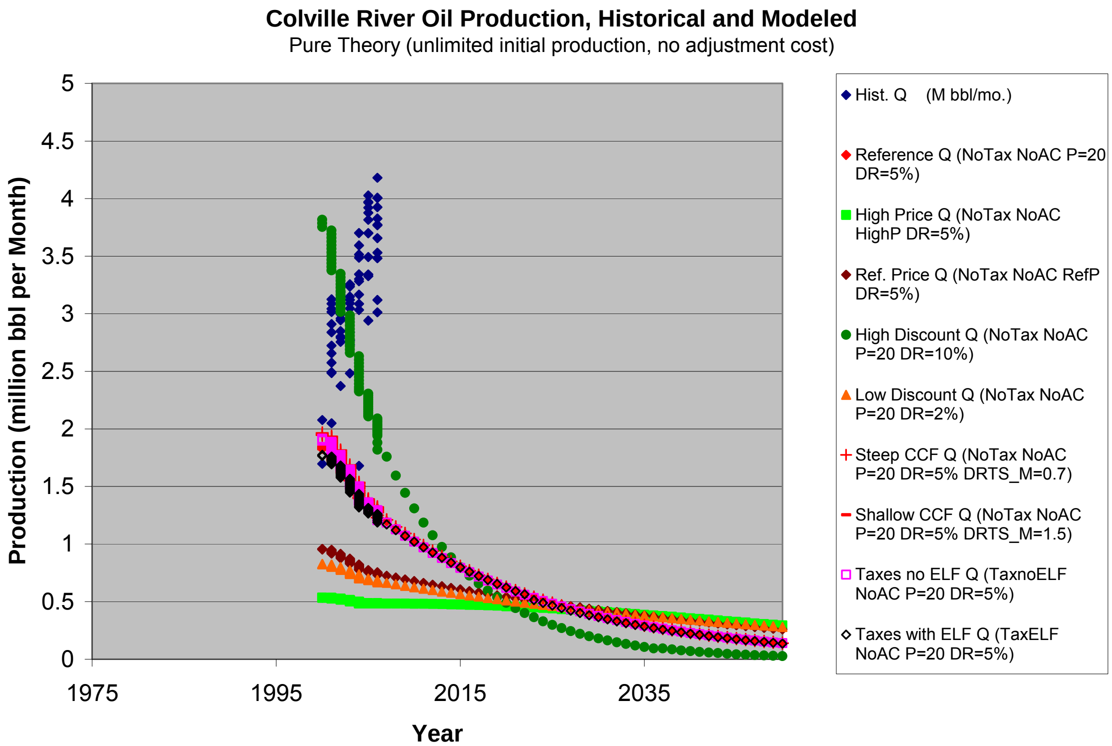


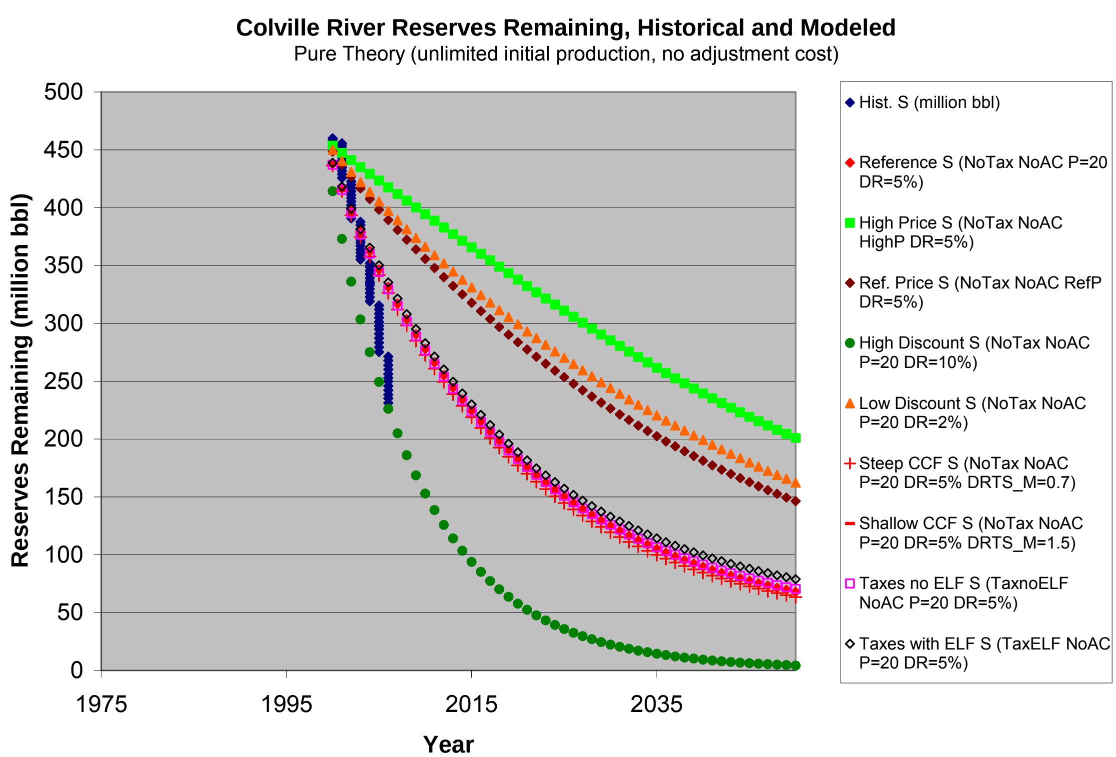

Milne Point Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes
($ million, 1982-84)
500
450
400
350
300
250
200
150
100
50
0
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
Best Fit Tax (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=9.5%)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=9.5%)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=9.5%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=9.5%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=9.5%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=9.5%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=9.5%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=9.5%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=9.5%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=9.5%)

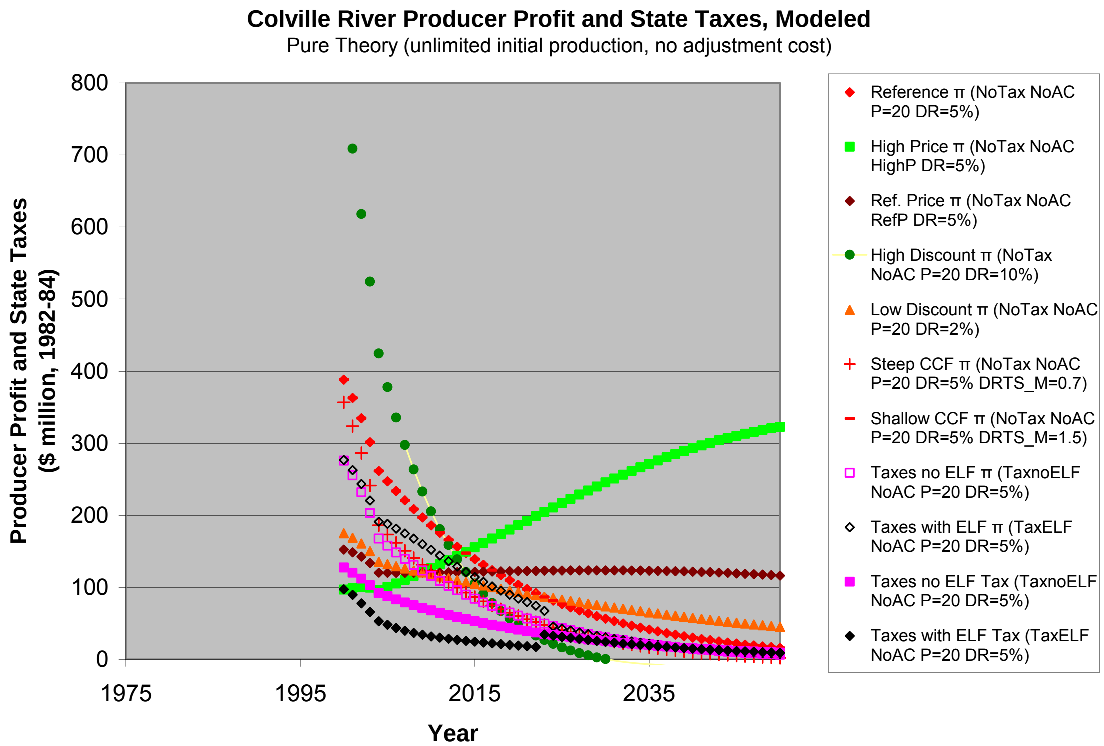

Milne Point Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost
($/bbl, 1982-84)
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=9.5%)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=9.5%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=9.5%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=9.5%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=9.5%)

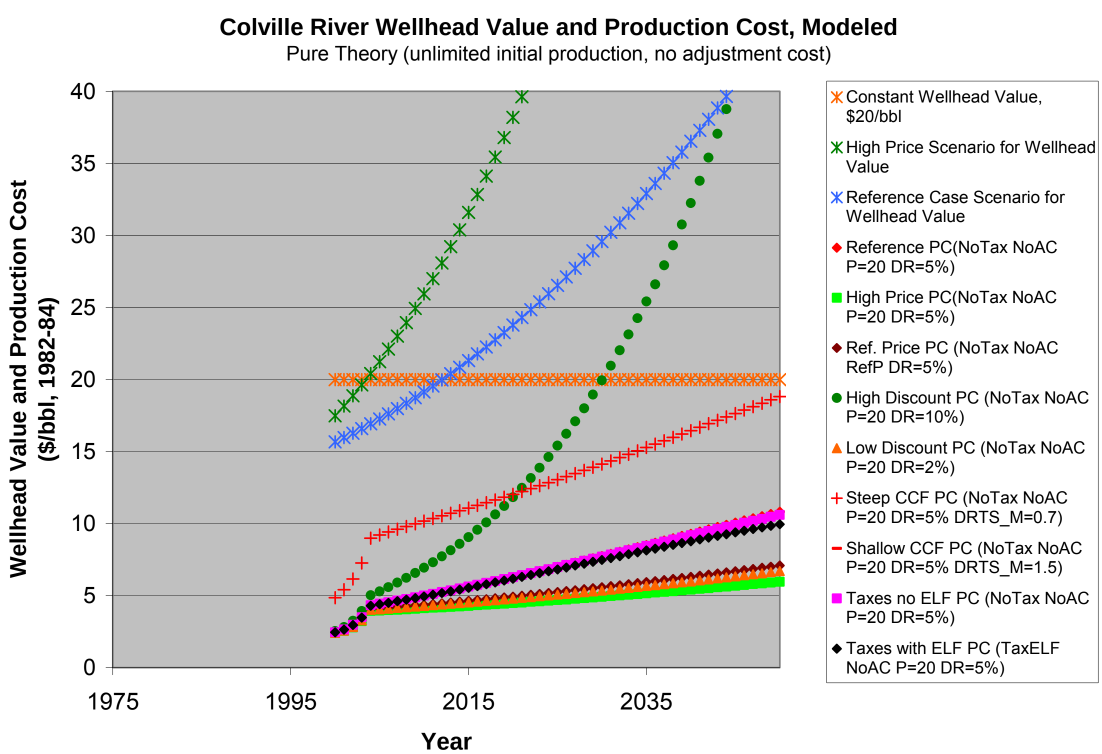

Milne Point Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
40000000
35000000
30000000
25000000
20000000
15000000
10000000
5000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=6*10^9, Ref. P, DR=9.5%)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=9.5%)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=9.5%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=9.5%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=9.5%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=9.5%)

Colville River: *uncalibrated model results*

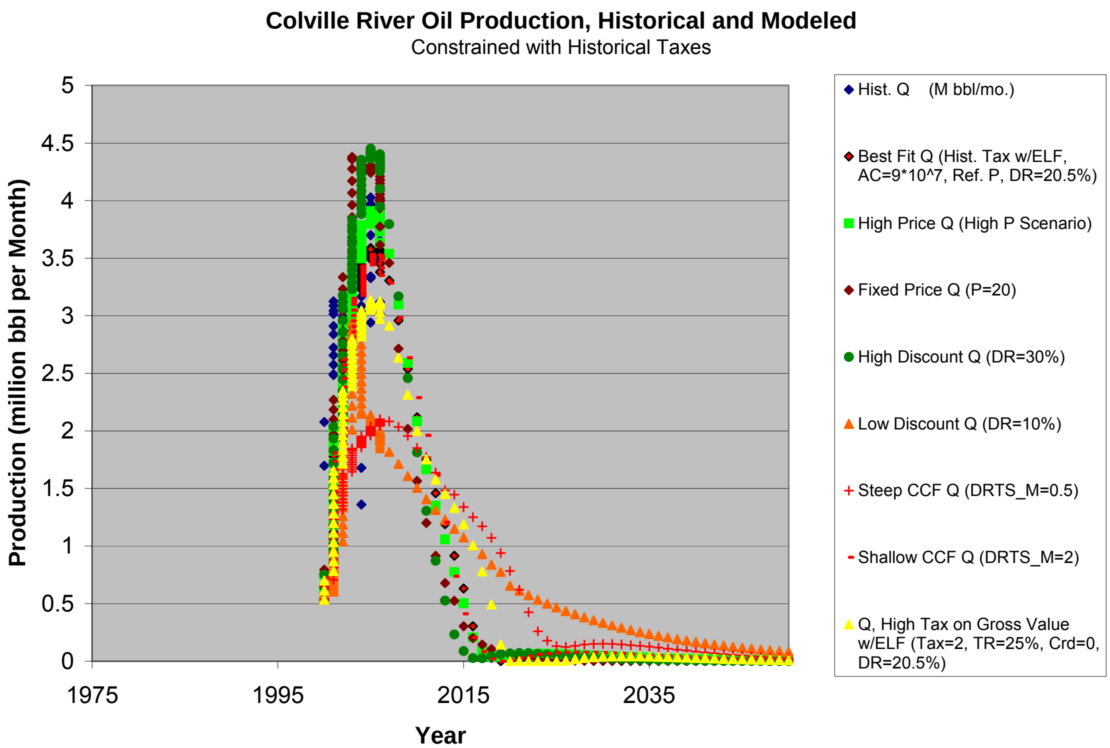


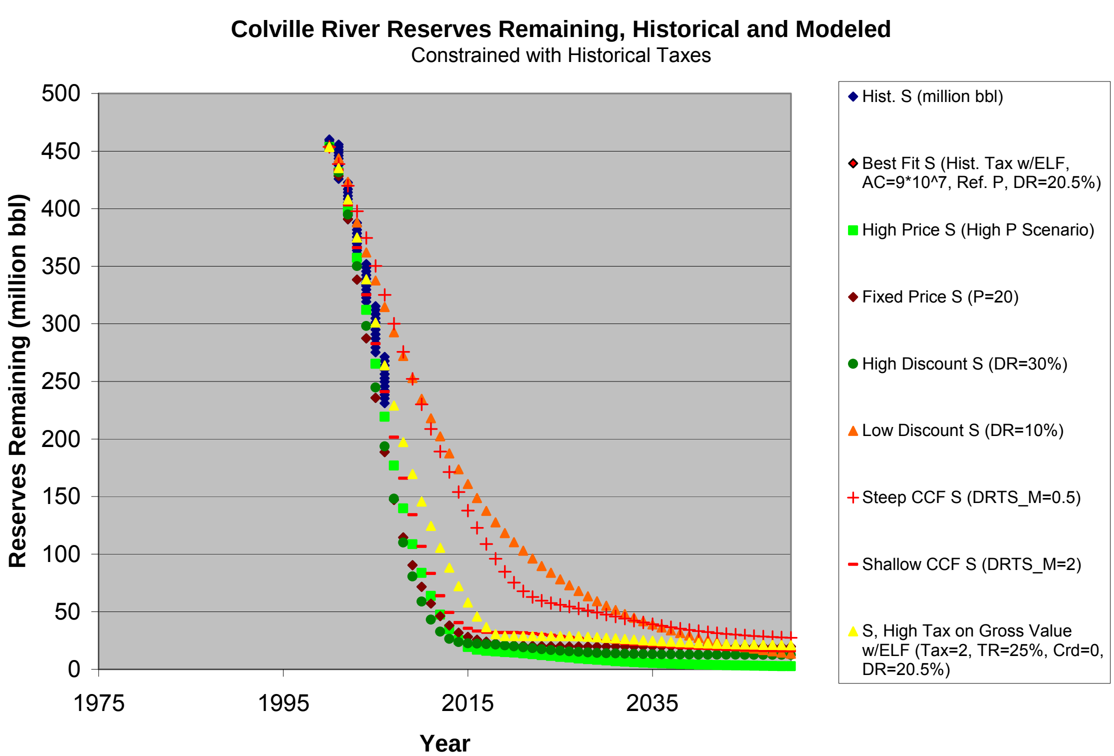


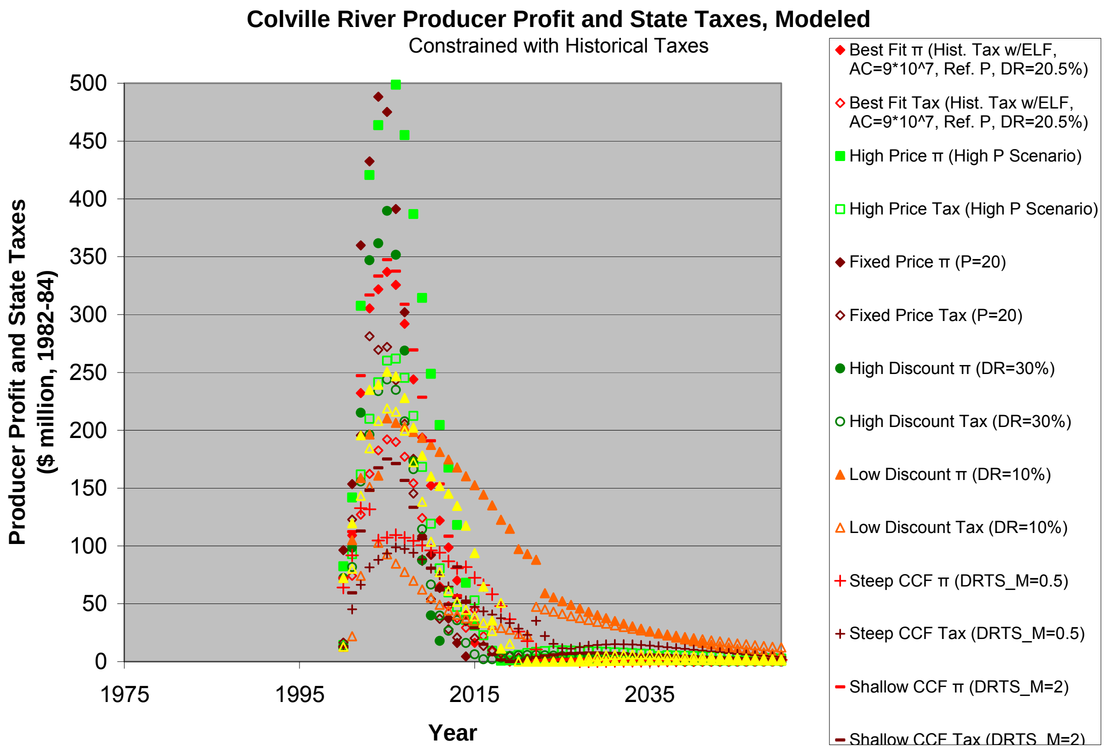

Colville River Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
800
700
600
500
400
300
200
100
0
1975
1995
2015
2035
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

