

Modeling Bicycle Facility Operation: a Cellular Automaton Approach

Gregory Gould

Department of Civil and Environmental Engineering
& Institute of Transportation Studies

University of California

1 Shields Ave, Davis, CA, 95616

Phone: (603) 781-1929 Fax: (530) 752-7872 ggould@ucdavis.edu

Alex Karner

Department of Civil and Environmental Engineering
& Institute of Transportation Studies

University of California

1 Shields Ave, Davis, CA, 95616

Phone: (530) 752-0586 Fax: (530) 752-7872 aakarner@ucdavis.edu

Abstract

State-of-the-art methods for the safe and efficient design of bicycle facilities are based on difficult to collect data and potentially dubious assumptions regarding cyclist behavior. Simulation models could offer a way forward, but existing bicycling models in the academic literature have not been validated using actual data. This paper attempts to address both of these shortcomings simultaneously by conducting a field study to obtain real-world bicycle data and implementing a simulation using a multilane and inhomogeneous cellular automata model to reproduce the observations. The resulting model is found to emulate field conditions while possibly under-predicting bike path capacity. The analysis indicates that the model's potential as for planning could be high given additional work on the underlying model specification and the collection of additional data.

Given current concerns surrounding regional air pollution, climate change and urban congestion, this research is timely. If we begin to see more substantial mode shifting to non-motorized modes, this and similar models could become standard tools in the city or regional planner's toolkit.

After a discussion of the context in which this research is being conducted, we review the relevant literature on bicycle facility design and bicycle traffic operation, before summarizing the real-world data collection methods and results. The simulation model is then presented along with results and discussion comparing the modeled to observed data. We conclude with suggestions for future work in data collection and model development.

Introduction

Traffic engineers typically concern themselves with designing intersections and roads which are safe and which provide a sufficient level of service (LOS) to users. For highways, LOS is ranked from A to F (best to worst) according to its volume to capacity (v/c) ratio as defined in the Highway Capacity Manual (TRB 2000b). The capacity of a highway is the point at which a marginal increase in vehicle density results in decreasing traffic flow rates and corresponds to LOS E. The relationships between highway traffic flow, density, and speed are well understood through observation, models and simulations (see, e.g., Gartner, Messer et al. 2001). The understanding of the relationship between these basic traffic quantities and the physical highway environment is essential to the traffic engineer in building properly designed highways.

When it comes to designing bicycle facilities, namely shared-use “bicycle”¹ paths and bike lanes, comparatively much less is known about the relationships between the basic traffic quantities mentioned above. Until fairly recently, most bicycle paths were designed as two lane, two-way facilities using some basic and arbitrary assumptions about the width required for safe bicycle travel. This approach does not allow for designing bicycle paths based on projected or observed bicycle demand, speed or other important factors. Given the lack of knowledge about bicycle operation, several studies have been conducted with the goal of quantifying the relationships between flow, density and speed, and several methods have been proposed to measure bicycle path LOS.

Given rising concern over regional air quality and climate change which are both linked to motor vehicle use, bicycling may become an increasingly attractive alternative to driving.² The latter set of concerns stems from the burning of fossil fuels to provide transportation energy resulting in the emission of the greenhouse gas carbon dioxide,³ the primary driver of climate change (IPCC 2007) whose magnitude is substantial and growing. The potential to use bicycles and

¹ Which typically carry at least pedestrian in addition to bicycle flow, although there are shared use paths for bicycles and mopeds (not pedestrians) in the Netherlands (Botma and Papendrecht 1991).

² There are substantial cultural barriers to overcome before the potential of the bicycle can be fully realized, however. The Secretary of Transportation, Mary Peters, in a recent television interview on the Public Broadcasting Station stated that, “there’s about probably[sic] some 10 percent to 20 percent of the current spending [derived from gas tax revenue] that is going to projects *that really are not transportation*...Some of that money is being spent on...bike paths [and] trails” (emphasis mine). See <http://tinyurl.com/2mxmzm> for the complete transcript.

³ Other greenhouse gases and air pollutants are also created, but carbon dioxide represents the largest proportion.

other non-motorized modes for climate change mitigation has long been recognized. In a report for the World Bank written in 1992, Replogle (1992) observes that without policy support, nonmotorized vehicular travel will decline precipitously in major Asian cities with disastrous climate change effects. It appears instead that many countries, but most notably China, have pressed ahead with motorization goals according to recent disaggregate international vehicle ownership forecasts presented by Dargay et al. (2007). They project an 11 percent annual average growth rate for vehicle ownership in China from 2002 to 2030. This is clear evidence that Replogle's (1992) advice was not heeded.

The attractiveness of the bicycle increases further when combining these concerns with urban congestion issues. If cycling is to become a significant mode of transportation, it will have to be accompanied by an increase in the quality and quantity of bicycle infrastructure. As an example, Kingham et al. (2001) surveyed United Kingdom commuters about what factors would drive a mode shift to bicycle from automobile. They found that improved infrastructure, security and safety combined with residential locations closer to work could effect this change. At least one of these components—improved infrastructure—will invariably be measured using some form of LOS. If provided at an appropriate level, the desired changes in land use could be achieved as a result.

In the near- to medium-term, with mode share dominated by motor vehicles, the focus of most construction and transportation funding is likely to remain on highways and related infrastructure. Therefore, research into the optimal design and operation of bicycle facilities is a timely venture since the bicycle facilities that we operate and design today under limited budgets will influence the perception of bicycling as a viable form of transportation tomorrow. Limited funding should not be wasted on overbuilt bicycle facilities, nor should that funding go to projects which provide such a poor LOS that users are unnecessarily inconvenienced or endangered.

In this paper we discuss the relationships and differences between vehicular flow on highways and bicycle traffic flow on shared-use paths. We review the historical and current literature covering bicycle traffic flow theory and how it is used in determining the LOS and the design

widths of bike paths. We find that available data and data collection methods present problems in bicycle traffic research caused by the difficulty of observing potentially important bicycle traffic parameters (such as the number of passings and delayed passings) and the relative infrequency of paths which receive a high volume of bike traffic, thus limiting opportunities to observe the full range of traffic conditions. Given these difficulties, we propose the use of a Cellular Automata simulation model to study some aspects of bicycle operation on a bike path which are difficult to do otherwise. We collect bicycle traffic data from the University of California, Davis (UC Davis) campus at three locations with different path widths and traffic characteristics at high flow times to compare with simulation results.

Literature Review

Bicycle traffic theory is not as advanced as highway traffic theory and has not received nearly as much attention (Botma and Papendrecht 1991). Further, “significant research is required in almost all areas” (Taylor and Davis 1999, p. 108) of bicycle traffic including traffic flow, intersection control, capacity and LOS, modeling and geometric design of cycling facilities. However, numerous studies on bicycle operation and path design have been conducted over the past 40 years. Most of the early studies were conducted in Europe where bicycling as a form of transportation (rather than recreation) makes up a larger mode share (Pucher, Komanoff et al. 1999). In the United States early studies were conducted around college campuses where dedicated bicycle facilities exist and bicycles make up a large portion of mode share.

The objective of most early bicycle traffic studies was to determine the optimal bike path or lane width, while others were conducted to assess safety, the impact of bicycles on vehicle traffic flow, and capacity of intersections (see Taylor and Davis 1999 for a review). However, our current research is only interested with operation of dedicated bicycle and pedestrian facilities so we will not mention these other studies further, unless they contribute additional information.