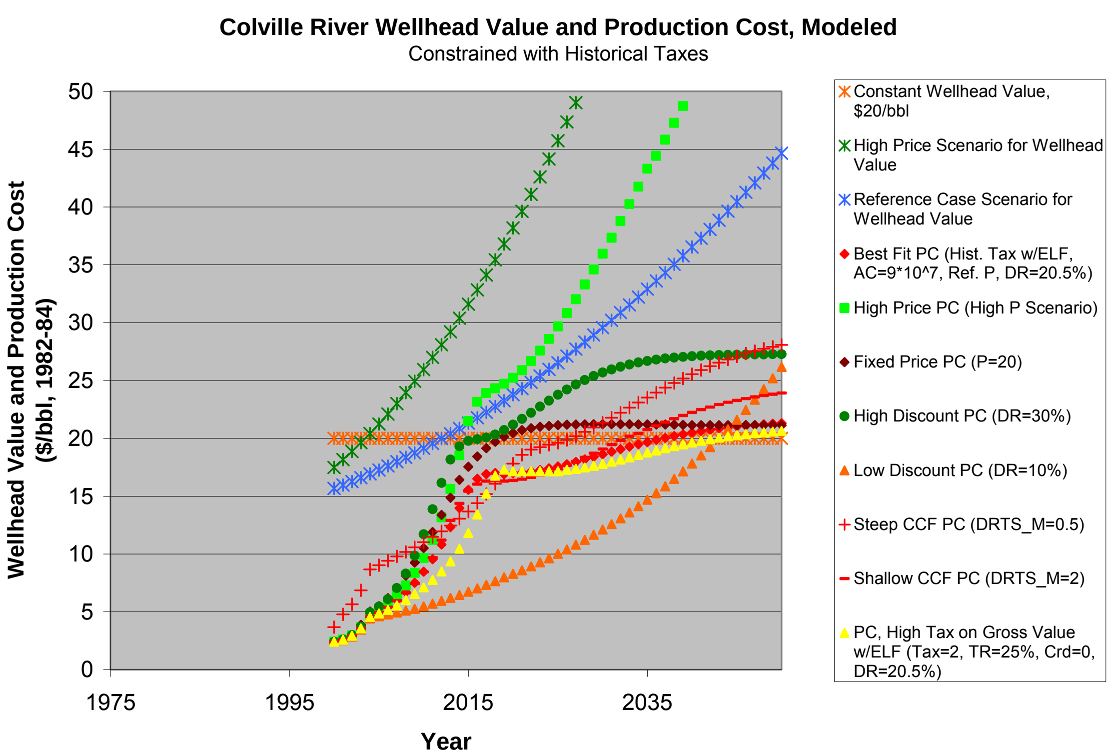

Colville River Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
40
35
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

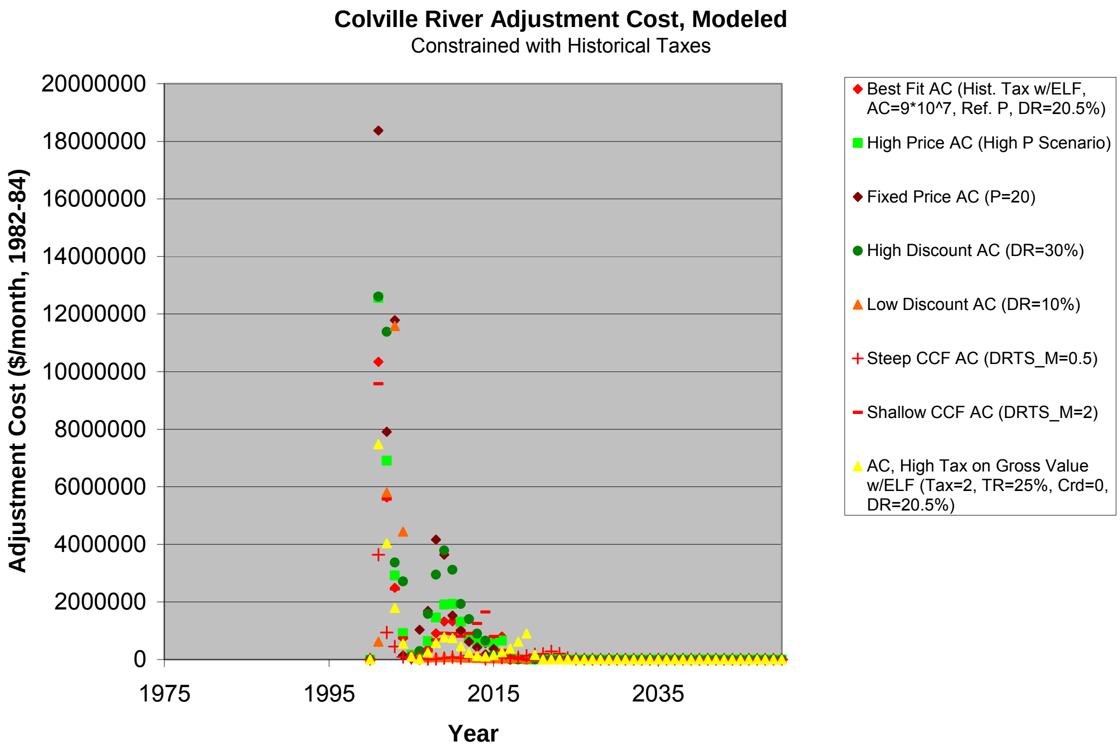
Colville River: *calibrated model results*



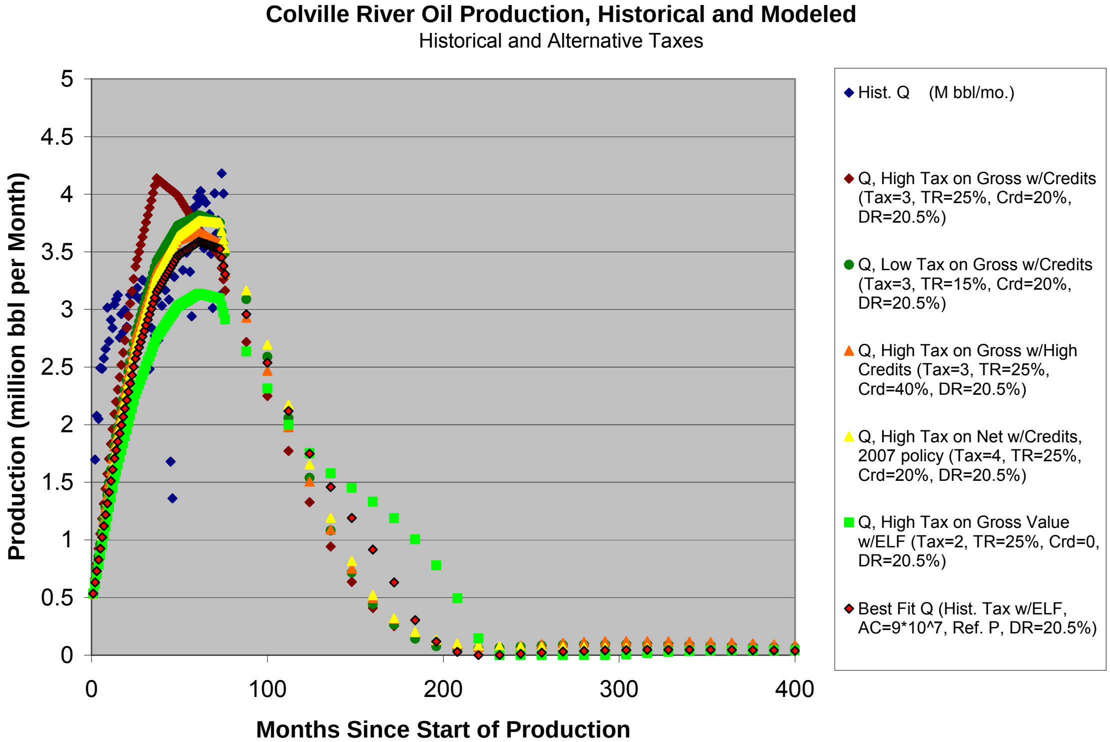


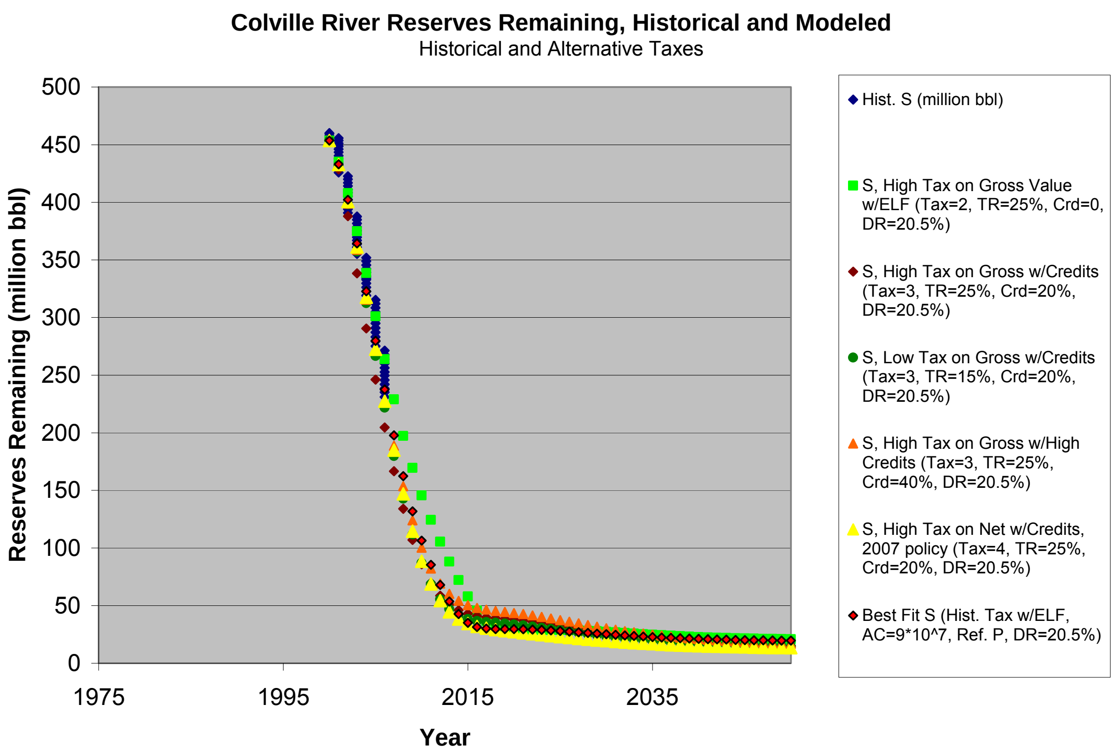

Colville River Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
0
50
100
150
200
250
300
350
400
450
500
1975
1995
2015
2035
Best Fit π (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
Best Fit Tax (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π (DR=30%)
High Discount Tax (DR=30%)
Low Discount π (DR=10%)
Low Discount Tax (DR=10%)
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)
Shallow CCF Tax (DRTS_M=2)

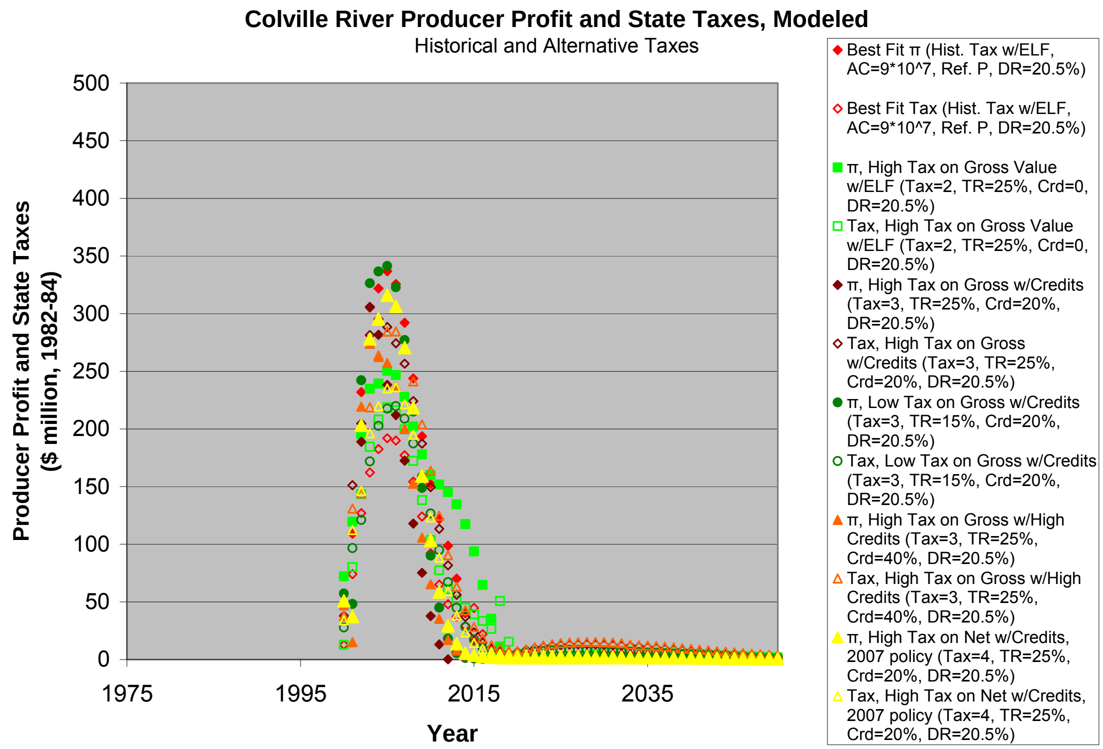

Colville River Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
0
5
10
15
20
25
30
35
40
45
50
1975
1995
2015
2035
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC (DR=30%)
Low Discount PC (DR=10%)
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=20.5%)


Colville River Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
20000000
18000000
16000000
14000000
12000000
10000000
8000000
6000000
4000000
2000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC (DR=30%)
Low Discount AC (DR=10%)
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=20.5%)

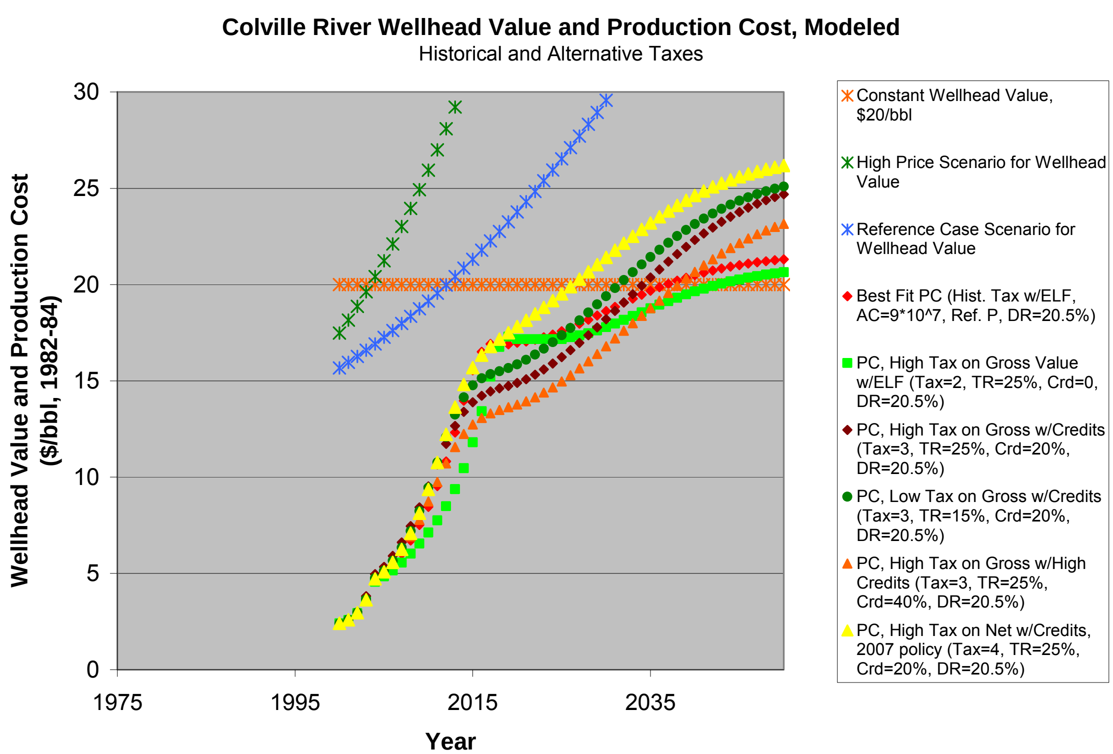
Colville River: *tax scenario model results*



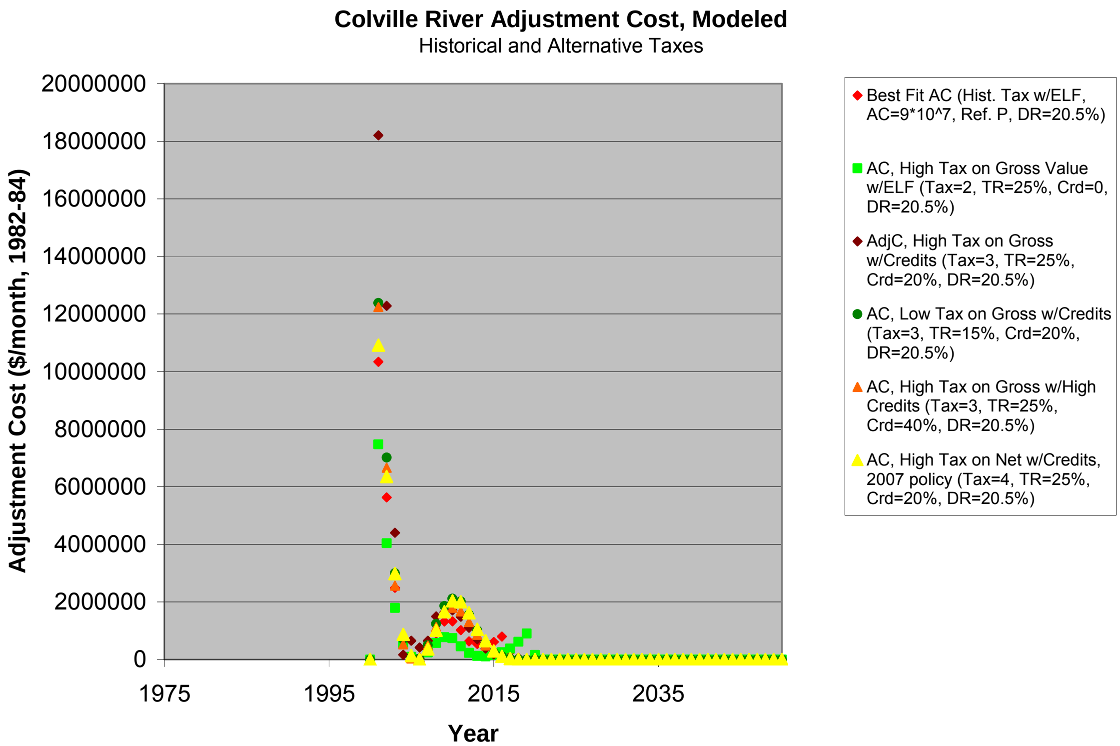




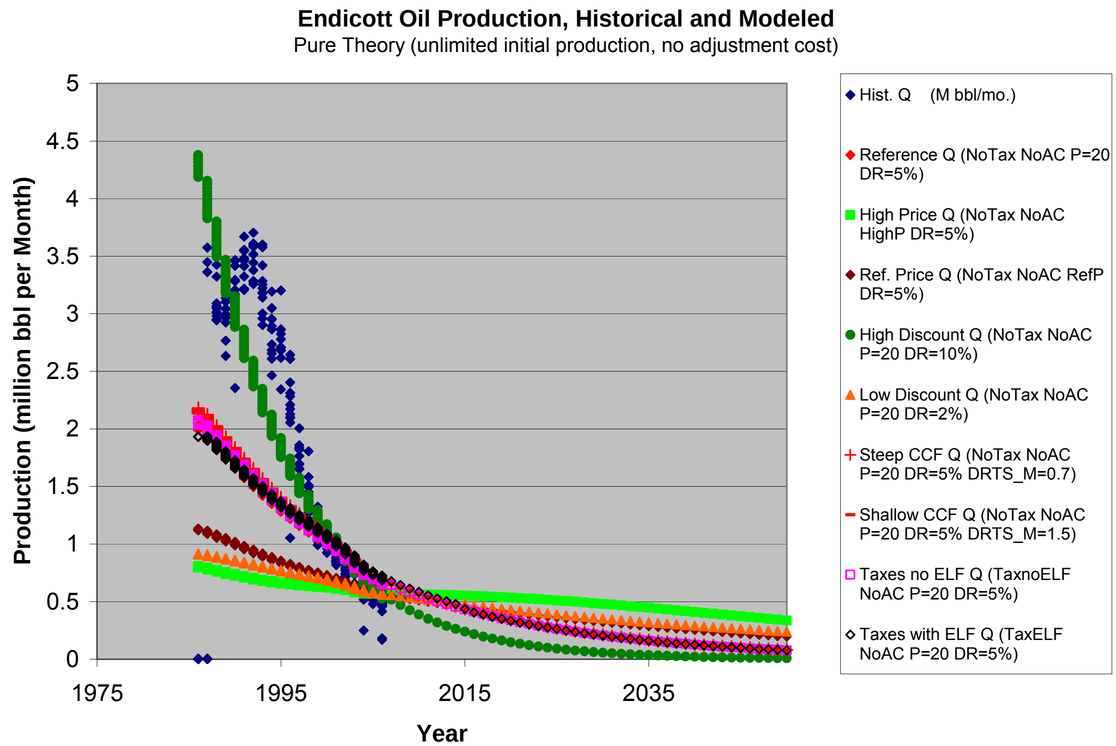
Colville River Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes ($ million, 1982-84)
500
450
400
350
300
250
200
150
100
50
0
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
Best Fit Tax (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=20.5%)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=20.5%)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=20.5%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=20.5%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=20.5%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=20.5%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=20.5%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=20.5%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=20.5%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=20.5%)