An early study of bicycle operation for the City of Davis, California and the University of California (Smith 1972) was conducted to determine the adequacy of then-current bicycle facilities and to plan for future bicycle facilities. The study begins with an overview of the state of knowledge of bicycle traffic theory at the time, which comes entirely from earlier European

research. The study reports that based on German design standards, which were the *de facto* standards at the time, a cyclist requires a minimum four foot travel lane due to the fact that a bicycle can not travel in a perfectly straight line and that a bicycle path or lane should then be at least twice this width to allow for passing. A review of the earlier European studies, summarized in a plot, also finds that bike path capacities are approximately 235 bikes/hr/ft, but vary across the studies. These two findings were the main pieces of information used to design many of the bike paths and lanes which exist in Davis today.

Given the lack of bicycle traffic theory at the time, and the increasing popularity of bicycles in the United States, a comprehensive study was conducted by Miller and Ramey (1975) at the University of California, Davis for the DeLeuw, Cather and Company (the company was previously commissioned by the University and City to conduct the initial bicycle flow study mentioned above). The report found that then-current standards for bicycle facility design were based on European standards which vary widely across country and study and had not been validated for conditions in the United States. Further, they found that the then-current design standards did not consider LOS, how the user perceives the quality of the facility. The researchers proposed a new approach to bicycle facility analysis based on the LOS paradigm for Highways (HRB 1965) which grades a facility from A to F (best to worst) based on v/c ratios. The v/c ratios are assumed to be correlated to a user's perception of the quality of service provided by a highway facility which depends on highway conditions including travel time, speed, safety and freedom to maneuver. In order to use this approach the relationship between flow and density is required. This requires the determination of the fundamental diagram of bicycle traffic.

To determine the fundamental diagram of bicycle operation Miller and Ramey (1975) used a radar gun to measure speed, and counted bicycle flow on a selection of bicycle paths around Davis and the American River path in Sacramento. They then determined density using the well known relationship between the three parameters and the path width as shown in equation 1.

$$(1) \quad k = \frac{q}{u \cdot width}$$

where:

q = flow (bikes/hr)

u = average speed (mph)

k = density (bikes/ft²)

$width$ = path width (ft)

A summary of Miller and Ramey's (1975) findings which were summarized in a report by Homburger (1976) are shown in Figure 1.

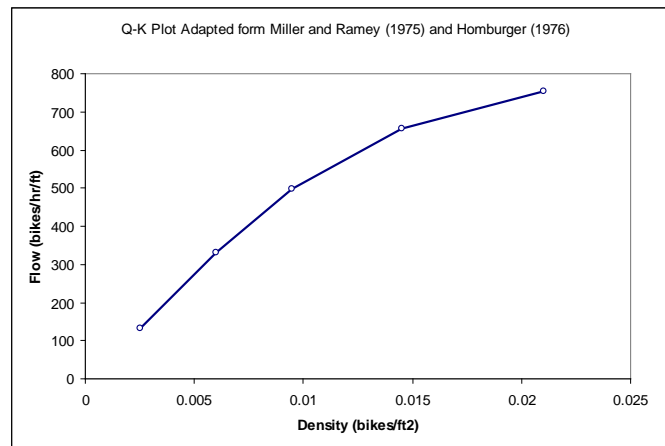


Figure 1: Fundamental diagram for bicycle flow

The results, as indicated by the plot, contradict the earlier estimate of bicycle path capacity of 235 bikes/hr/ft and appears to indicate that capacity is somewhere above 700 bikes/hr/ft. The authors find, based on fitting a polynomial function to the data, that capacity is reached at 792 bikes/hr/ft.

An alternative approach to characterize bicycle flow relationships was undertaken by Navin (1994) who analyzed cyclists under experimental conditions. Children (aged 11-14) were recruited from elementary schools and were filmed while riding on a 2.5 m wide ovular track behind a lead cyclist. Different groups were selected, each at a different lead cyclist speed. Data collection included speeds of cyclists (compared to the pacesetter to ensure that stability had been achieved) and density measured as the sum of bicycles entirely and partially on the observed area, divided by the area of the section. Flow was counted as the number of bikes passing a point per unit time. Turning radii and grade effects on speed were also observed. Navin (1994) concludes that the maximum flow achievable on a 2.5m wide bike path is 10,000 bikes/h,

or 1220 bikes/hr/ft, somewhat higher than the previous estimate likely due to the orderly manner in which the experiment was conducted.

For validation of his experimental results, Navin (1994) utilizes data from Botma and Papendrecht (1991) who studied bicycle traffic operations on a mixed use bike path (bicycles and mopeds) under real-world conditions using a specially designed mat containing detectors capable of measuring moments of passage and lateral positions of cycles. No location where capacity was reached could be found and mean speed was found to be independent of path width, and speed measurements followed a normal distribution. Using a definition of capacity based on an extrapolation of the volume-density ratio, they find an approximate capacity of 6000 bikes/hr on a 250cm wide bike lane, or 732 bikes/hr/ft while noting that “this value is only an indication of the order of magnitude of bicycle path capacity” (p. 68) since it is not clear that their data follows a quadratic relationship, but they still proceed to fit one to it. Alternately, they define capacity as reached when all cyclists are “nonfree” or constrained to follow a bike in front of them instead of proceeding at their desired speed. Using a decomposition of bikes into free and nonfree based on previously defined methods for automobiles, they determined that capacity varied between 1170 and 1400 bikes/hr/ft largely in agreement with Navin (1994).

To determine LOS, Miller and Ramey (1975) divided the area under the curve in Figure 1 to the left of the critical density (found to be 0.024 bikes/ft^2) into six sections and assigned them the familiar A – F grades. LOS A was chosen to be the area where free flow speeds were observed, with the other grades being assigned based on v/c ratios reported for Highway traffic in the 1965 Highway Capacity Manual (HRB 1965). Based on this data, and findings that flow-density relationships are not linear with path width, minimum path widths can be determined to meet each level of service (see Smith 1976, Table 32 for an example of minimum path width determination).

Navin (1994) defines LOS to “reflect riding comfort and freedom to move laterally” (p. 34). As such, he proposes LOS based on the free area surrounding a bicycle which he breaks into three zones representing shrinking distances to the reference cyclist: circulation, comfort and collision.

In LOS A, no zones overlap whereas in LOS F, collisions are eminent. The author then calculates v/c ratios for each LOS, returning to similar conclusions as above.

The LOS approaches proposed above never caught on, and most paths were constructed based on guidelines provided by AASHTO's "Guide for the Development of Bicycle Facilities," the latest edition of which was published in 1999 (AASHTO 1999). Earlier editions form the basis of the 1985 Highway Capacity Manual's bicycle recommendations (TRB 1985). The AASHTO guide recommends a 10 foot wide bike path and under expectations of high usage up to 14 foot wide bike paths. These guidelines appear to be based on earlier research indicating that a cyclist requires a minimum of four feet per lane, so that a two lane bicycle path (two direction bike paths) should be at least eight feet wide. An extra two feet are added to accommodate service vehicles and allow some extra room for passing.

Nearly two decades after the first LOS approach was proposed, Botma (1995) and Allen et al. (1998) proposed a new LOS approach based on the idea of hindrance. Botma's (1995) idea was that the quality of a bicycle path trip was based on how constrained a cyclist's movements are. This is similar to the ideas of Miller and Ramey (1975) and that of the LOS used for highways in the Highway Capacity Manual (TRB 2000b) but differs in that Botma (1995) does not assume density is the most important factor in determining hindrance. Instead, he proposes that the number of passing and meeting (meeting a cyclist traveling in the opposite direction) events are better proxy of hindrance and hence of LOS. This may be a better measure of LOS due to several important differences between bicycle facility operation and highway operation. The first is that not all cyclists attempt to travel at the same maximum speed. Slow cyclists often impede faster cyclists as the faster cyclist must wait for an opportunity to pass. Each acceleration to pass a slower cyclist adds to the fatigue and delay of the faster cyclist. The second is that bicycle paths are often in reality "mixed-use" paths which also serve pedestrians and other non-motorized vehicles which impede the cyclist.

Both papers (Botma 1995; Allen, Roupail et al. 1998) provide similar methods for estimating the number of passing and meeting events based on some assumptions about bicycle operation and the path. The major assumption are that slow cyclists do not impede faster cyclists, the path

only contains two lanes, meeting events provide 50 percent less hindrance than passing events and that cyclist speed is normally distributed. The first two assumptions severely limit the usefulness of the proposed methods since cyclists do not actually travel in lanes and a planner may want to design a bicycle path with more than two lanes. The second assumption, that slower cyclists do not impede faster cyclists, is plainly unrealistic and likely to be important. Additionally, the third assumption is based solely on the opinion of the researchers. However, at least one study (Virkler and Balasubramanian 1998) found that the proposed methods did a good job at predicting meeting and passing events.

As in previous cases, LOS is assigned a grade from A-F with A being fewer passing and meeting events and F being many. The most current Highway Capacity Manual (TRB 2000a) adopts this methodology as its recommendation for determining the LOS of a bicycle path. In the 2000 Highway Capacity Manual, the formulas presented to determine passing and meeting events include an additional assumption that the mean speed of bicycle traffic is 11.2 mph with a standard deviation of 1.9 mph. No guidance is provided on how to incorporate knowledge of different average speeds.

Due to the above mentioned limitations a new study, the most recent available, "Evaluation of Safety, Design, and Operation of Shared Use Paths" (Hummer, Roupail et al. 2006) has suggested a new method of determining LOS and bike path width. The study's stated goals are to produce an objective measure of bicycle facility LOS which is verified with actual field data, including cyclist's opinions, and is easy to implement with readily available or easy to collect data. The proposed method extends the methods of Botma (1995) and Allen et al.(1998) to account for passive passing events, delayed passing events, and variable path width. Passive passing events occur from the point of view of the cyclist being passed. Delayed passing events are those that a faster cyclist wishes to make, but must wait for a suitable opportunity.

In order to determine an objective LOS measure, cyclists were surveyed about their opinions of the quality of service provided by bike paths of different designs and under different traffic conditions. This was accomplished by showing a group of 105 respondents, mostly from local bicycle user groups, 36 one minute video clips of bike paths of different designs and traffic

conditions and then asking them to rate on a five-point scale their assessment of the adequacy of lateral and longitudinal separation, ability to pass and their overall opinion of quality of service. The results were then analyzed to obtain a regression equation which relates the respondent perceptions of the quality of service to the physical attributes and traffic condition of the displayed paths.

The resulting equation is able to determine a LOS score between one and five based on the number of weighted events (active and passive passing and meeting), the path width and whether or not the path has a center line. However, the survey instrument was not able to capture user perception of delayed passing events because they would be impossible to observe in the 60 second video clips. Accordingly, a correction factor was added based on the judgment of the researchers which reduced the LOS score in proportion to the number of delayed passing events. The resulting LOS was assigned the usual A-F grades, with a LOS score of three defining the boundary between LOS C and D.

To make practical use of this method, an engineer or planner must know the number of active, passive and delayed passing events to determine the LOS or, based on a desired LOS, to determine the proper path width. However, these metrics are extremely difficult to measure in the field. The study by Hummer et al. (2006) used a floating bicycle observer method, complete with onboard video, speed and audio recording devices. Additionally, if a new path is being designed, the engineer or planner needs a method to estimate the required variables. Hummer et al. (2006) developed a model to estimate the required variables given the path width, whether or not there is a center line, flow rate and mode split (ratio of cyclists to pedestrians). The model estimates the number of events that a cyclist traveling at the average speed (12.5 mph) would experience.

While the method proposed by Hummer et al. (2006) expands and improves on the earlier hindrance methods, it still relies on assumption of average speed and speed distribution, assumes a 50/50 directional split in flow and relies on a complicated probabilistic model to estimate the required variables. The validity of the survey technique to determine user perception of bicycle operational quality is also subject to question—the use of bicycle club members instead of a

more representative cyclist or general populations sample has likely biased their results. Slower cyclists or cyclists with different objectives may value path attributes differently. A LOS measure dependent on such an unrepresentative segment of the population cannot be justified for use in a nation wide design standard.

While simulation models cannot tell us anything about a user's perception of bicycle facility quality, they could provide insight into how various parameters, including cyclist behavior, can impact facility operation. Field data, especially potentially important variables such as passing and delayed passing, can be difficult to observe. Additionally, field observations are limited to the conditions that exist today. For example, we rarely observe congested bicycle facilities and are limited to studying bike paths that are no more than 20 feet wide. Simulation models can help fill in the gaps and explore alternative designs. For these reasons, we propose a simulation model to aid in bicycle facility design. The remainder of this paper describes the bike flow data collected on the UC Davis campus and our attempts to reproduce key features of the data using simulation modeling.

Methods: UC Davis data collection

The UC Davis campus provides a good environment for the study of bicycle operation due to a large network of shared-use bike paths and dedicated lanes, and the popularity of using bicycles for non-recreational purposes. Bicycle traffic data was collected between November 30, 2007 and December 5, 2007 at three locations on campus during peak traffic conditions. The sites were chosen based on the following criteria:

- High bike flows
- Designated bike paths with minimal pedestrian and motor vehicle cross traffic
- Far enough from intersections or other obstacles which could impede traffic

The bicycle paths and data collection sites (red boxes) are shown in the images below.