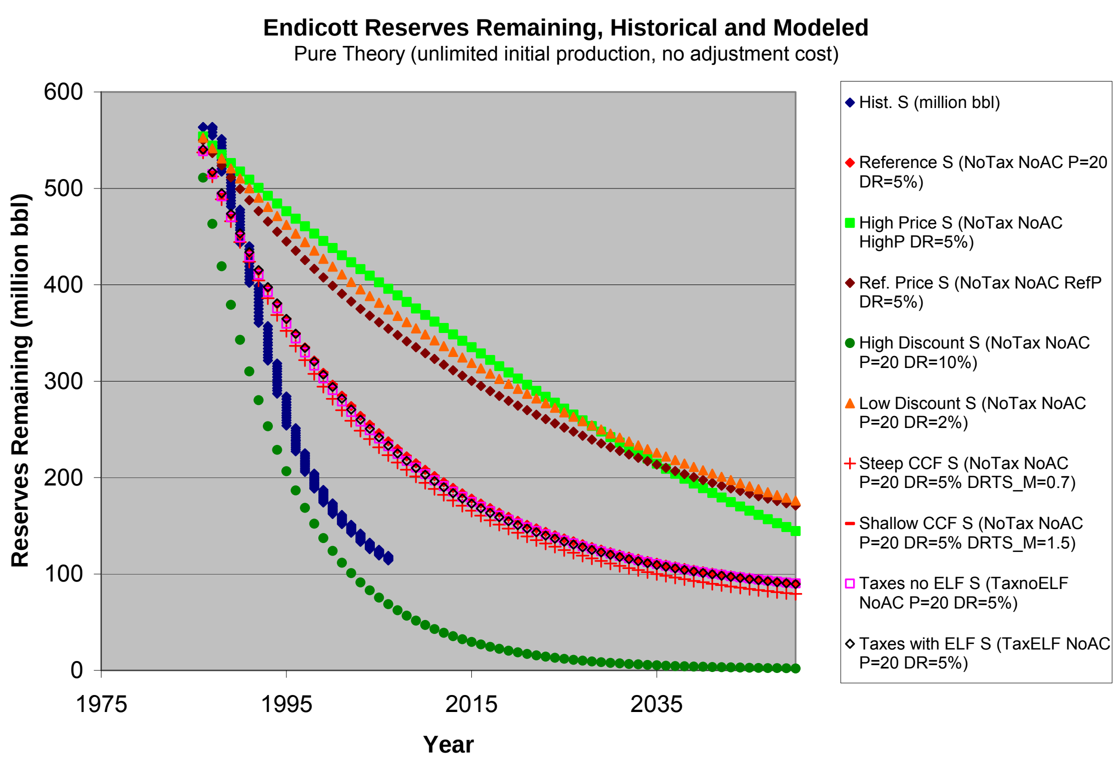


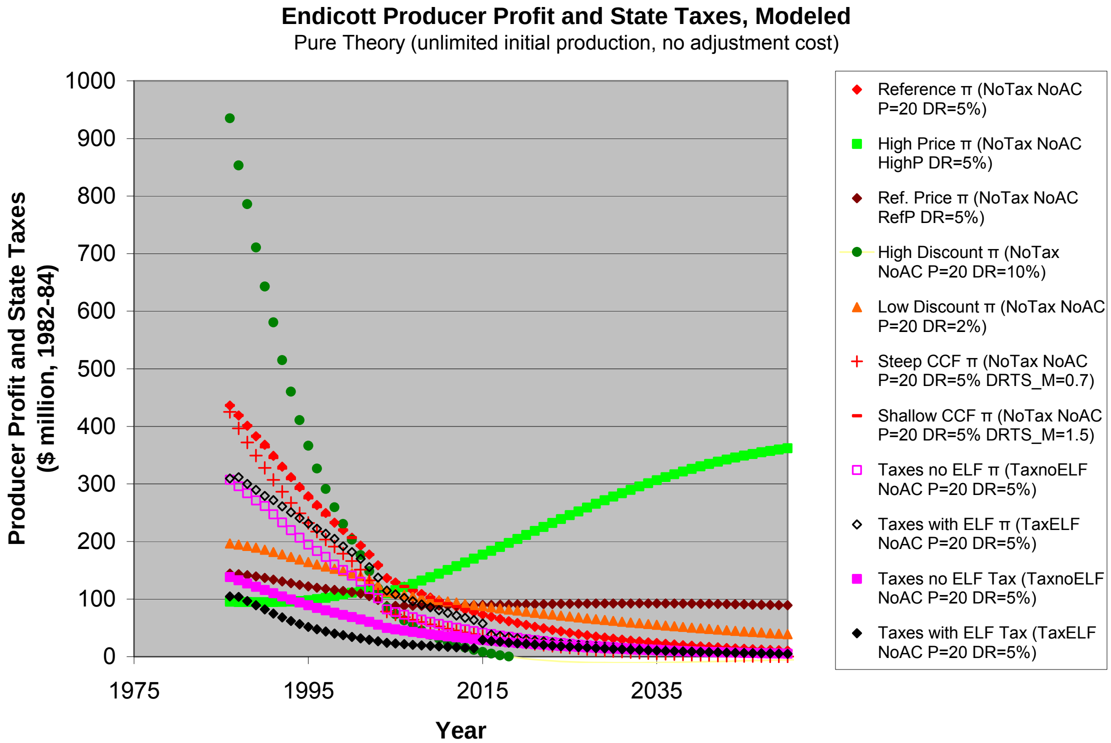
Colville River Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=20.5%)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=20.5%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=20.5%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=20.5%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=20.5%)

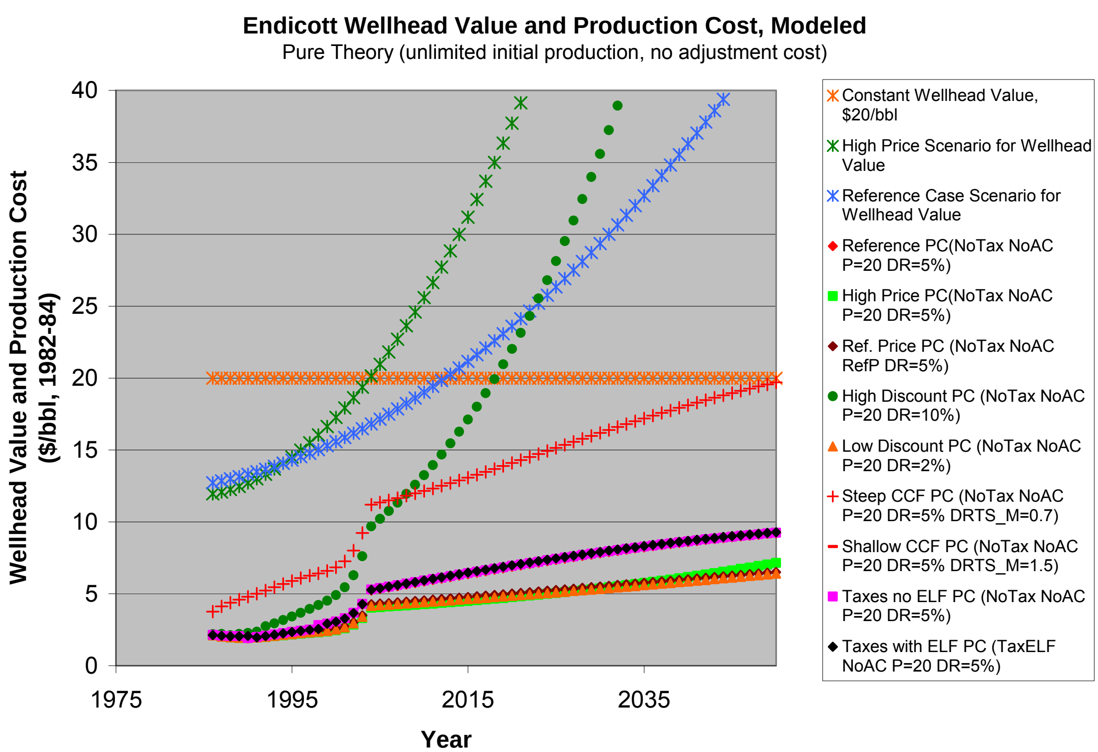

Colville River Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
20000000
18000000
16000000
14000000
12000000
10000000
8000000
6000000
4000000
2000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=9*10^7, Ref. P, DR=20.5%)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=20.5%)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=20.5%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=20.5%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=20.5%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=20.5%)

Endicott: *uncalibrated model results*

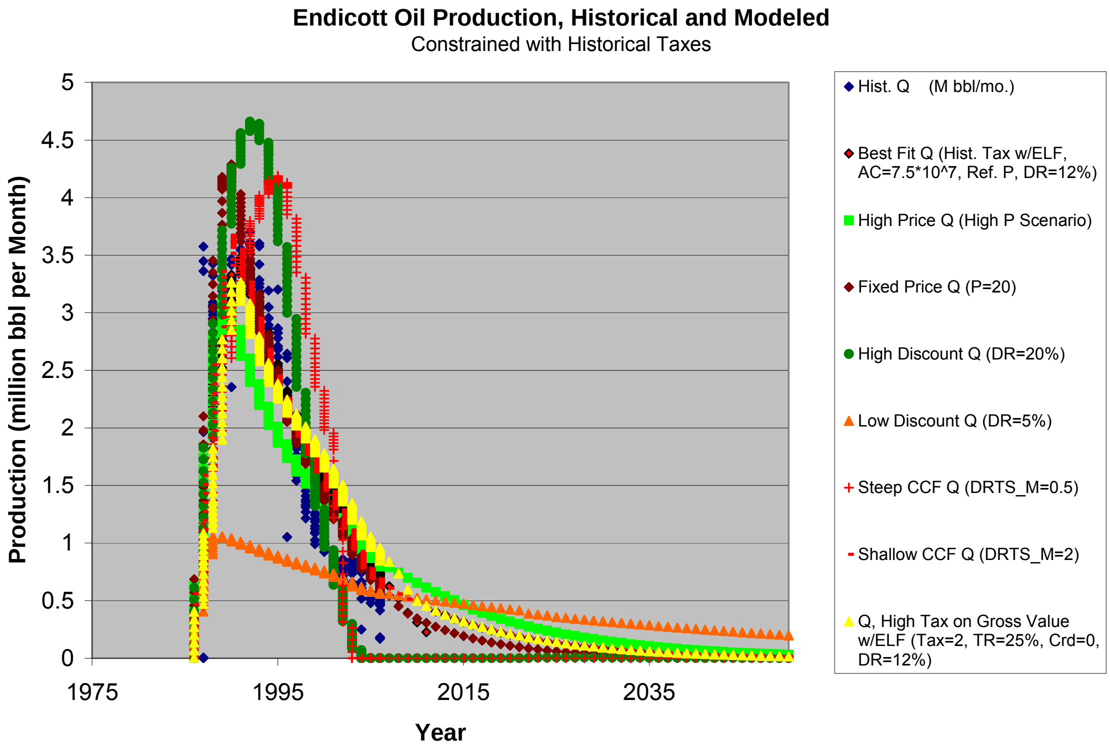


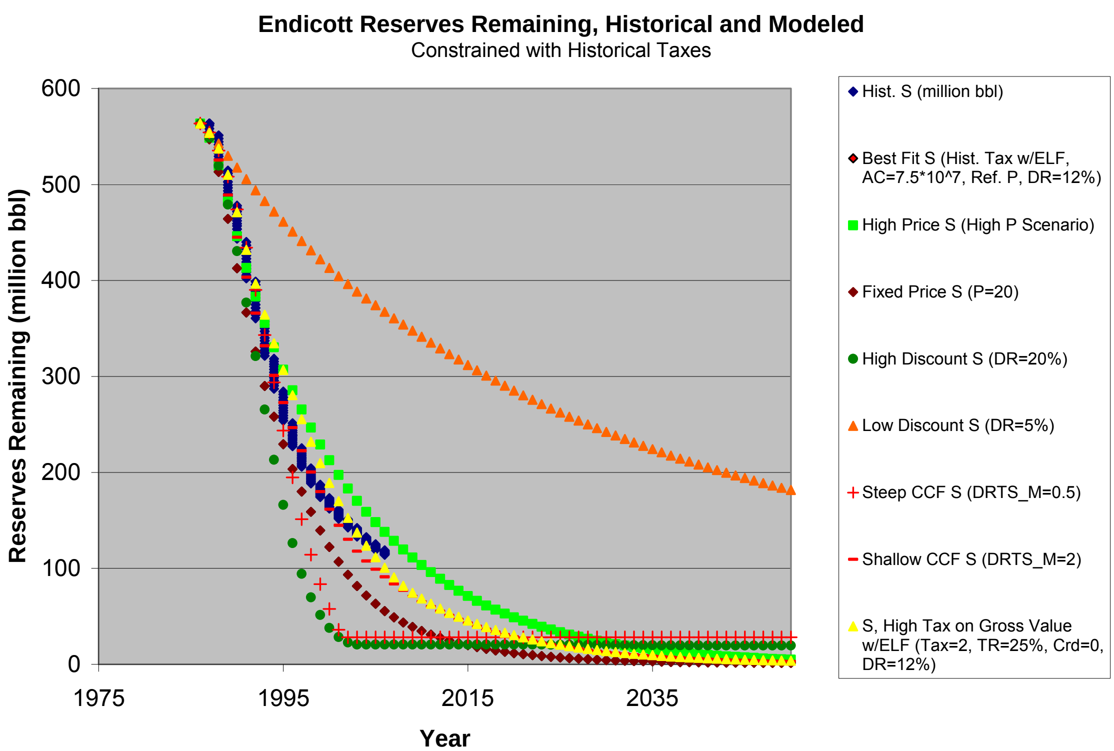


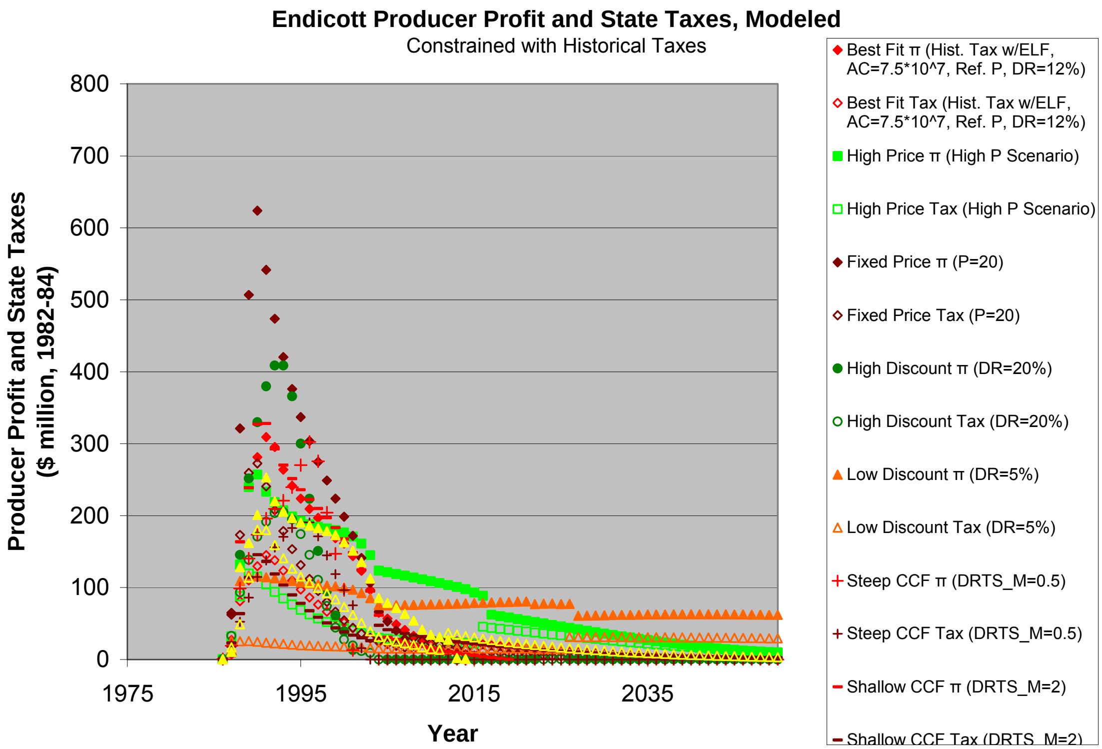

Endicott Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
1000
900
800
700
600
500
400
300
200
100
0
1975
1995
2015
2035
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