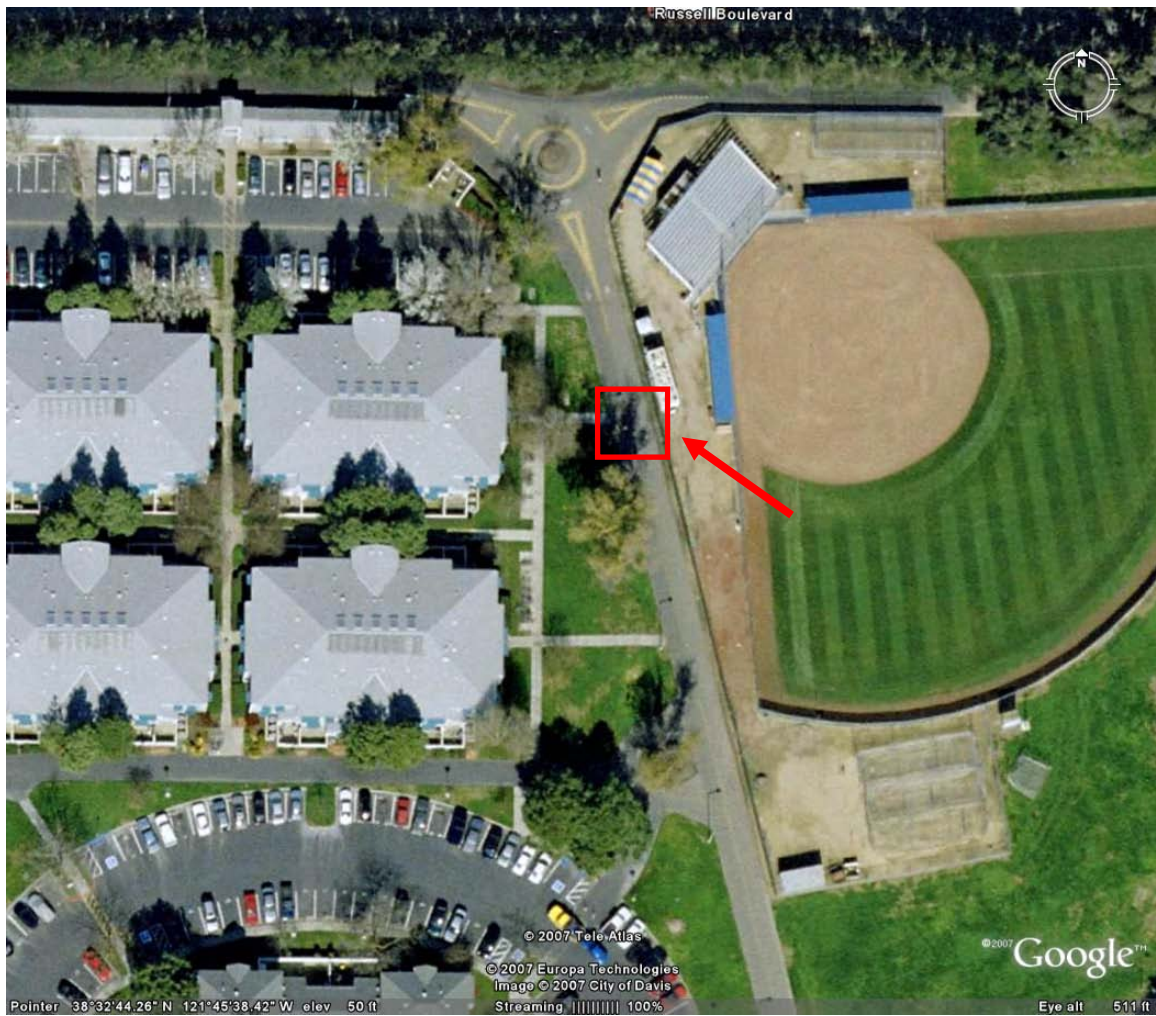


Figure 2: Site 1 – Bird’s eye view, Russell



Figure 3: Site 1 - Ground level view, Russell



Figure 4: Site 2 – Bird’s eye view, Bio



Figure 5: Site 2 - Ground level view, Bio

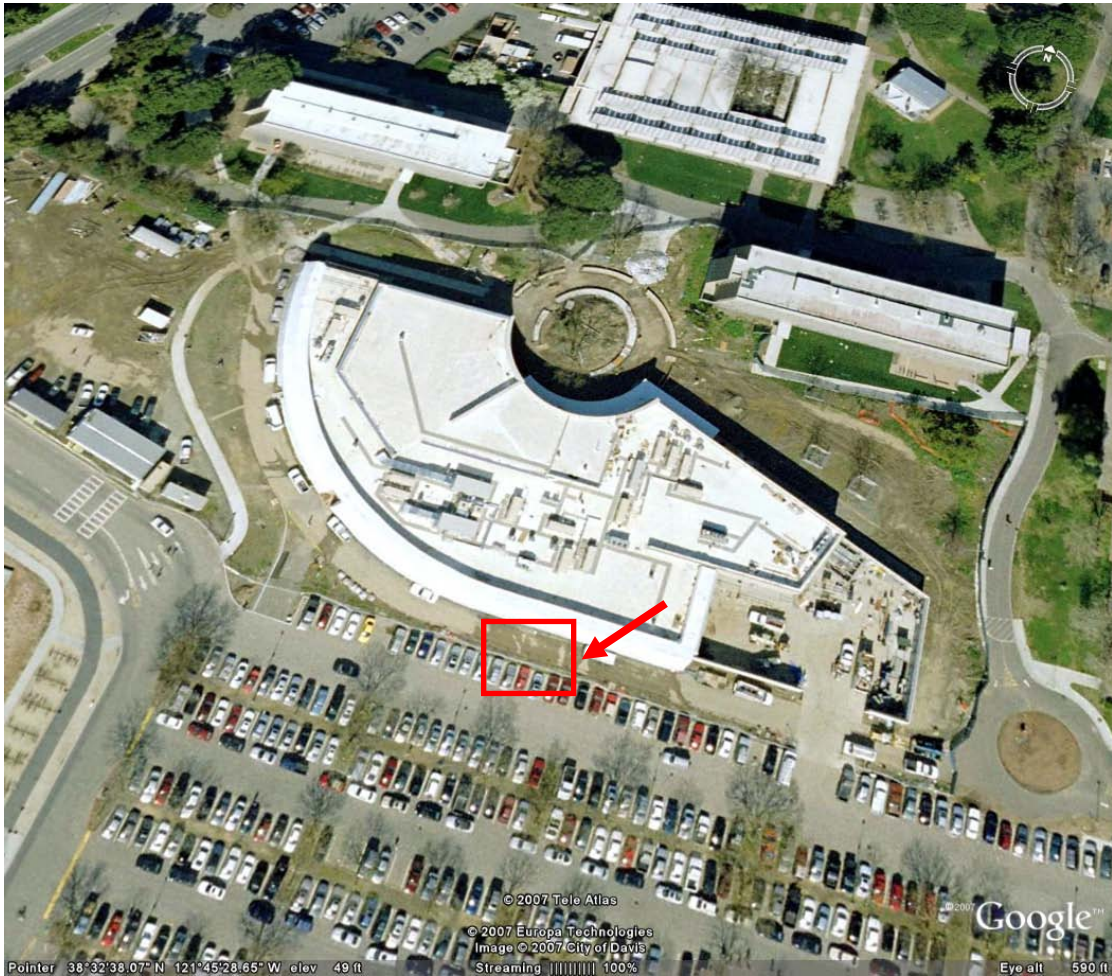


Figure 6: Site 3 – Bird’s eye view, Arc



Figure 7: Site 3 - Ground level view, Arc

The Russell and Arc sites typify the larger style bicycle paths around the campus while the Bio site typifies a narrower style path. Each of the bicycle paths in this study has a yellow center line and a fairly straight sampled section. The Arc path is concrete while the Russell and Bio paths are asphalt. The Russell and Arc sites are largely used for traveling from residential areas to the campus core, resulting in heavy morning traffic flows and relatively little pedestrian traffic. The Bio site is closer to the campus core located near a number of large lecture halls and experiences heavy traffic between courses. The Bio site also has a fair number of pedestrians, though many choose to walk on a dirt track on the side of the path to avoid bicycles (see Figure 8).



Figure 8: Dirt pedestrian track at Bio site

The details of the data collection sites and times are provided in Table 1 below. The peak flow periods were short, lasting only 10 to 15 minutes, and usually occurred just before the hour during morning commutes and during class changes. Traffic during other time periods was generally much lower. Accordingly, we chose to record during peak periods.

Table 1: Data collection site details

Data Collection					
Site	Date	Time	Length	Path Width	Sample Area Length
Russell	12/5/2007	8:45am	16min	9.6 ft (east) / 7.75 ft (west)	30ft
Bio	11/30/2007	11:45am	14min	12 ft	19.5 ft
Bio	12/3/2007	8:40am	24min	12 ft	19.5 ft
Arc	11/30/2007	8:45am	12min	20 ft	20ft

Bicycle traffic data was collected using a hi-definition digital video camera (Sony HDR-SR1) mounted on a tripod along the side of each path providing a perpendicular view of same. At each site we moved the camera as far back from the path as possible to maximize observation area. This area is shown in Table 1 as the Sample Area Length. Reference lines were placed across each path using pink duct tape or ribbon to clearly mark the sample area.

The bicycle traffic videos were then downloaded onto PC workstations and analyzed using Media Player Classic v6.4.9.0 video playback software.⁴ This software allowed us to set discrete time intervals for observation. From the videos, bicycle flow and density and pedestrian flow were recorded in each direction. Flow was recorded by counting the number of bicycles or pedestrians entering the sample area during 30 second intervals and then dividing by the path width. Unlike highway traffic flow which is reported in flow per lane, bicycle paths do not have lanes and so flows are reported per unit of path width. For directional flows, the path width was assumed to be the distance from the center line to the edge and for combined flows the width of the entire path. This assumption was made based on our observations that bicycles generally observed the center line, keeping to its right side. The corresponding density was recorded in one second time steps by counting the number of bicycles or pedestrians within the sample area at a point in time and then dividing by the sample area. Similar to the flow calculation, the area for directional density was assumed to be the distance from the centerline to the edge multiplied by the sample area length. The average density over 30 seconds was then estimated by averaging 30 density observations. Speed data was not collected due to the labor intensity involved with hand clocking (using a stopwatch to time bicycles cross a known distance) and that our small sample

⁴ <http://sourceforge.net/projects/guliverkli>

width areas are not ideally suited for such measurements (a larger distance would provide more accurate average speed measurement).