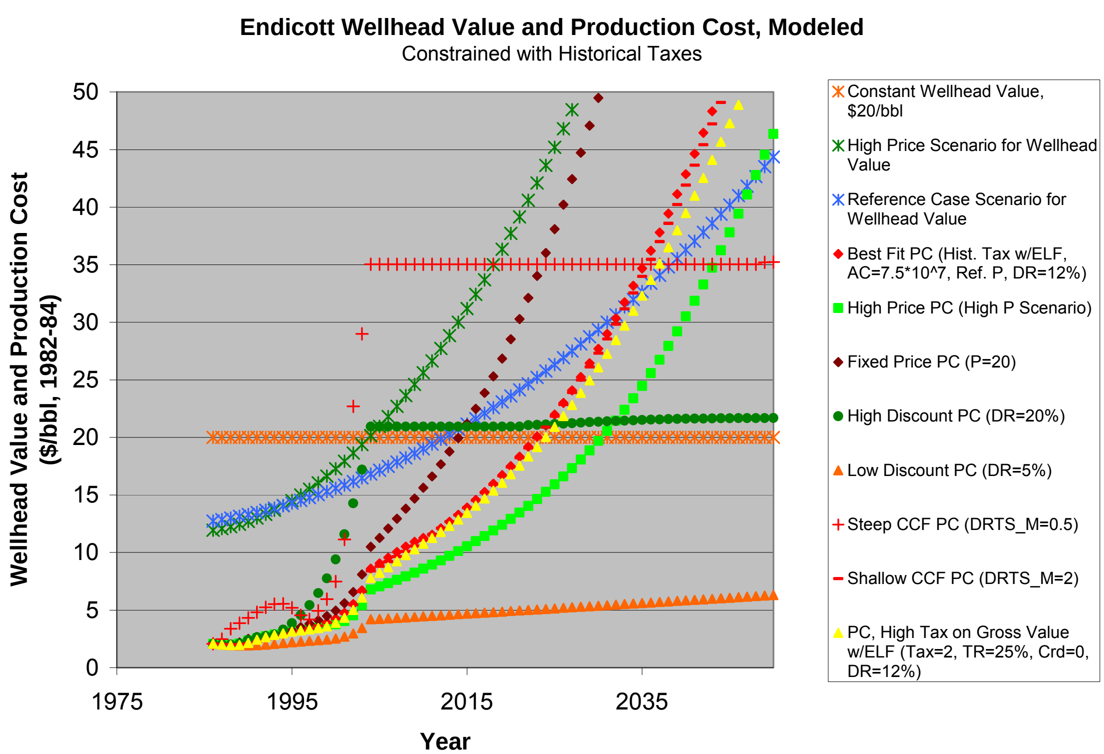

Endicott Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
40
35
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

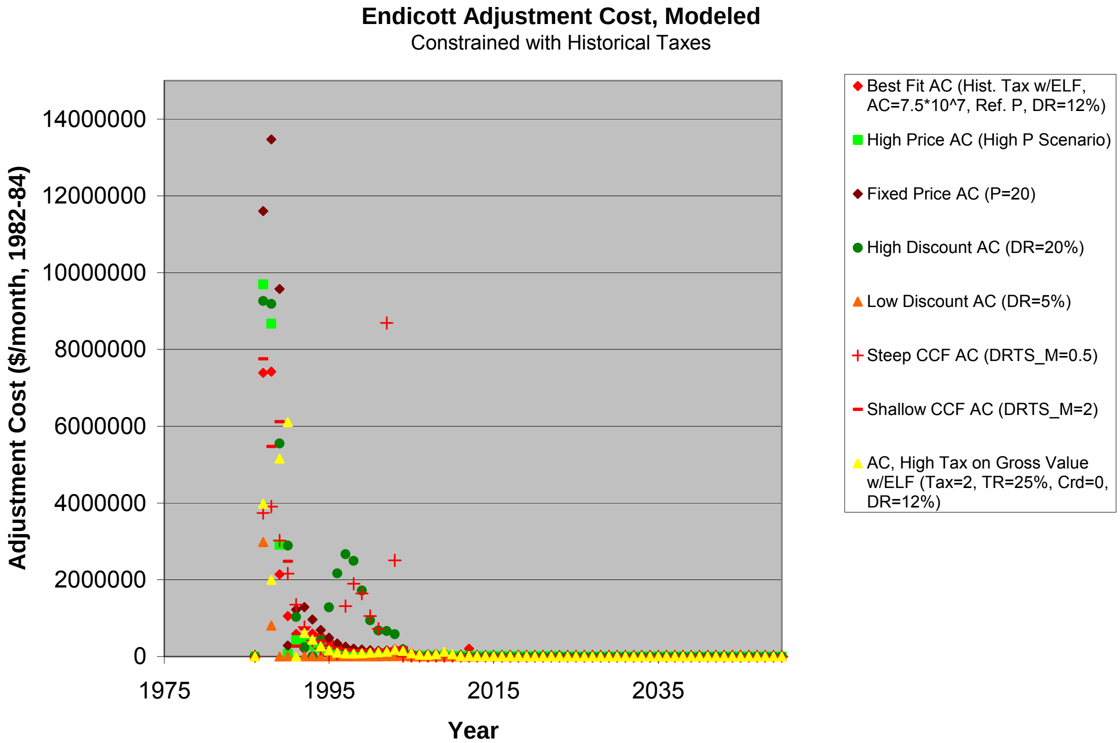
***Endicott:** calibrated model results*





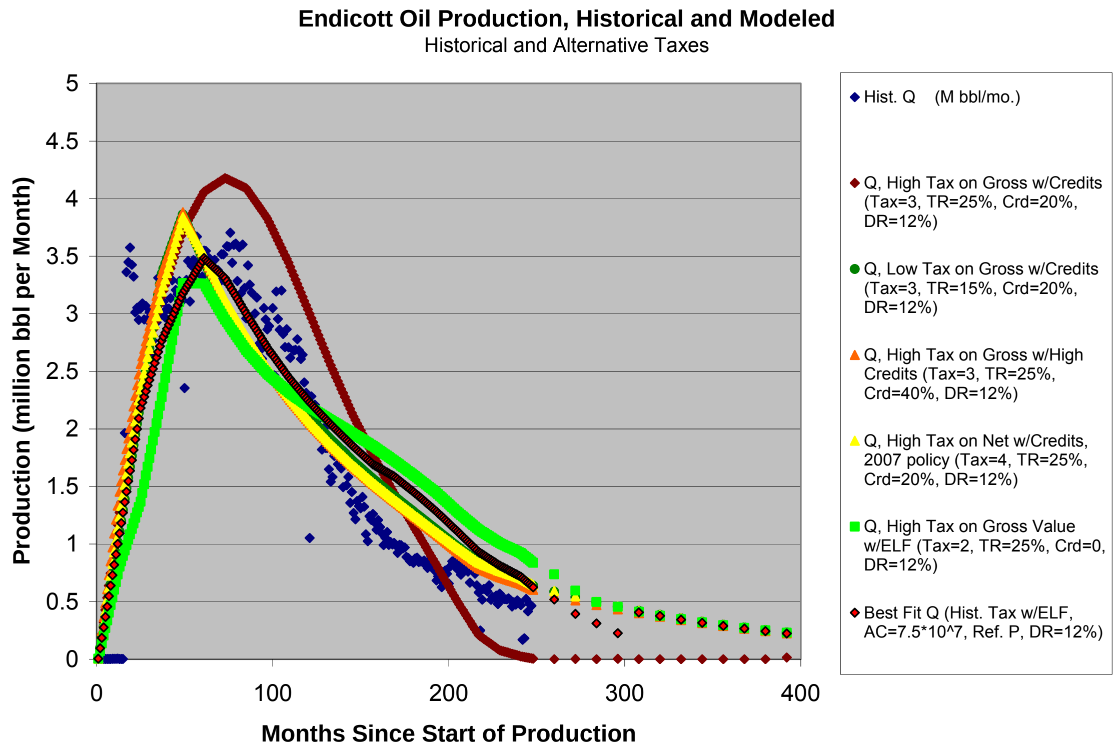

Endicott Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
Best Fit π (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
Best Fit Tax (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π (DR=20%)
High Discount Tax (DR=20%)
Low Discount π (DR=5%)
Low Discount Tax (DR=5%)
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)
Shallow CCF Tax (DRTS_M=2)

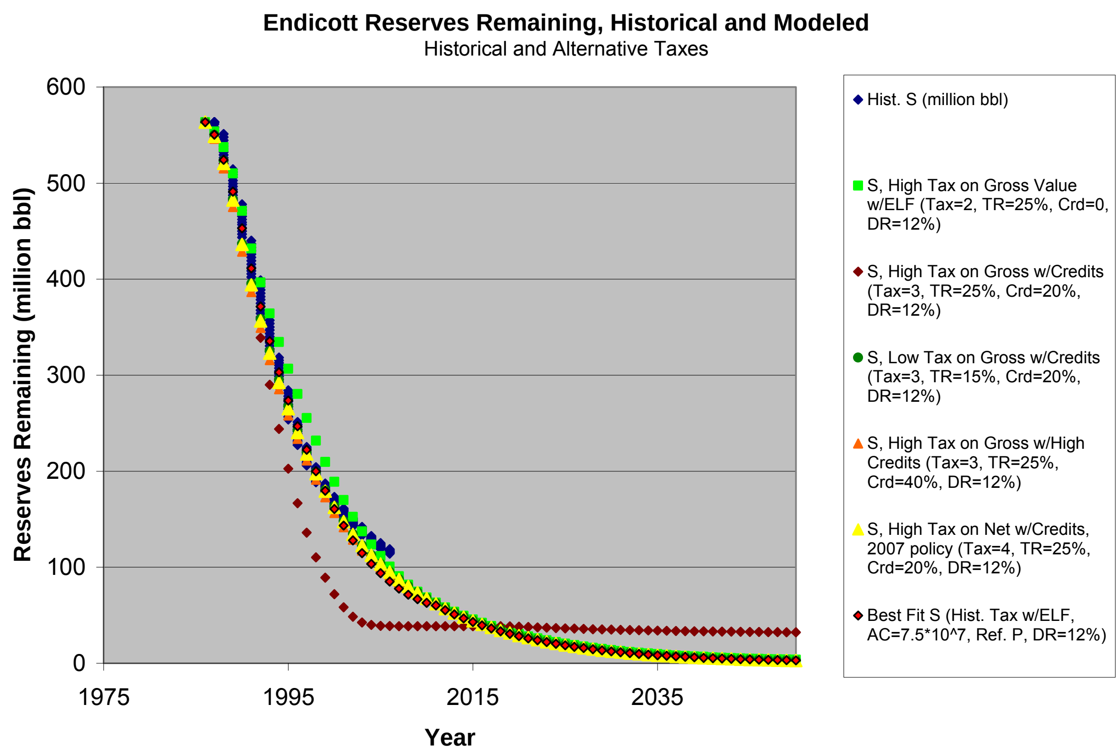

Endicott Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC (DR=20%)
Low Discount PC (DR=5%)
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=12%)

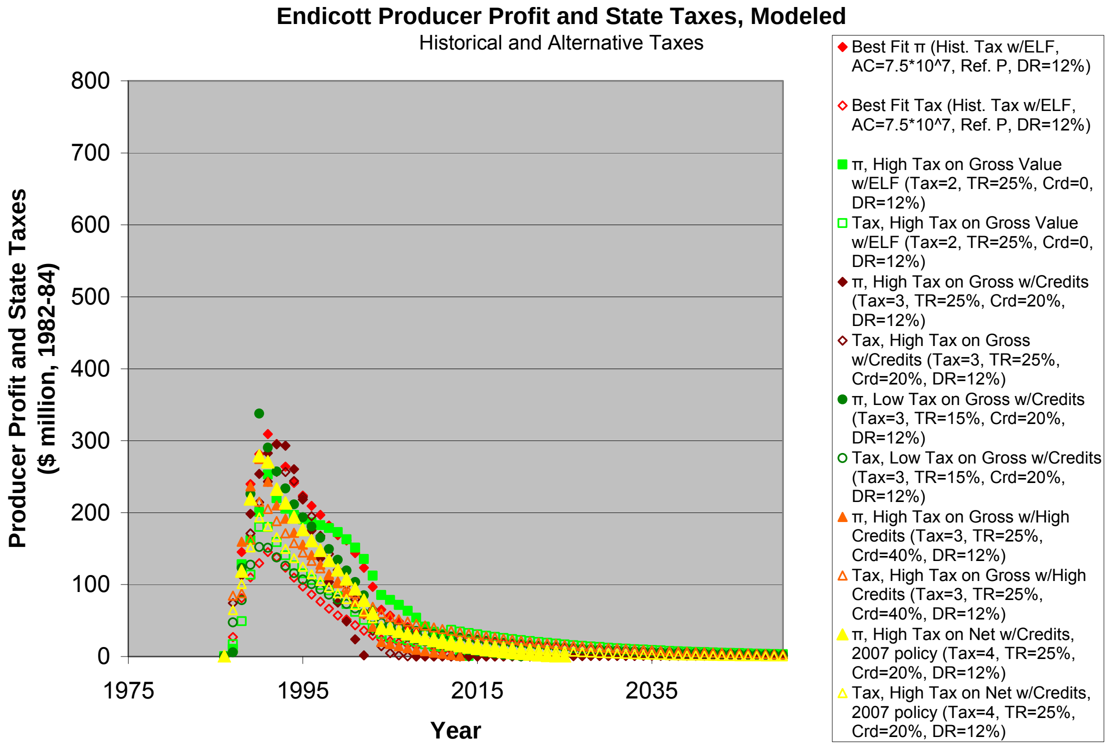


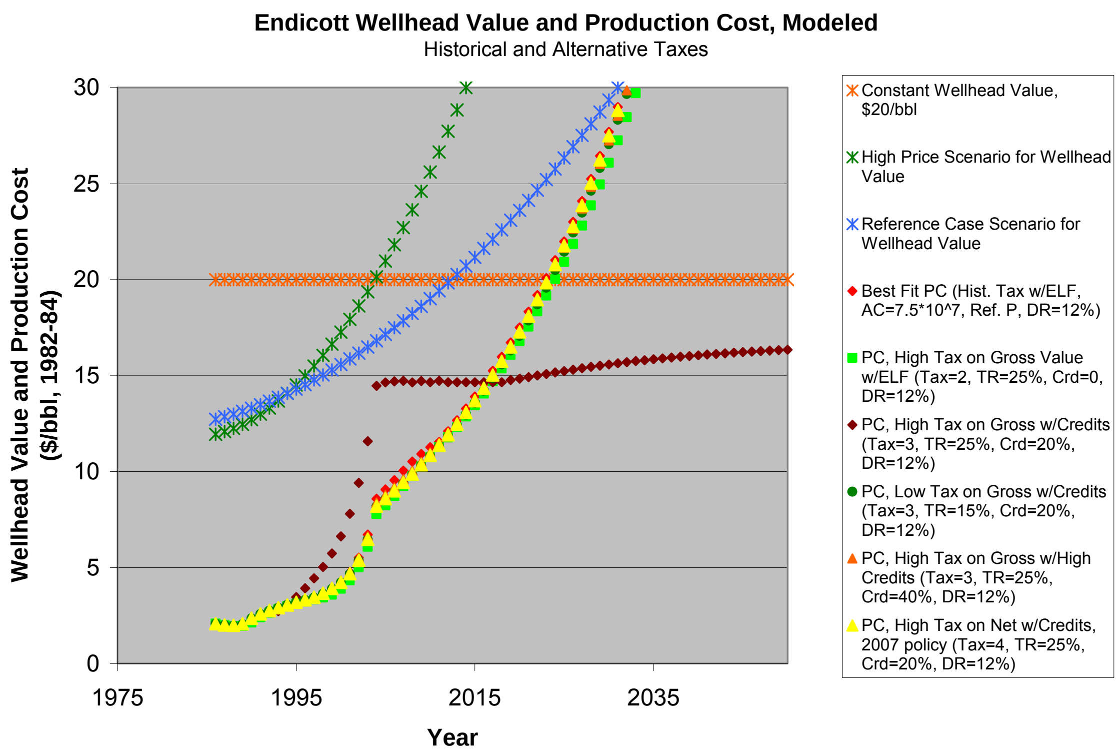
Endicott Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
14000000
12000000
10000000
8000000
6000000
4000000
2000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC (DR=20%)
Low Discount AC (DR=5%)
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=12%)

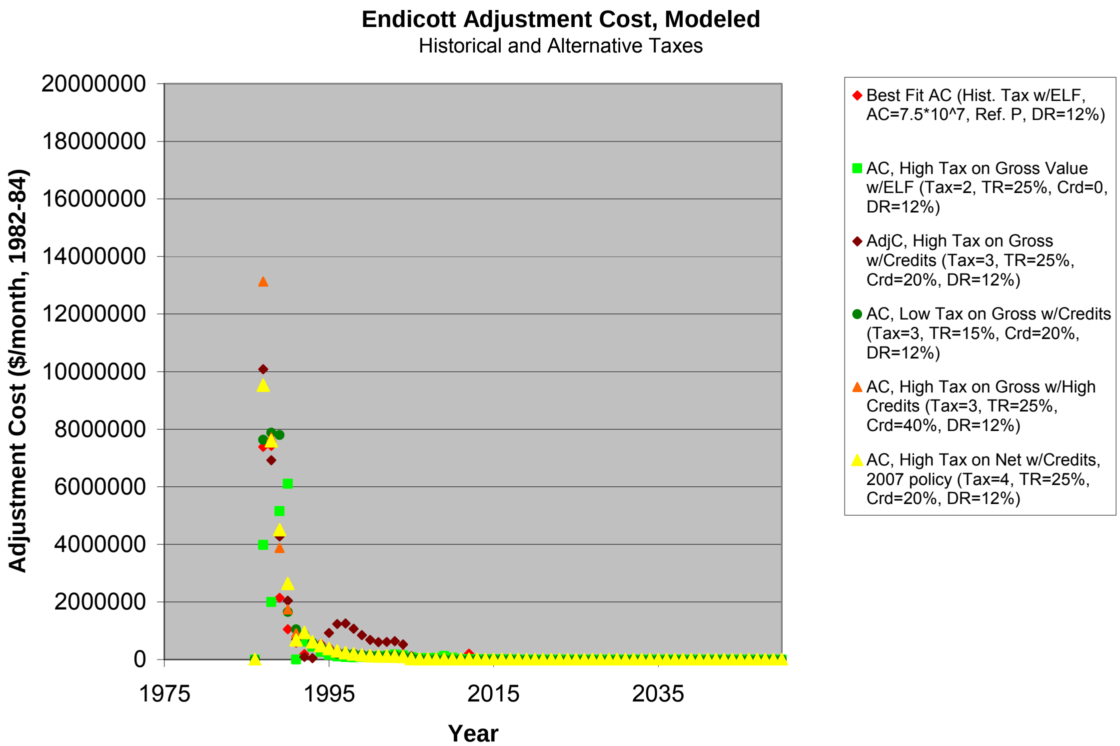
Endicott: *tax scenario model results*



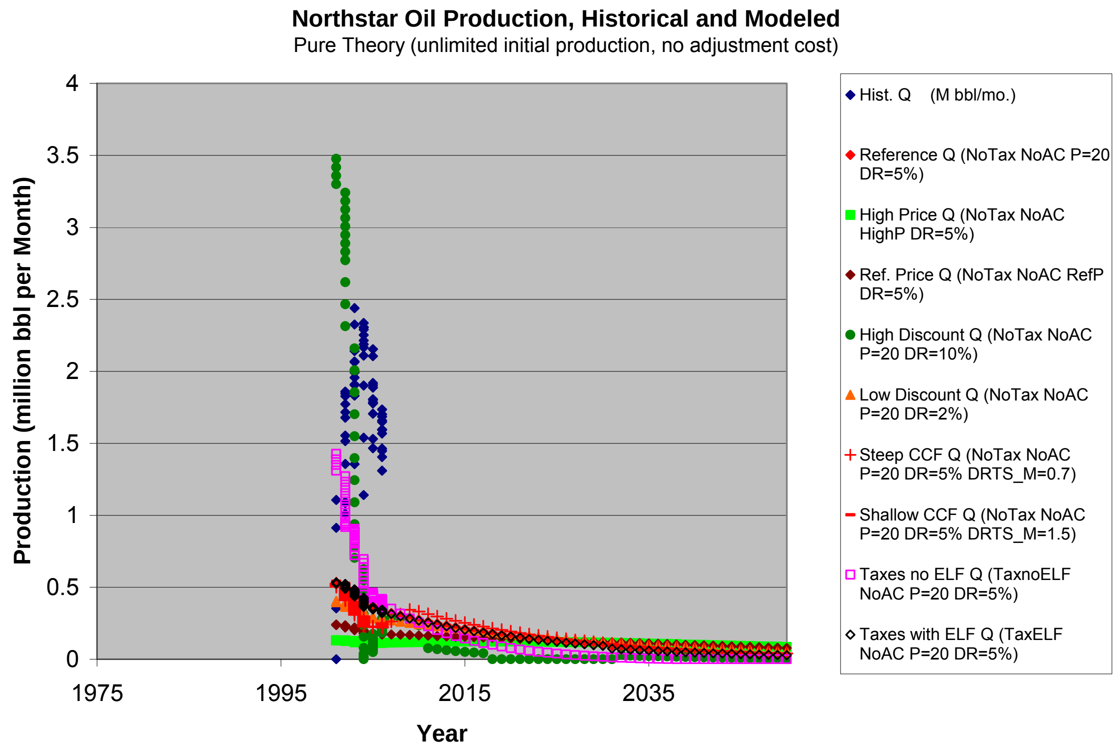


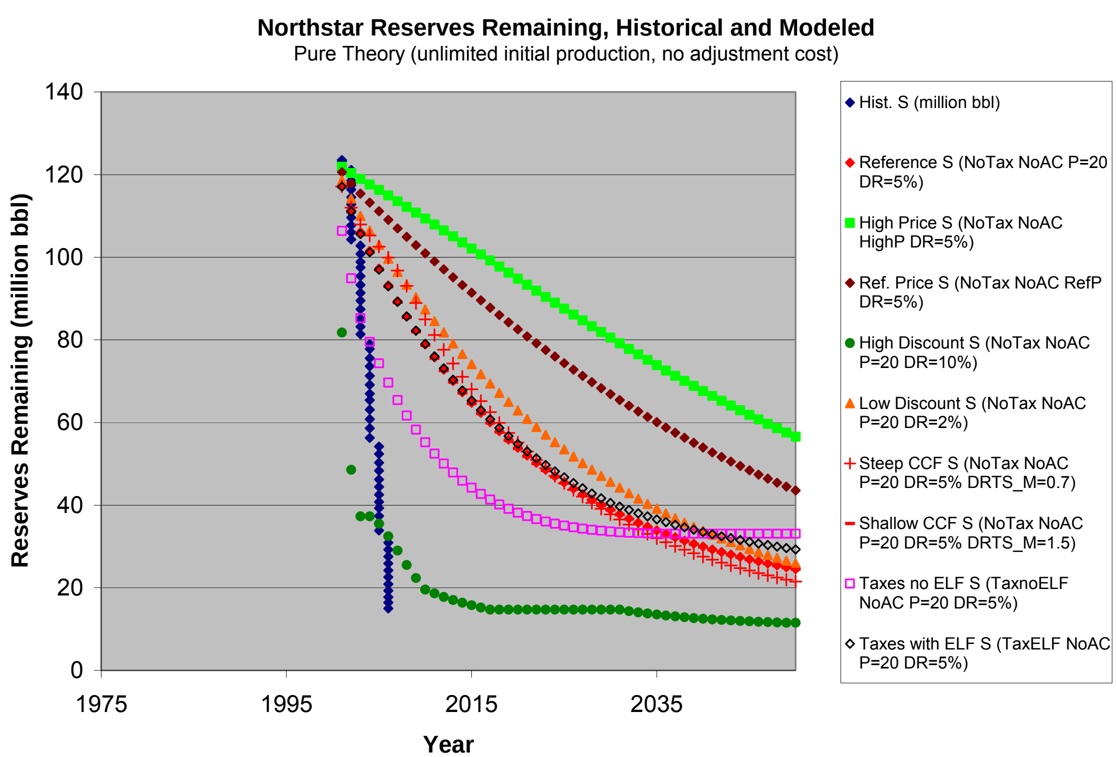

Endicott Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes ($ million, 1982-84)
800
700
600
500
400
300
200
100
0
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
Best Fit Tax (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=12%)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=12%)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=12%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=12%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=12%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=12%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=12%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=12%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=12%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=12%)

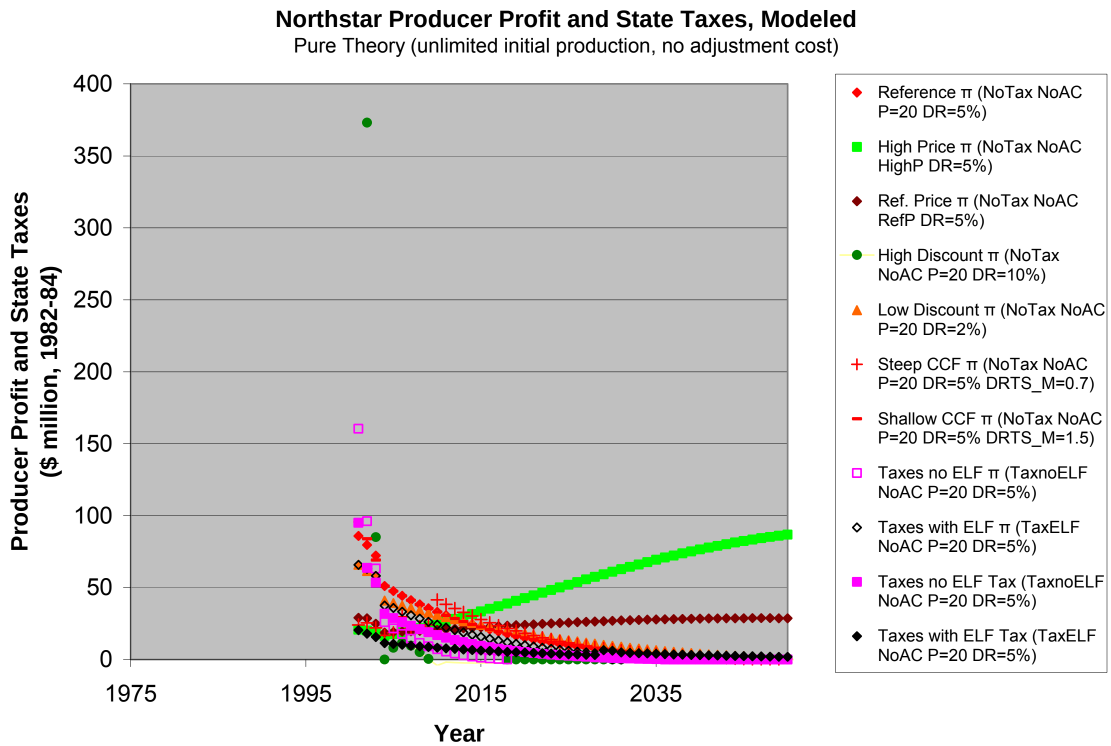

Endicott Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=12%)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=12%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=12%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=12%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=12%)

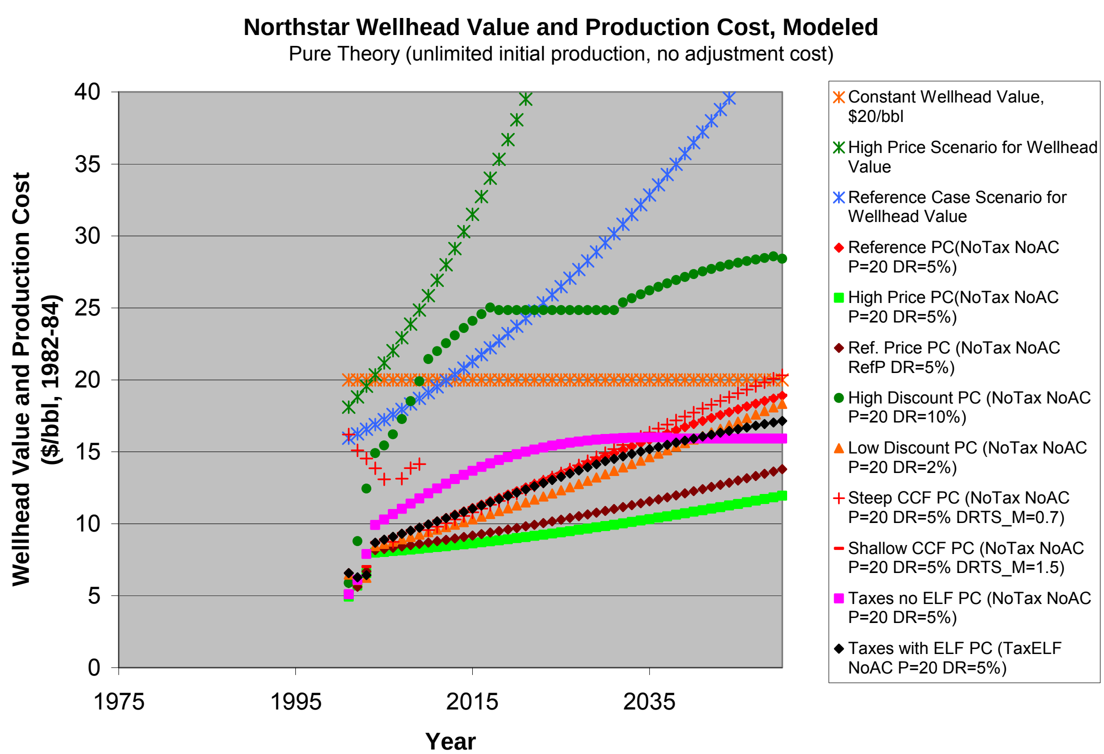

Endicott Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
20000000
18000000
16000000
14000000
12000000
10000000
8000000
6000000
4000000
2000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=7.5*10^7, Ref. P, DR=12%)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=12%)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=12%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=12%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=12%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=12%)

Northstar: *uncalibrated model results*

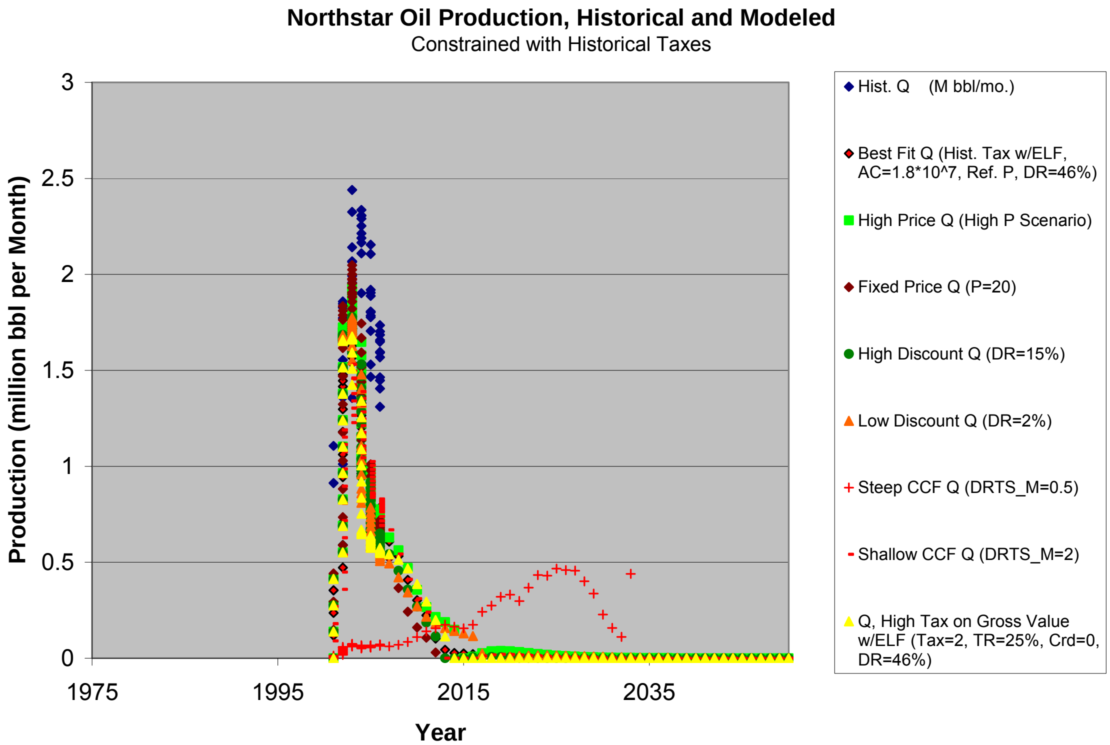


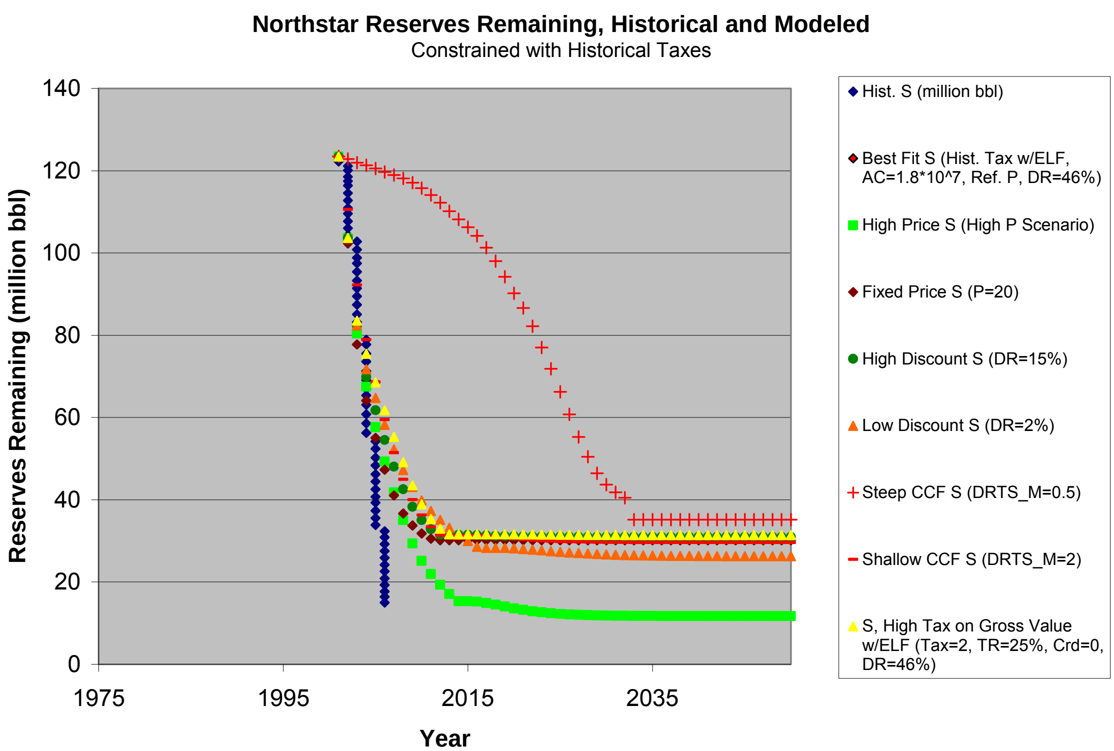


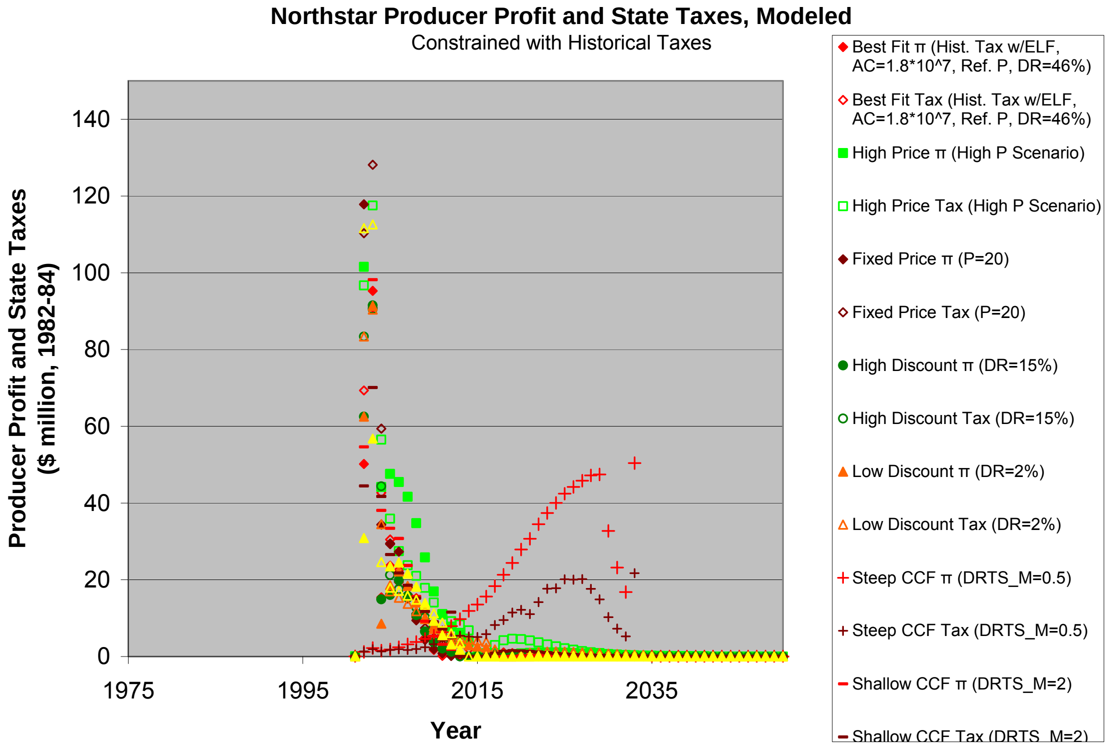

Northstar Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
400
350
300
250
200
150
100
50
0
1975
1995
2015
2035
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

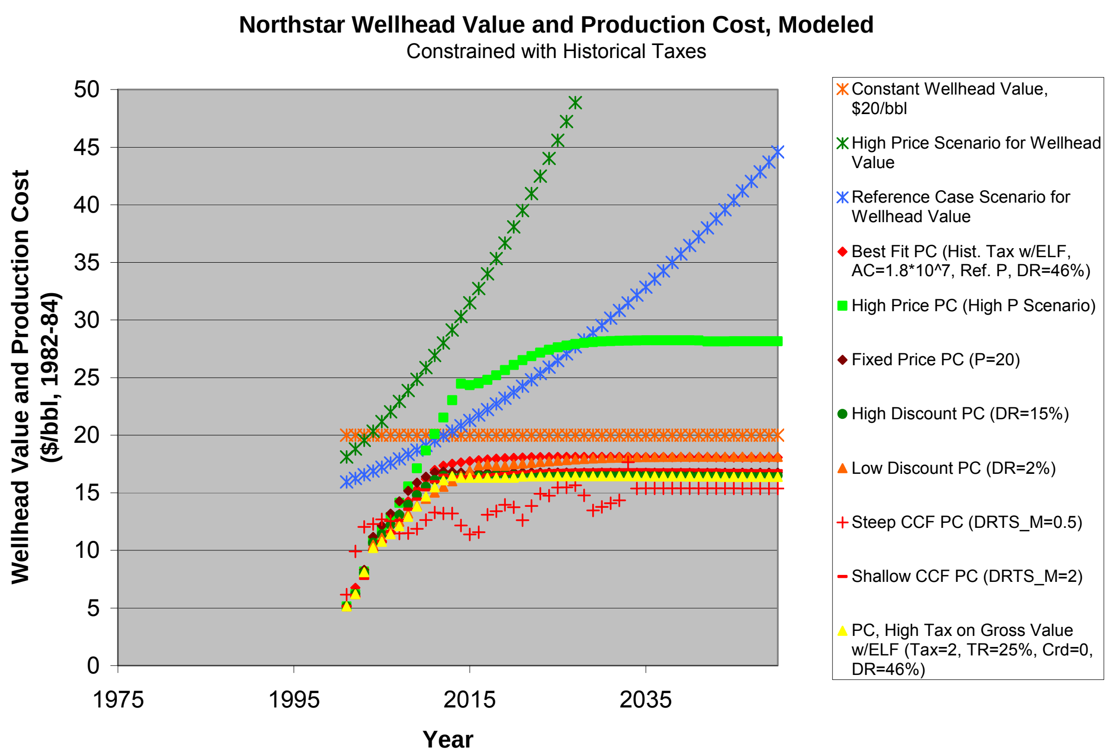

Northstar Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
40
35
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