Results and discussion: UC Davis data collection

The plots in Figure 9 show the 30 second counts and cumulative counts of bikes at two of the sites. The Russell site is not displayed because it was counted differently. At the Russell site the flows moved in platoons due to a signalized intersection upstream of the data collection area. To reduce the burden of the bicycle counting and density estimation steps we only took measurements of the platoons ignoring the flow in-between them. However, the trends displayed in Figure 9 and Figure 10 describe the traffic patterns observed at the Russell site.

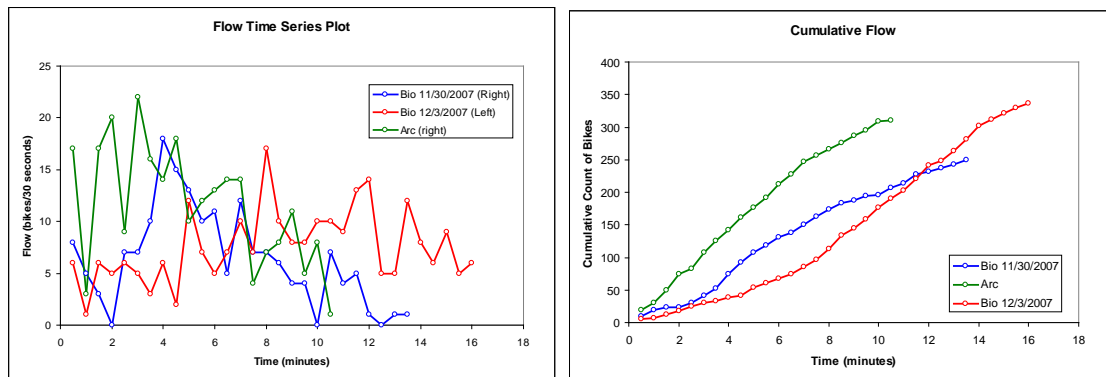


Figure 9: 30 second bike counts and cumulative bike counts

The plots in Figure 9 show that bicycle flows were variable, most notably at the Arc site were the affect of the up stream traffic signal that impacted the Russell site can still be noticed. The plots also indicate that peak flows only lasted several minutes and that at each site we observed between 250 and 300 bikes.

The flow-density relationships at each site are shown in Figure 10. The blue points indicate traffic moving to the right and green points to the left. At each site, for each direction, flow was positively correlated to density, indicating that at no time did we observe densities above capacity. Fitting a line through the combined right and left moving traffic observations and finding the slope provides an estimate of the average speed and is also displayed in Figure 10.

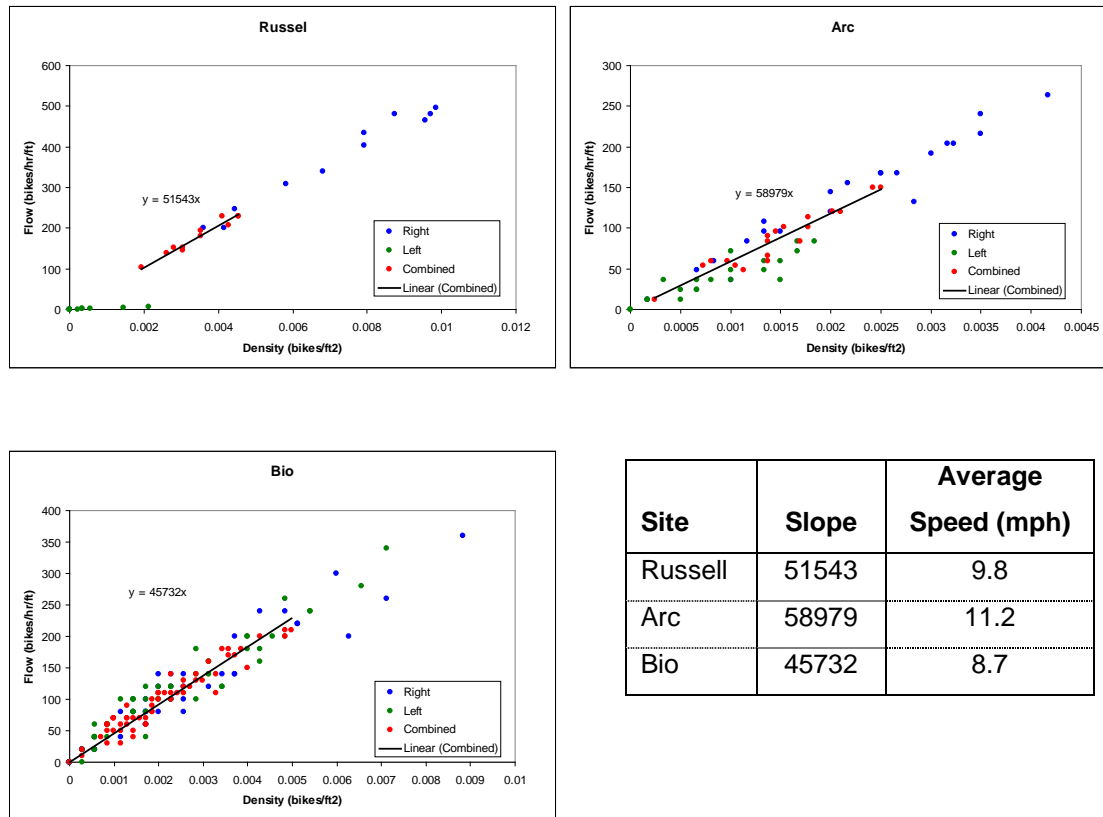


Figure 10: Flow-density plots and average speed estimates

The estimated average speeds are close to those reported in the literature (Miller and Ramey 1975; Homburger 1976; Smith 1976; Allen, Rouphail et al. 1998) though they vary across the three sites and appear to increase with increasing path width.

Methods: simulation modeling

Cellular automata (CA) models are widely used across many disciplines to explore interactions between agents that possess a finite set of changeable characteristics. CA models are discrete since the interactions occur on a grid of finite size and number of locations (cells). Such models have recently seen increasing academic attention particularly in physics, mathematics and computer science.⁵ The application of a CA approach to traffic flow was proposed by Nagel and Schreckenberg (1992) who modeled a single-lane under free flow and congestion conditions.

⁵ Indeed, a query for “cellular automaton or cellular automata” returns slightly over 6000 results with 54% published in 2001 or later.

In a CA model for traffic flow, each cell represents a discrete section of the roadway of a specified distance. In any given time step, that cell may either be occupied, or not. An iteration of the model begins by updating all speeds in the network. If the speed of a vehicle is less than the limit and the distance to the next vehicle is greater than the current speed plus one, it is incremented. If the space between the current vehicle and the next is greater than the current vehicle's speed, it decelerates to the distance between vehicles minus one. Since human beings do not operate deterministically, and we are attempting to model human behavior, a random element is then imposed—if a certain (usually) small probability threshold is exceeded, then the current vehicle reduces its speed by one. This property keeps the system from quickly entering a deterministic state (Nagel and Schreckenberg 1992). Finally, the positions of all vehicles are updated based on their speeds, and the model is iterated again until the desired number of time periods has been modeled.

The utility of CA models for traffic flow stems from the shortcomings of existing models. Traffic flows can be investigated at a variety of levels ranging from the most aggregate (traditional four step travel demand models) to the most disaggregate (microscopic simulation models). However, it is difficult, costly and sometimes impossible or meaningless to extract macro-level traffic characteristics from micro-level models and vice versa (Benjaafar, Dooley et al. 1997). CA methods can explicitly include individual driver behavior, extend to multi-lane cases, and are relatively simple and cheap in terms of computing time and effort required (Benjaafar, Dooley et al. 1997). The results of many CA model runs can be aggregated to study the macro-level traffic characteristics of a given roadway including plots of the fundamental diagram, traffic characteristic-time, and space-time (trajectory) plots.

Several parameters must be specified for CA model implementation. For traffic flow these include length of roadway, cell length (vehicle length since they must be equal as a cell can either be occupied or not), speed limit (in miles/hour or cells/update time), and the time increment under study.

Due to the simplicity of the CA model, research on its implementation under various conditions has been extensive. Of particular relevance to the present study, recent attempts have included

models which are purported to represent bicycle flow (see, e.g., Jiang, Jia et al. 2004; Jia, Li et al. 2007). These models are not CA in the strictest sense—they are multi-value CA models in the sense that each cell may be occupied by a number of vehicles greater than one. In both example papers, the maximum number of bicycles occupying a given cell is four. They also introduce inhomogeneous vehicle fleets to account for the differences in speeds observed between cyclists of different ability, age and equipment. Two maximum speeds were used: two cells/update for fast bicycles, and one cell/update for slow bicycles. Neither model was empirically validated using actual bicycle data which makes their relevance to bicycle planning applications highly questionable. Additionally, we argue that the additional utility gained by switching to a multi-value CA model may outweighed by the additional model complexity introduced. For these reasons, this paper implements a multi-lane inhomogeneous CA model to investigate bicycle flow characteristics on a one mile loop with varying parameter values. We utilized Matlab R2006a (The MathWorks Inc. 2006) for model implementation. The model parameters are discussed below.

The approach taken here is shown to be computationally simple while providing rich behavioral data for comparison to the empirical results described above. It is expected that this tool could be used to determine LOS measures for new bike path construction.⁶

The extension of the single lane CA to the two lane case was originally proposed by Rickert et al. (1996). The algorithm described below borrows from their work. It involves much the same logic as the single lane case with several exceptions. Before the acceleration step, both lanes are examined to evaluate lane changing opportunities. Irrespective of the lane changing rule employed,⁷ the action of changing lanes is taken to be strictly parallel in the sense that every vehicle decides *simultaneously* at the beginning of each time step whether to change lanes. This ensures that no vehicles are favored in the lane changing process, and that nothing changes in between time steps that would affect the resultant outcome. Unfortunately, this adds to the

⁶ Of course, without accurately forecasted bike travel demand, these models will not produce reliable results. The travel demand modeling of bicycles is an interesting field substantially overlapping the study of transportation and land use interactions. The interested reader is referred to Cervero and Kockelman (1997) for a discussion of how the built environment can affect the generation of non-motorized trips and to Porter et al. (1999) for an overview of pedestrian and bicycle forecasting issues.

⁷ To be discussed further below.

computational burden, since many operations could be rolled into one if sequential updates were instead used.

The actual implementation of the multilane case takes the form of several if statements combined with functions which are capable of looking forward and backwards in the opposite lane. The case of three or more lanes would require many additional statements and added complexity (Daoudia and Moussa 2003). For our initial lane changing rule we implemented the following conditions which must all be true in order for a vehicle to change lanes, following Rickert et al. (1996):

- The gap in the current lane to the next vehicle is less than or equal to some desired threshold, x ($\text{nextVeh} \leq x$)
- The gap ahead in the next lane is greater than or equal to some desired threshold, x_0 , which may or may not be equal to x ($\text{nextVehO} \geq x_0$)
- Looking backwards, the closest vehicle is sufficiently far away ($\text{backDist} \geq y$)
- A random number is less than the probability of lane change ($\text{rand} < p\text{Lane}$)

The relevant quantities are illustrated in Figure 11. Rickert et al. (1996) note that the omission of the final condition leads to rapid lane changing behavior if all vehicles are initially located in the same lane and density is relatively high. This is because of parallel updating. Since they all look ahead and see a gap smaller than desirable, they all decide to change lanes. The same thing happens in the next time step and the process repeats ad infinitum. However, reducing the probability of lane changing to 0.5 from 1.0 reduces the number of these “ping-pong” changes by a factor of five (Rickert, Nagel et al. 1996). In any case, the random properties of the CA model are what make it interesting from a behavioral perspective; the extension of this idea to lane changing behavior is no exception. As an additional mitigation measure, all vehicles were initially randomly distributed across the two lanes with speed equal to zero to ensure that no steady state patterns would form upon model initialization.

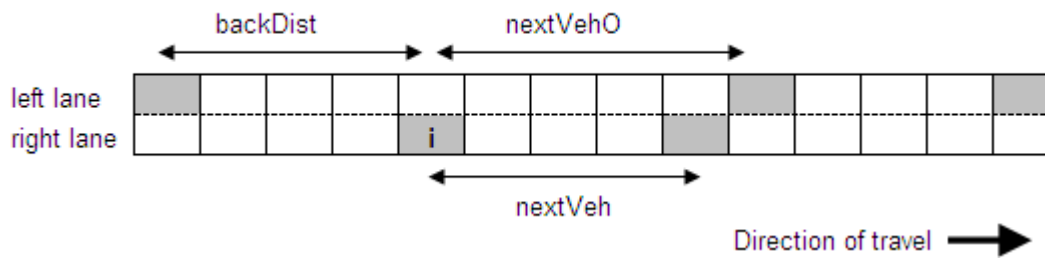


Figure 11: Illustration indicating relevant quantities for lane changing rule, adapted from Knopse et al. (2002)

Note: Shaded cells are occupied by vehicles

i = vehicle for which we are determining lane changing quantities

$backDist$ = distance from the present vehicle to the closest vehicle opposite the direction of travel in the opposite lane

$nextVehO$ = distance from the present vehicle to the closest vehicle in the direction of travel in the opposite lane

$nextVeh$ = distance from the present vehicle to the closest vehicle in the direction of travel in the same lane

Parameters

The value of undertaking a modeling exercise in this case lies in the calibration of the model results to the observed data. To this end, there are several parameters which were held constant across model runs as they reflect more or less physically constant values as observed in the literature. These are listed along with their assigned values in Table 2. Above a certain value, the length of roadway considered should not affect model results, as long as there is sufficient distance for the bicycles to reach a steady state. Preliminary testing revealed that one mile was an appropriate length. Using seven feet as the cell length is justified in combination with the maximum speeds used for slow and fast vehicles. Average cycling speeds reported in the literature, as discussed above all fall around 12 mph which translates approximately to 2.5 cells/s given a 7 foot cell length. Similarly, faster bicycles seldom travel faster than 15 mph, or approximately 3 cells/s. Given these observations, we choose speeds of 2 (9.5 mph) and 3 cells/s for slow and fast cyclists respectively. Our observations also indicated that bicycle speed was relatively constant with little probability of random slowdowns, save for the occasional cellular phone using bicyclist.

Table 2: Constant parameters

Parameter	Value
Length of road	1 mile
Length of cell	7 feet
Max speed 1	3 cells/s
Max speed 2	2 cells/s
Slowdown probability	0.1
Symmetry	No

Other model parameters were varied since they were expected to have a significant effect on the results. These are listed in Table 3.