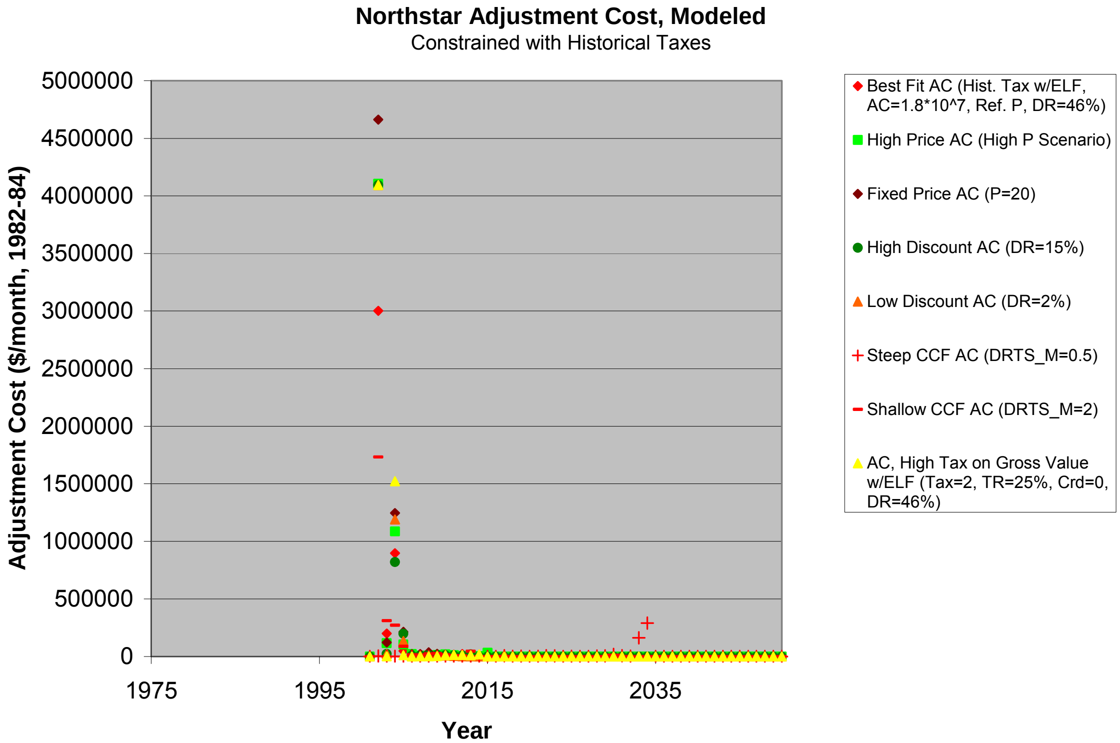
Northstar: *calibrated model results*





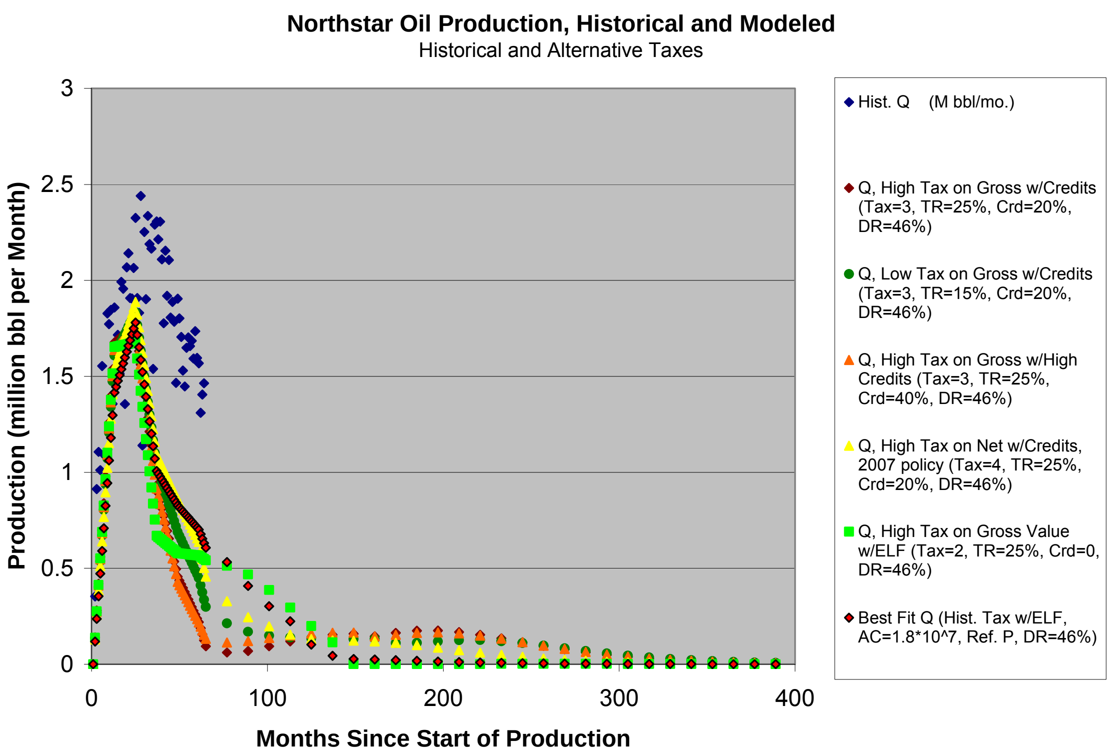

Northstar Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
Year
0
20
40
60
80
100
120
140
1975
1995
2015
2035
Best Fit π (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
Best Fit Tax (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π (DR=15%)
High Discount Tax (DR=15%)
Low Discount π (DR=2%)
Low Discount Tax (DR=2%)
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)
Shallow CCF Tax (DRTS_M=2)

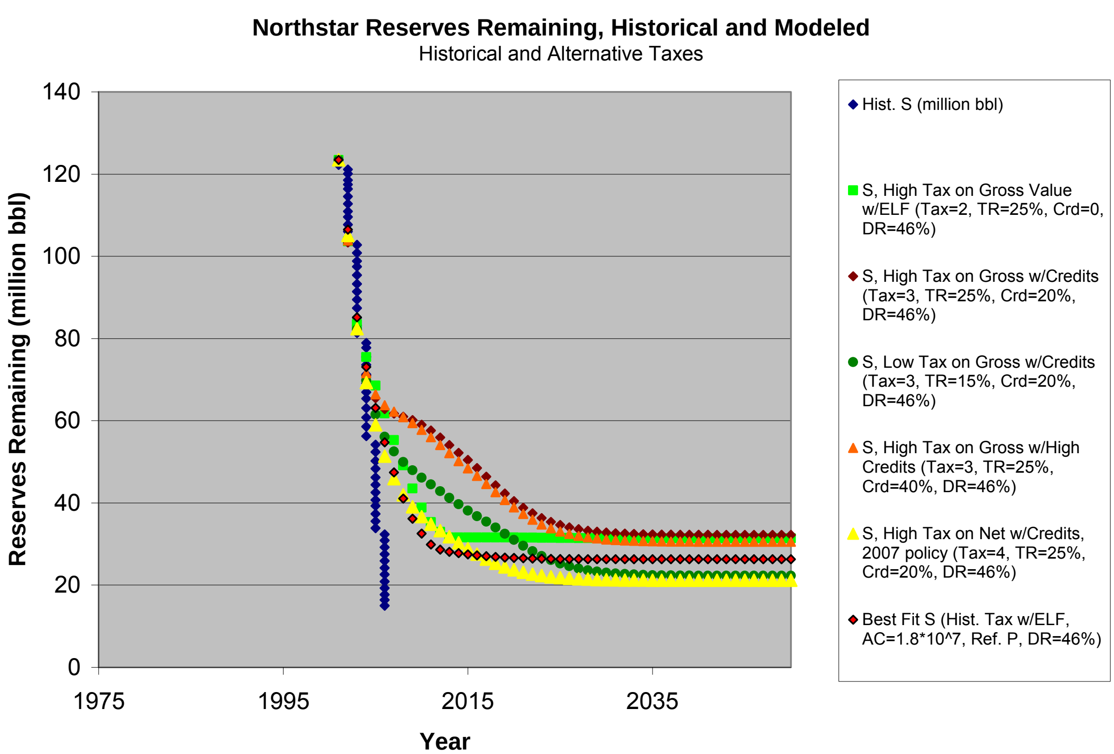

Northstar Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
Year
0
5
10
15
20
25
30
35
40
45
50
1975
1995
2015
2035
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC (DR=15%)
Low Discount PC (DR=2%)
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=46%)

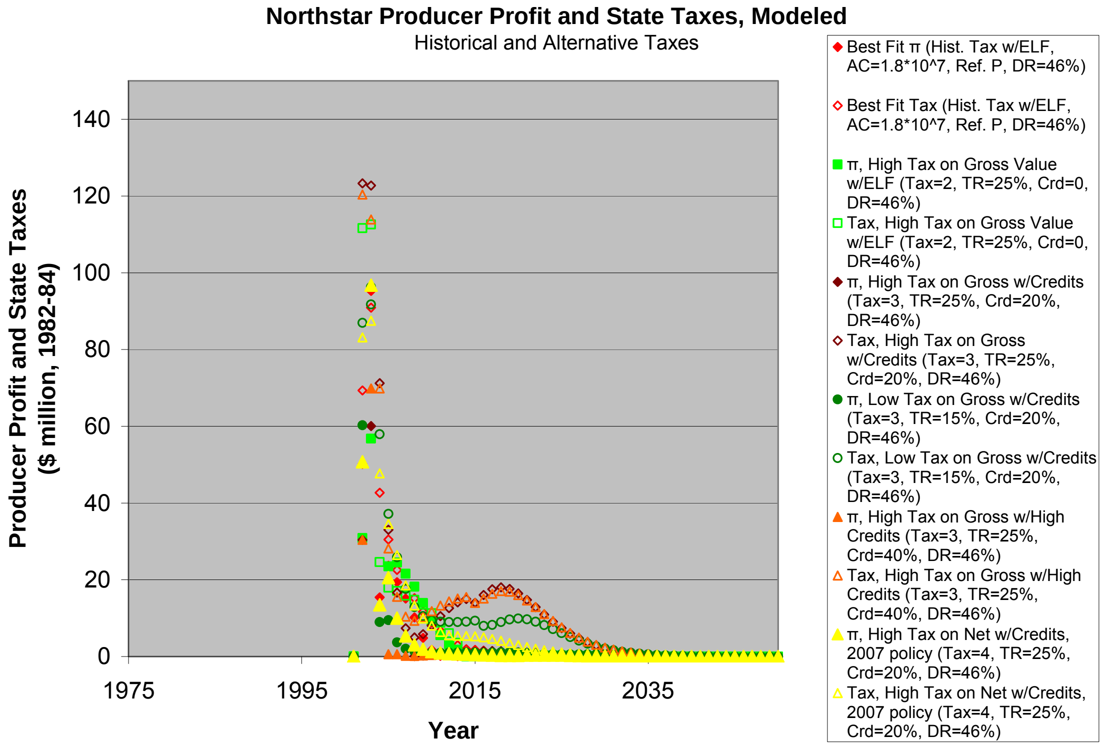


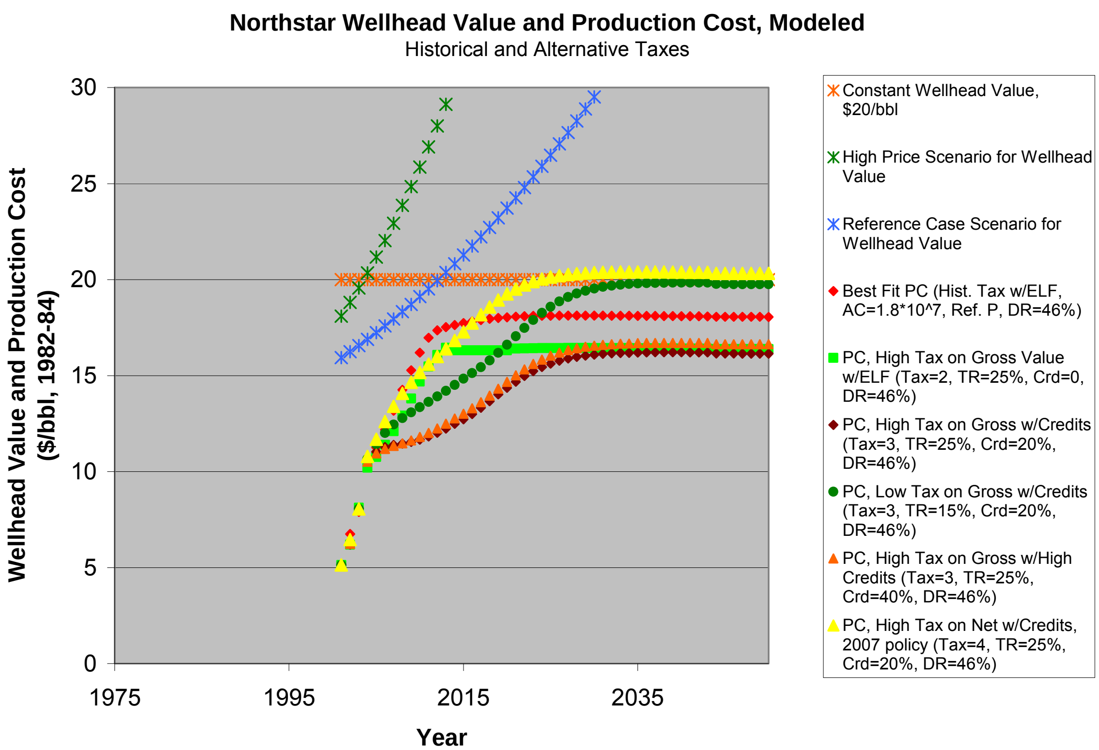
Northstar Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
5000000
4500000
4000000
3500000
3000000
2500000
2000000
1500000
1000000
500000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC (DR=15%)
Low Discount AC (DR=2%)
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=46%)

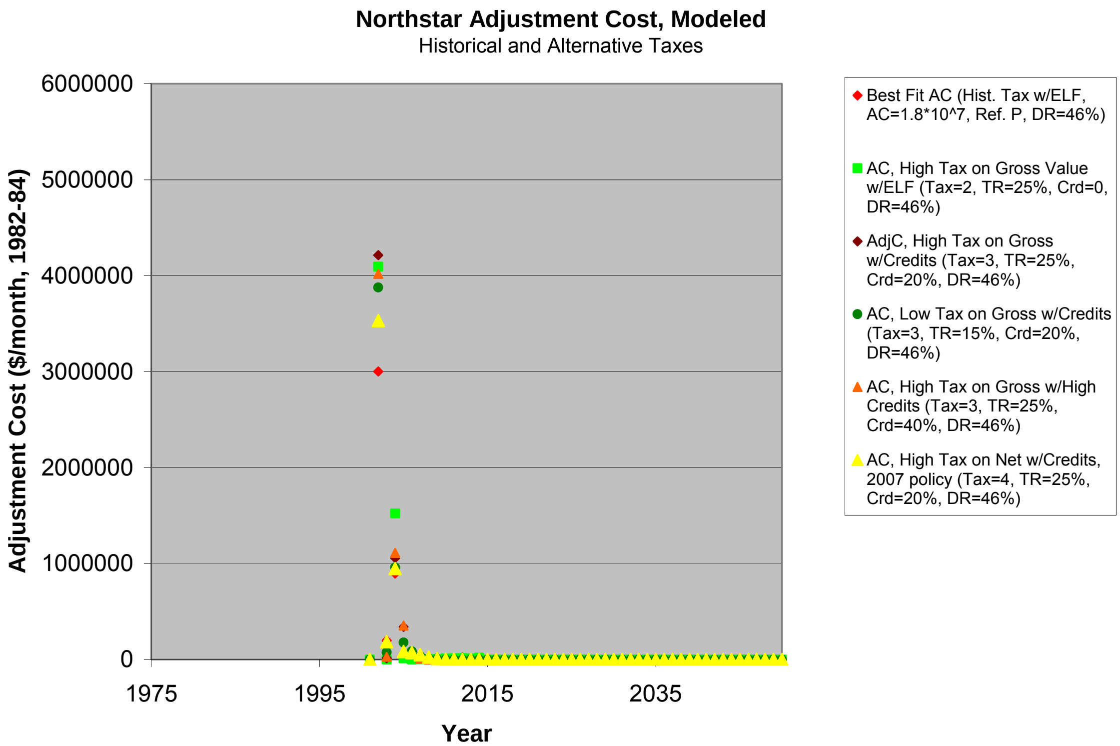
Northstar: *tax scenario model results*



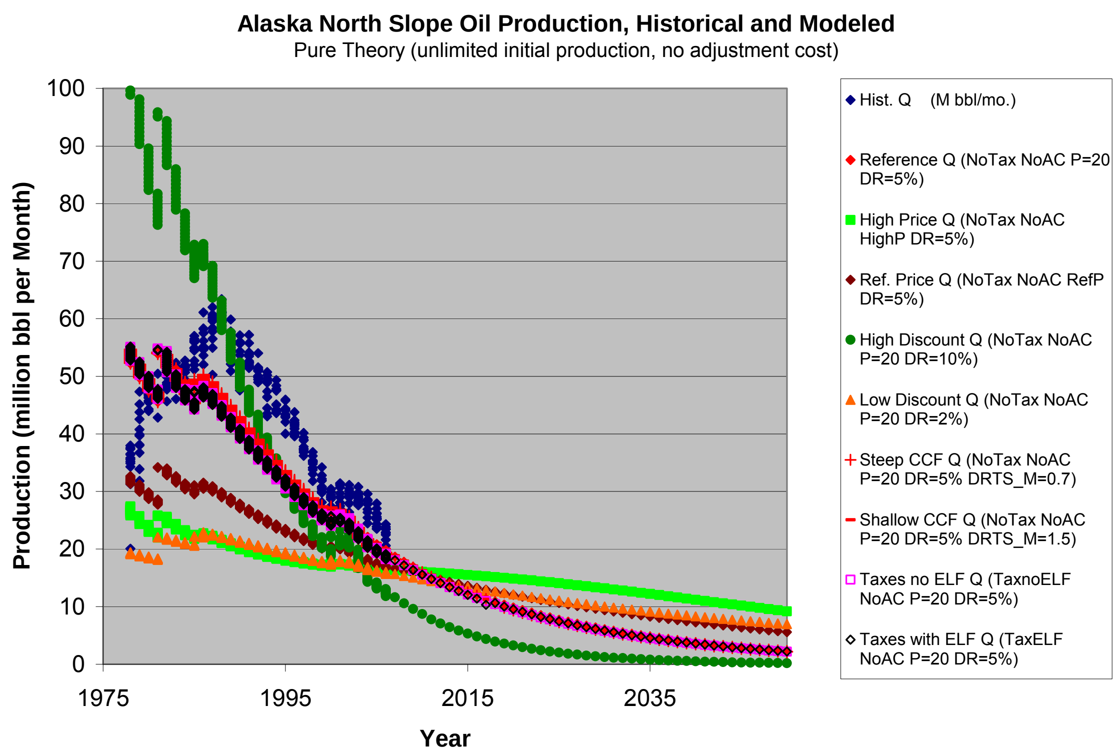


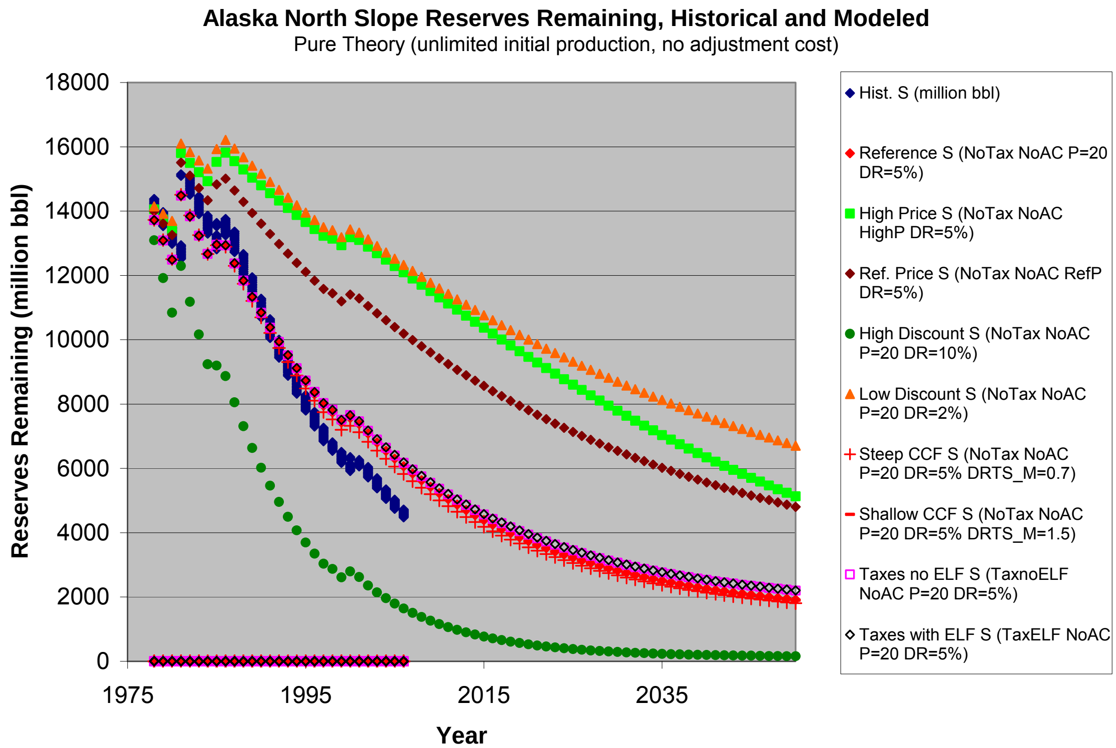

Northstar Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes ($ million, 1982-84)
140
120
100
80
60
40
20
0
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
Best Fit Tax (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=46%)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=46%)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=46%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=46%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=46%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=46%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=46%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=46%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=46%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=46%)

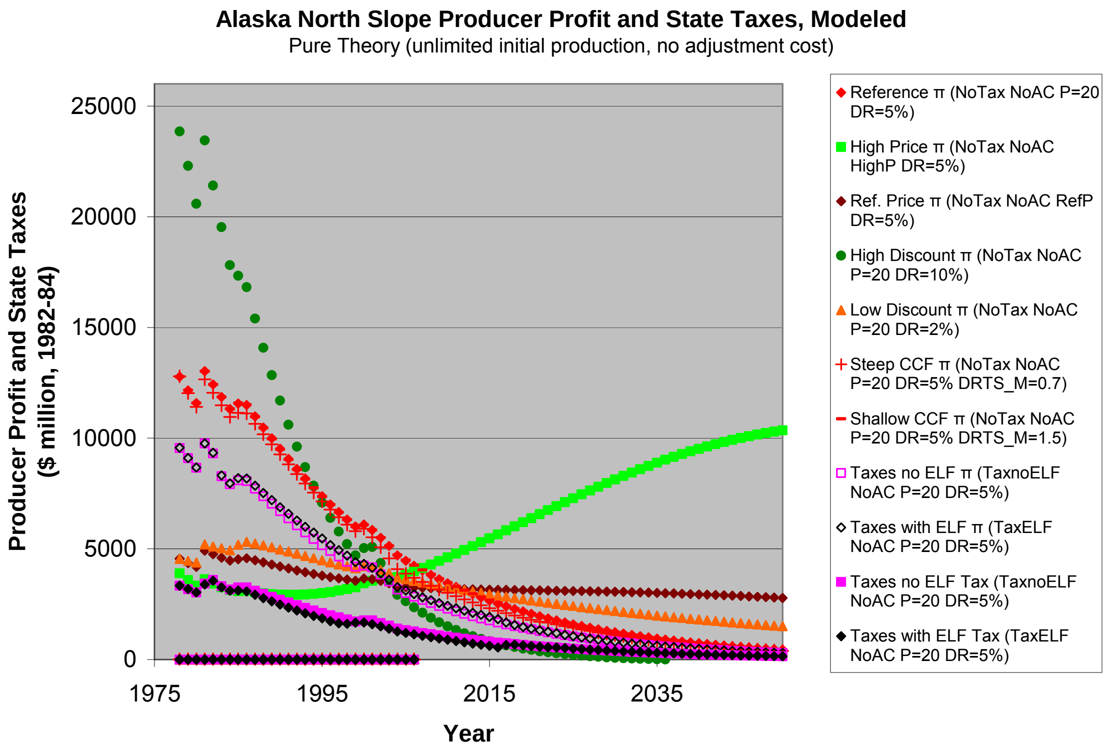

Northstar Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=46%)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=46%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=46%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=46%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=46%)

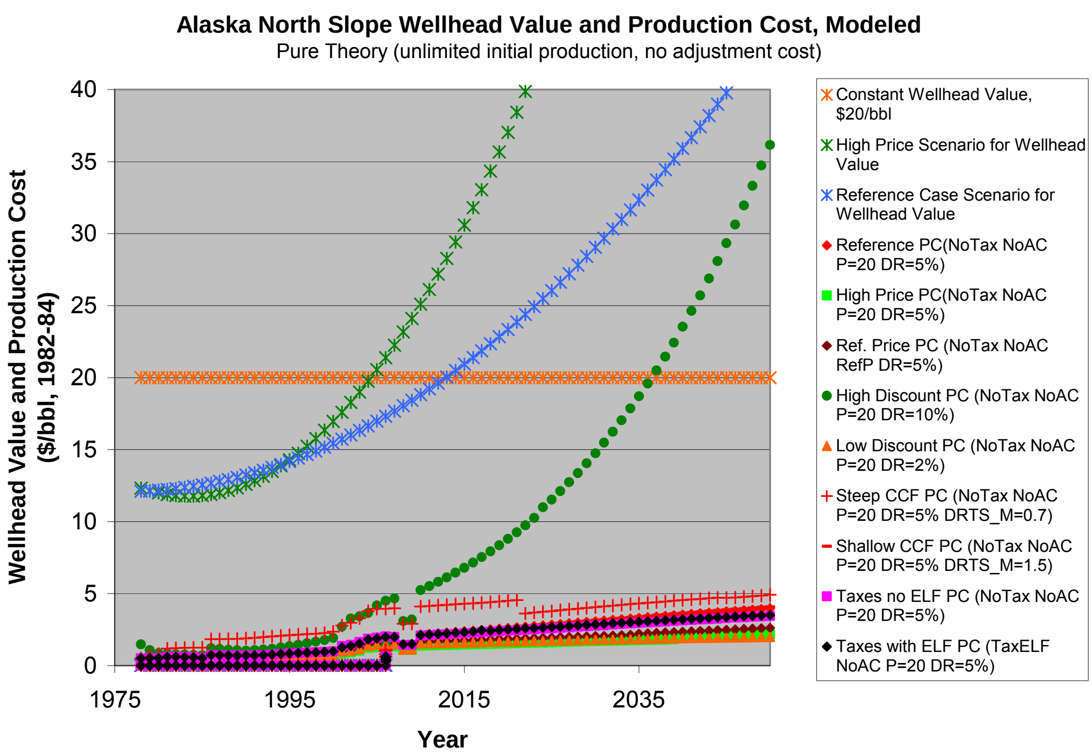

Northstar Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
6000000
5000000
4000000
3000000
2000000
1000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, AC=1.8*10^7, Ref. P, DR=46%)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0, DR=46%)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%, DR=46%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%, DR=46%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%, DR=46%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%, DR=46%)

***North Slope Total:** uncalibrated model results*

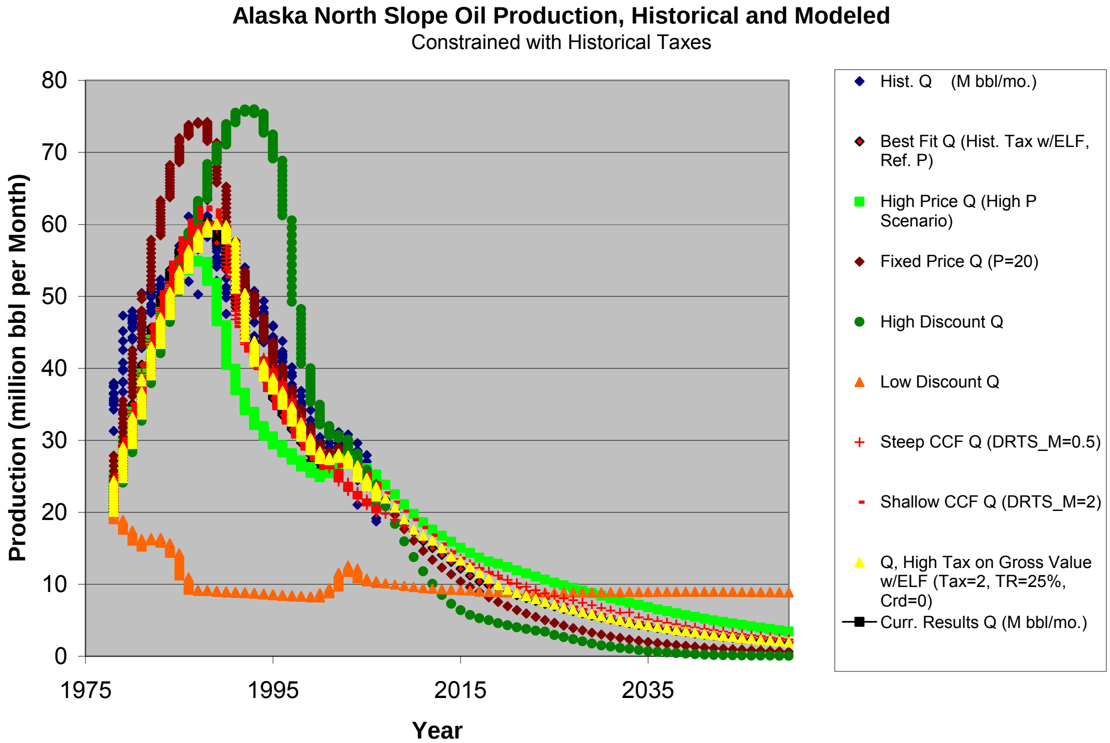


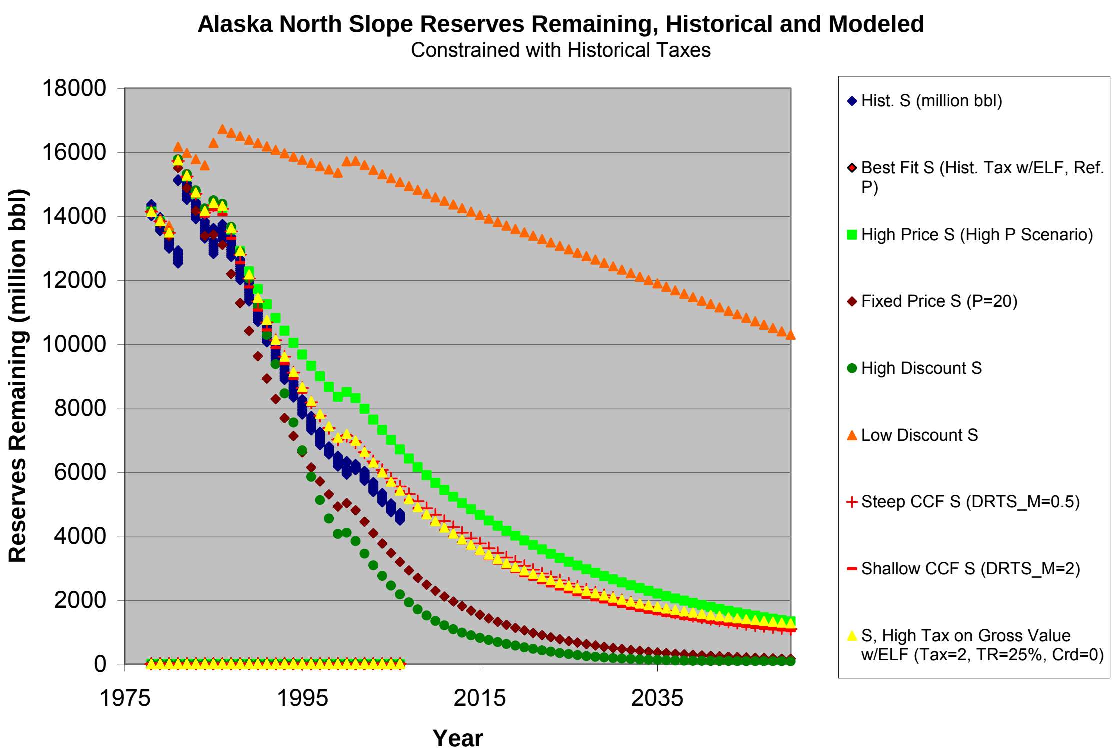


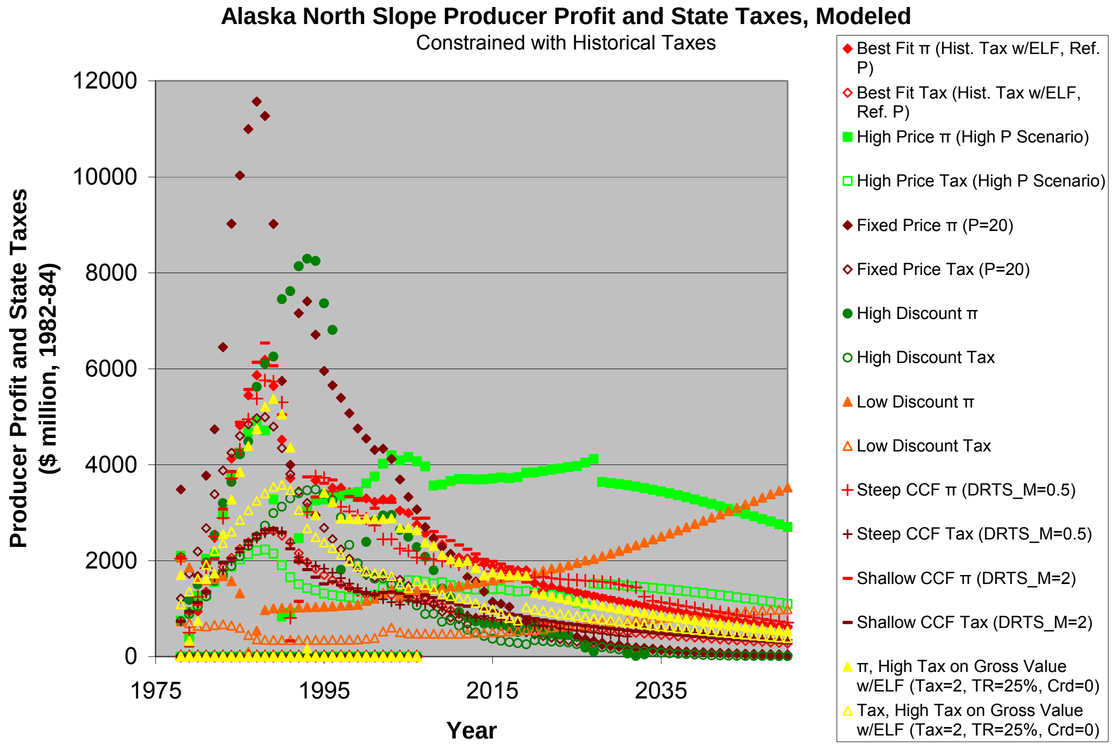

Alaska North Slope Producer Profit and State Taxes, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Producer Profit and State Taxes ($ million, 1982-84)
25000
20000
15000
10000
5000
0
1975
1995
2015
2035
Year
Reference π (NoTax NoAC P=20 DR=5%)
High Price π (NoTax NoAC HighP DR=5%)
Ref. Price π (NoTax NoAC RefP DR=5%)
High Discount π (NoTax NoAC P=20 DR=10%)
Low Discount π (NoTax NoAC P=20 DR=2%)
Steep CCF π (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF π (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF π (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF π (TaxELF NoAC P=20 DR=5%)
Taxes no ELF Tax (TaxnoELF NoAC P=20 DR=5%)
Taxes with ELF Tax (TaxELF NoAC P=20 DR=5%)

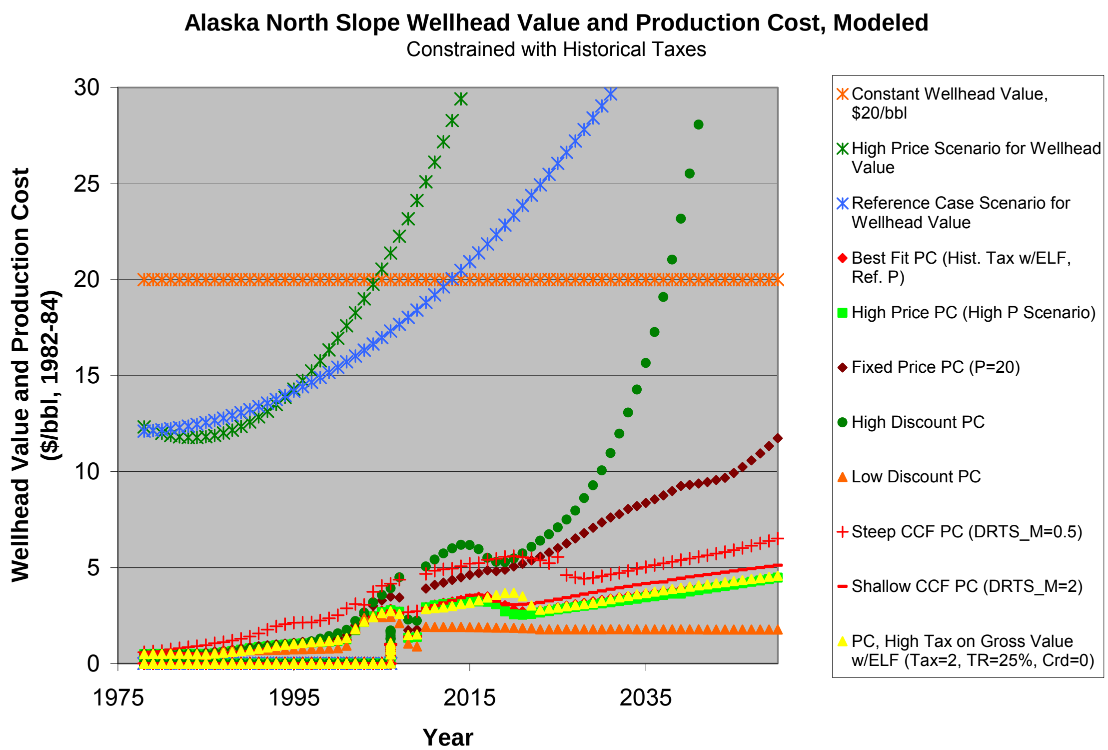

Alaska North Slope Wellhead Value and Production Cost, Modeled
Pure Theory (unlimited initial production, no adjustment cost)
Wellhead Value and Production Cost ($/bbl, 1982-84)
40
35
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Reference PC(NoTax NoAC P=20 DR=5%)
High Price PC(NoTax NoAC P=20 DR=5%)
Ref. Price PC (NoTax NoAC RefP DR=5%)
High Discount PC (NoTax NoAC P=20 DR=10%)
Low Discount PC (NoTax NoAC P=20 DR=2%)
Steep CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=0.7)
Shallow CCF PC (NoTax NoAC P=20 DR=5% DRTS_M=1.5)
Taxes no ELF PC (NoTax NoAC P=20 DR=5%)
Taxes with ELF PC (TaxELF NoAC P=20 DR=5%)