Table 3: Variable parameters

Parameter	Values
Look back distance	0, 1
Probability of lane change	0, 0.7, 0.9, 1
Proportion of slow vehicles	0.25, 0.5, 0.75

The lane changing rule as currently described gives no preference to occupying either lane. That is to say that once a vehicle is traveling in either lane, they stay there until another vehicle in that lane appears as though it will slow them down. This condition is known as *symmetry* and is not how cyclists behave. Instead, they attempt to overtake slower vehicles and then shift back to a position in front of the vehicle just passed. Thus, even though there are no ‘lanes’ per se, cyclists behave as if there is one lane for passing, and one lane for through traffic, especially on narrow lanes such as that at the Bio site. To implement the asymmetric condition in the model, the first lane changing rule is omitted when determining desirability of lane change from the left lane to the right hand lane. This means that a vehicle in the left hand lane need not be obstructed by a vehicle in front, but will try to get back into the right hand lane at the earliest opportunity.

Following the absence of symmetry from cyclist behavior, the third lane changing rule involving the look-back threshold was altered between low values. Since the consequences of obstructing another cyclist are low compared to the obstruction of an automobile, the threshold must be set much lower than for the vehicle case to emulate actual cycling behavior.

Finally, the proportion of slow vehicles was varied. Since we did not measure speed distributions in the field, we have only obtained average values for platoons of flow lasting 30 seconds. It is not possible to observe relative proportions from the video recordings.

Results and discussion: simulation modeling

Seven scenarios of the model were run varying over the parameters described in Table 3. Each model run is defined in Table 4.

Table 4: Scenario definitions

Scenario	Probability of Lane Change	Proportion of Slow Bikes	Look Back
Run 1	0.9	0.5	No
Run 2	0.9	0.25	No
Run 3	0.9	0.75	No
Run 4	1	0.5	No
Run 5	0.7	0.5	No
Run 6	0	0.5	No
Run 7	0.9	0.5	Yes (1 cell)

For each run the usual plots were created: flow-density (Q-K), speed-density (U-K) and speed-flow (U-Q), along with plots of the amount of lane changing across densities.

Flow and speed were calculated for each lane using the boundary method, which is analogous to a set of two loop detectors on a highway. In our case the boundary was the interface between two neighboring cells. If a vehicle had a velocity of 1 cell/second at cell i and time t , it passed through the boundary between cell i and cell $i+1$ by time $t+1$ second. Similarly, a vehicle with a velocity of 2 cells/second at cell $i-1$ at time t also passed through the boundary between cell i and cell $i+1$ by time $t+1$ second, and a vehicle with no velocity in cell i at time t did not pass through the boundary. The traffic simulation was broken into 30 second observation periods (which corresponded to the observation periods used in our field data collection) during which boundary observations were accumulated and averaged. Average velocities were estimated using a harmonic average to generate a space-mean speed.

Density could be estimated using its relationship with the calculated flow and speed above; however, since we can not observe bikes with a velocity of zero crossing the boundary these bikes would not be reflected in the estimate. This turns out to be an increasingly important consideration at high densities when many bikes have velocities of zero. Alternatively, density was estimated at the cell to the left of the boundary used above. If a bike was present the density was unity, otherwise it was zero. The observed densities over a 30 second period were summed and averaged.

The flow and density for each lane were then added together to obtain the density for the bike path. The average speed of the bike path was computed using a harmonic average of the average speed from each lane. The results were then standardized for comparison to our field data and the literature. Flow was converted from bikes/hr to bike/hr/ft by dividing by eight feet based on the assumption that each “lane” in our CA model was four feet wide. This follows from the literature, where bikes lanes are often assumed to be four feet wide. Density was converted from bikes/mile to bikes/ft² by dividing by the area of the bike path, again assuming that each lane was four feet wide. Speed was reported in miles per hour and is assumed to not be a function of the bike path width.

Figures 12-14 show the flow-density (Q-K) and velocity-density (U-K) plots for runs 1 – 3. The three scenarios show the effect of increasing the proportion of slow bikes in the CA model. There are two noticeable effects. With an increasing number of slow bikes the capacity of the bike path is first reached at a higher density (see the Q-K plots) and the average speed is lower at low densities under free flow conditions (see the U-K plots). These results make intuitive sense. Average free-flow speeds should be lower if there are a greater proportion of slow vehicles and capacity should be reached at higher densities given the slower free-flow speed.