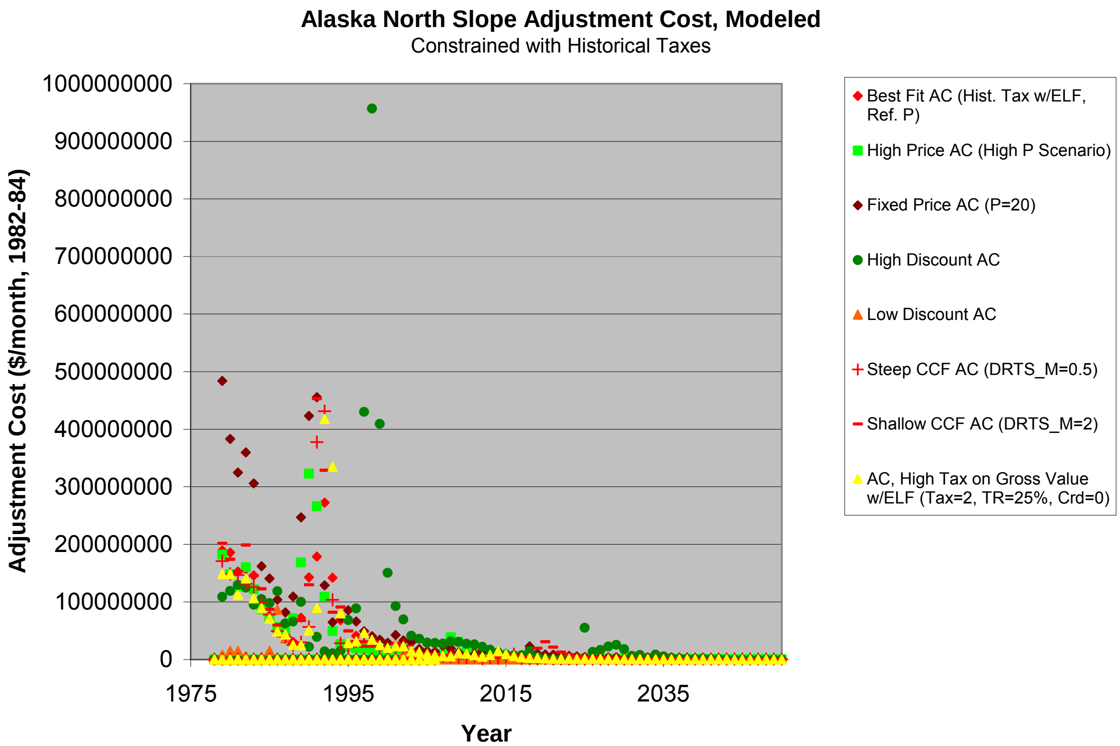
North Slope Total: *calibrated model results*



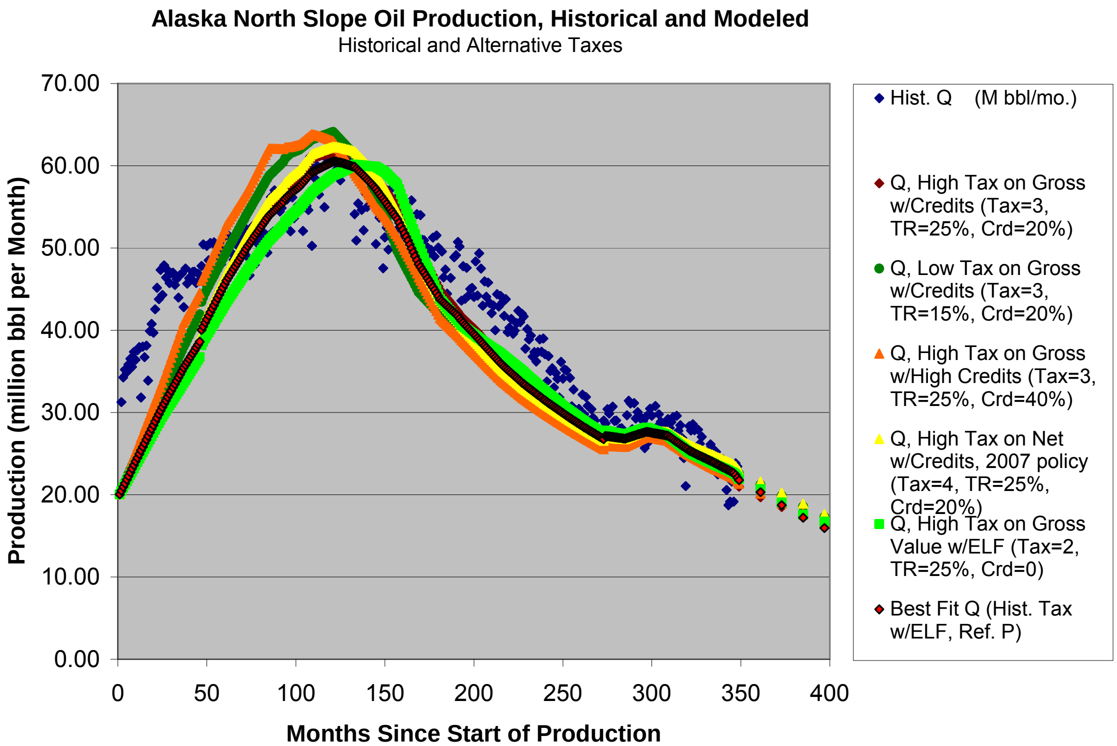


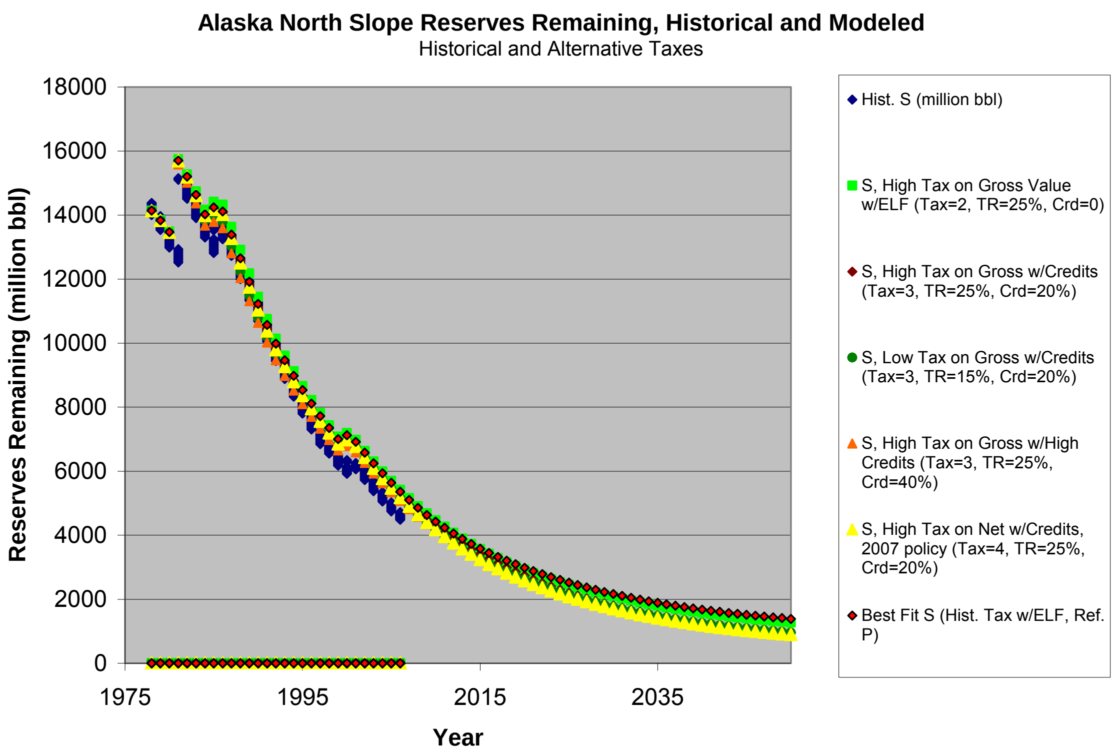

Alaska North Slope Producer Profit and State Taxes, Modeled
Constrained with Historical Taxes
Producer Profit and State Taxes ($ million, 1982-84)
12000
10000
8000
6000
4000
2000
0
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, Ref. P)
Best Fit Tax (Hist. Tax w/ELF, Ref. P)
High Price π (High P Scenario)
High Price Tax (High P Scenario)
Fixed Price π (P=20)
Fixed Price Tax (P=20)
High Discount π
High Discount Tax
Low Discount π
Low Discount Tax
Steep CCF π (DRTS_M=0.5)
Steep CCF Tax (DRTS_M=0.5)
Shallow CCF π (DRTS_M=2)
Shallow CCF Tax (DRTS_M=2)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)

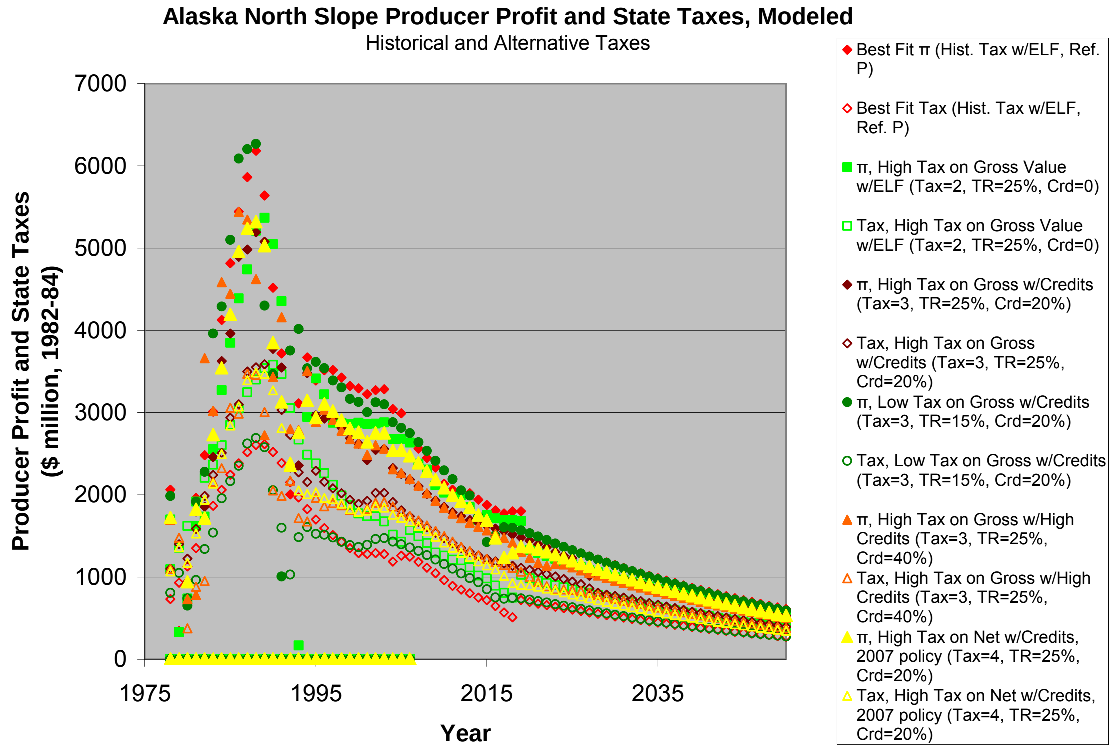

Alaska North Slope Wellhead Value and Production Cost, Modeled
Constrained with Historical Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
30
25
20
15
10
5
0
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, Ref. P)
High Price PC (High P Scenario)
Fixed Price PC (P=20)
High Discount PC
Low Discount PC
Steep CCF PC (DRTS_M=0.5)
Shallow CCF PC (DRTS_M=2)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)

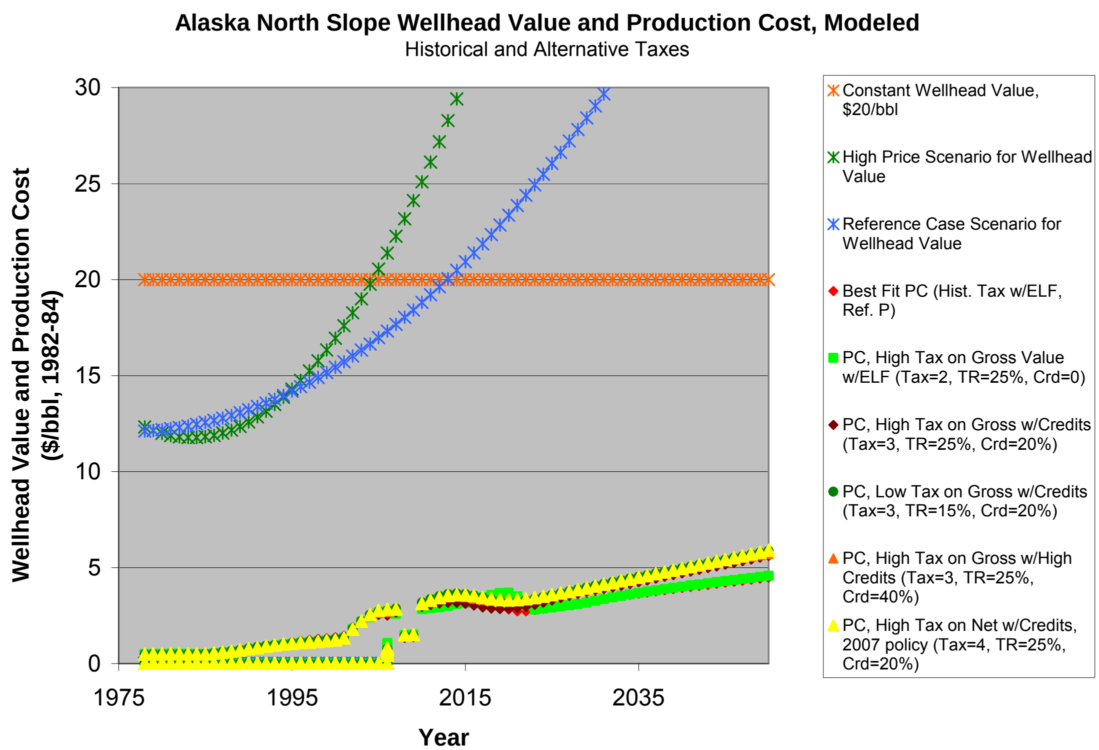

Alaska North Slope Adjustment Cost, Modeled
Constrained with Historical Taxes
Adjustment Cost ($/month, 1982-84)
1000000000
900000000
800000000
700000000
600000000
500000000
400000000
300000000
200000000
100000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, Ref. P)
High Price AC (High P Scenario)
Fixed Price AC (P=20)
High Discount AC
Low Discount AC
Steep CCF AC (DRTS_M=0.5)
Shallow CCF AC (DRTS_M=2)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)

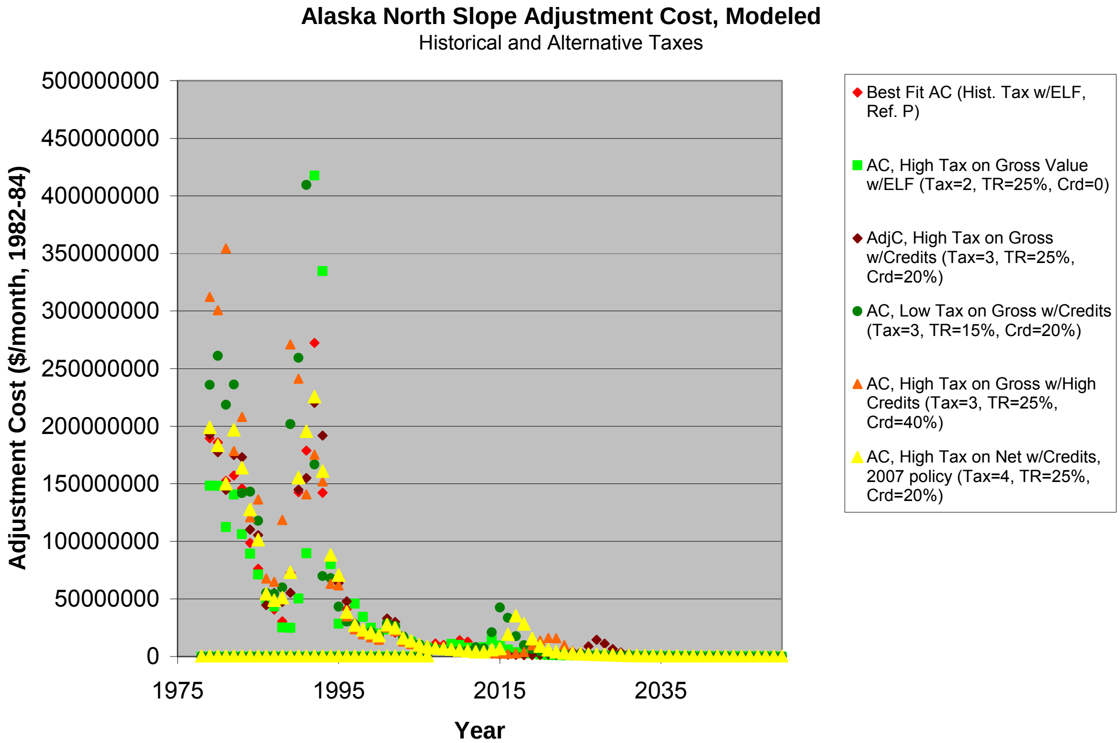
North Slope Total: *tax scenario model results*






Alaska North Slope Producer Profit and State Taxes, Modeled
Historical and Alternative Taxes
Producer Profit and State Taxes ($ million, 1982-84)
0
1000
2000
3000
4000
5000
6000
7000
1975
1995
2015
2035
Year
Best Fit π (Hist. Tax w/ELF, Ref. P)
Best Fit Tax (Hist. Tax w/ELF, Ref. P)
π, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)
Tax, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)
π, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%)
Tax, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%)
π, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%)
Tax, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%)
π, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%)
Tax, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%)
π, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%)
Tax, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%)


Alaska North Slope Wellhead Value and Production Cost, Modeled
Historical and Alternative Taxes
Wellhead Value and Production Cost ($/bbl, 1982-84)
0
5
10
15
20
25
30
1975
1995
2015
2035
Year
Constant Wellhead Value, $20/bbl
High Price Scenario for Wellhead Value
Reference Case Scenario for Wellhead Value
Best Fit PC (Hist. Tax w/ELF, Ref. P)
PC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)
PC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%)
PC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%)
PC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%)
PC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%)



Alaska North Slope Adjustment Cost, Modeled
Historical and Alternative Taxes
Adjustment Cost ($/month, 1982-84)
500000000
450000000
400000000
350000000
300000000
250000000
200000000
150000000
100000000
50000000
0
1975
1995
2015
2035
Year
Best Fit AC (Hist. Tax w/ELF, Ref. P)
AC, High Tax on Gross Value w/ELF (Tax=2, TR=25%, Crd=0)
AdjC, High Tax on Gross w/Credits (Tax=3, TR=25%, Crd=20%)
AC, Low Tax on Gross w/Credits (Tax=3, TR=15%, Crd=20%)
AC, High Tax on Gross w/High Credits (Tax=3, TR=25%, Crd=40%)
AC, High Tax on Net w/Credits, 2007 policy (Tax=4, TR=25%, Crd=20%)