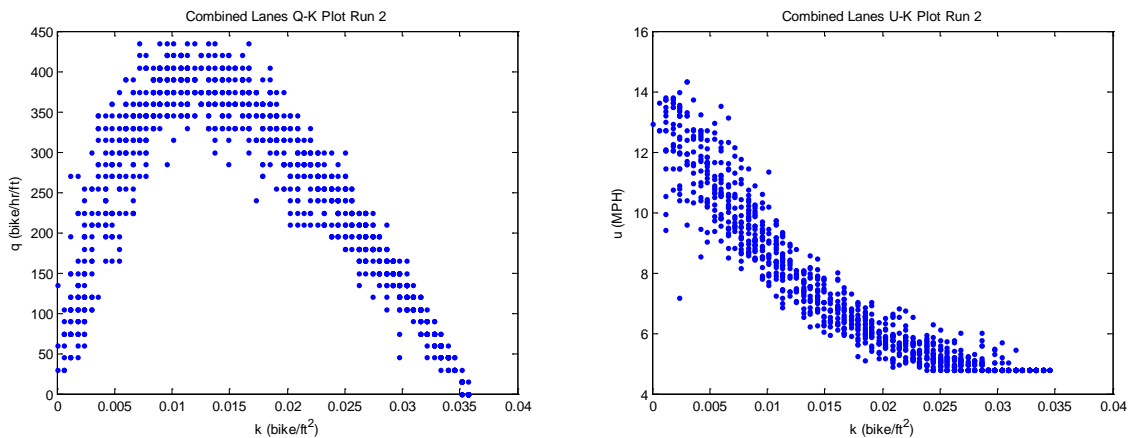


Figure 12: 25 percent slow bikes

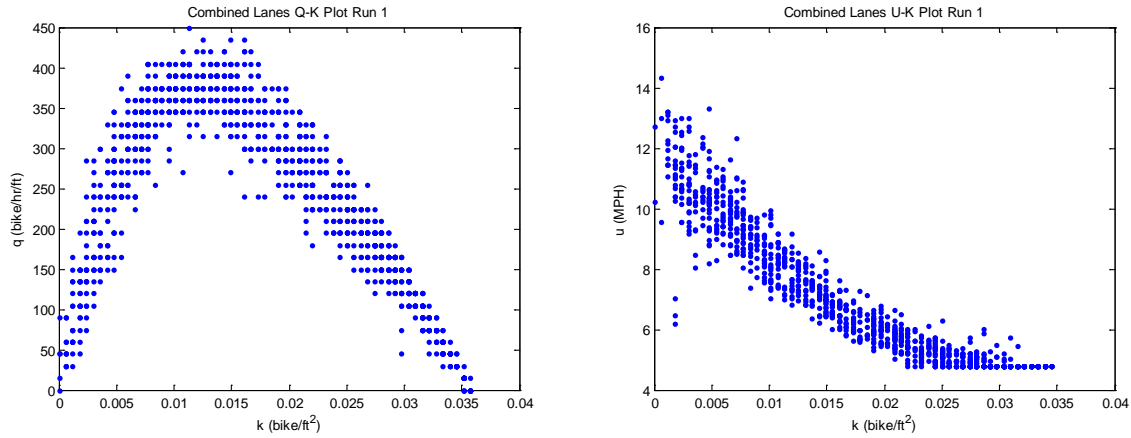


Figure 13: 50 percent slow bikes

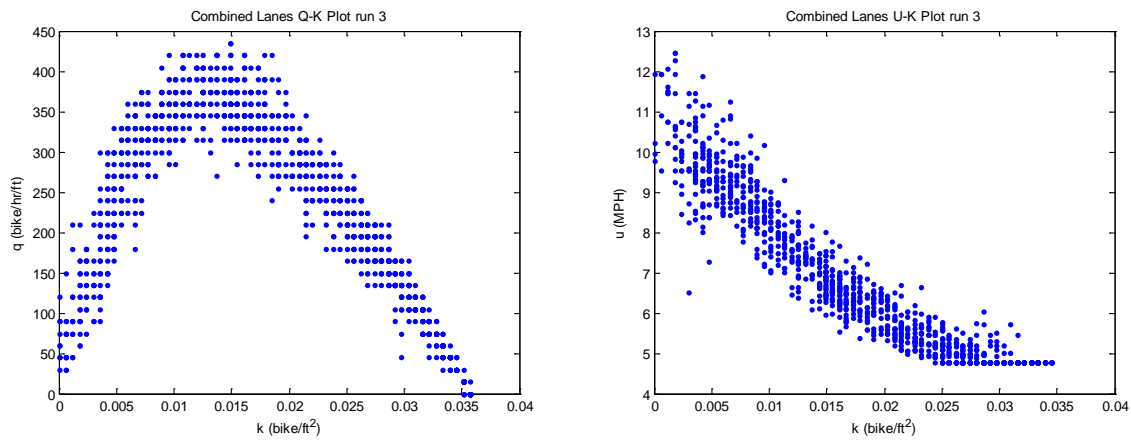


Figure 14: 75 percent slow bikes

Figures 15-17 show the flow-density (Q-K) and velocity-density (U-K) plots for runs 4 – 6. The three scenarios show the effect of changing the probability of a lane change in the CA model. Again, there are two noticeable effects. Allowing lane changing reduces the capacity of the bike path (compare Figure 15 and 17). Without lane changing, capacity is approximately 500 bikes/hr/ft while with 100 percent lane changing, capacity is about 425 bikes/hr/ft. Furthermore, increasing the probability of lane changing reduces capacity. The second effect is that without lane changing, the average free flow speed is reduced. Free flow speeds range from 10 mph to 14 mph with lane changing but only from 8 mph to 11.5 mph without lane changing. These results also make sense. Not allowing lane changing reduces slow downs caused by passing vehicles “cutting off” vehicles behind them (recall that these scenarios do not have “look-back”).

Additionally, by not allowing passing, the speeds of faster vehicles are limited by the slower moving vehicles.

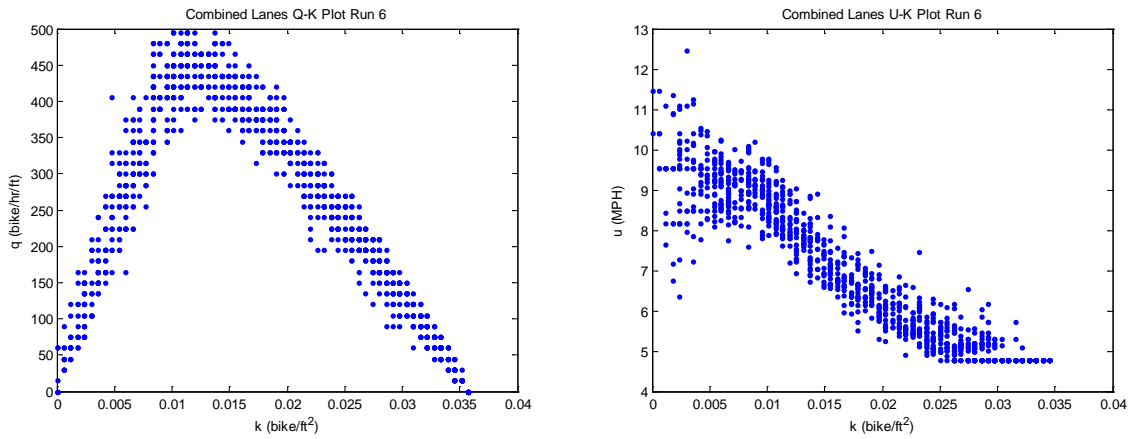


Figure 15: 0 percent probability of lane change

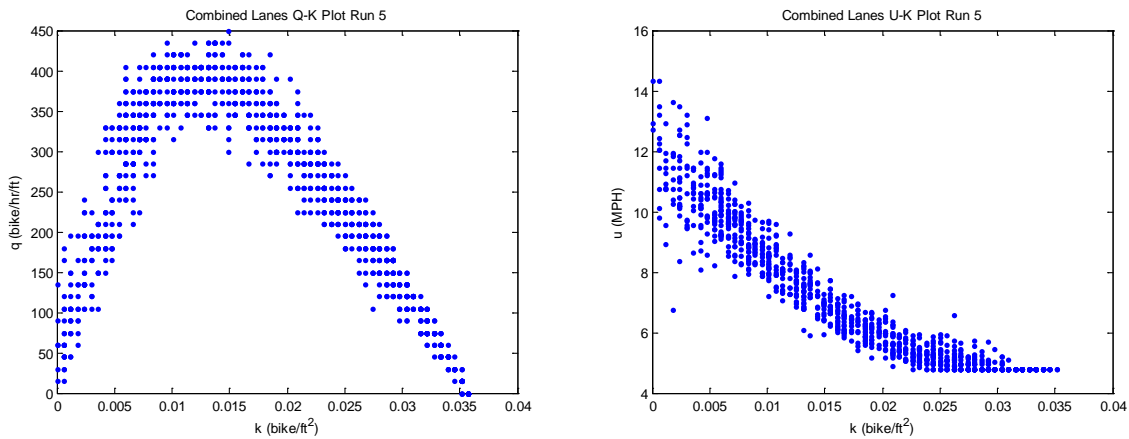


Figure 16: 70 percent probability of lane change

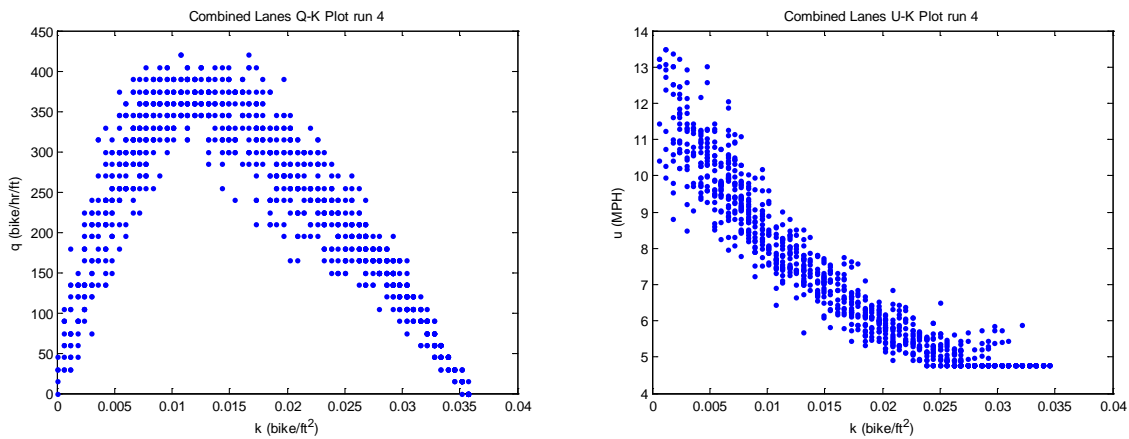


Figure 17: 100 percent probability of lane change

The last scenario, examining the effect of look-back, is displayed in Figure 18. The effect of allowing cyclists to “look-back” one cell (7 feet) before changing lanes allows both a high flow rate (just over 500 bikes/hr/lane) and maintenance of free flow speeds. The higher flow rate and speed is achieved since cyclists reduce the amount of slow downs caused by “cutting off” other cyclists and may maintain their desired free-flow speeds as long as they have a one cell gap in the left lane to move into.

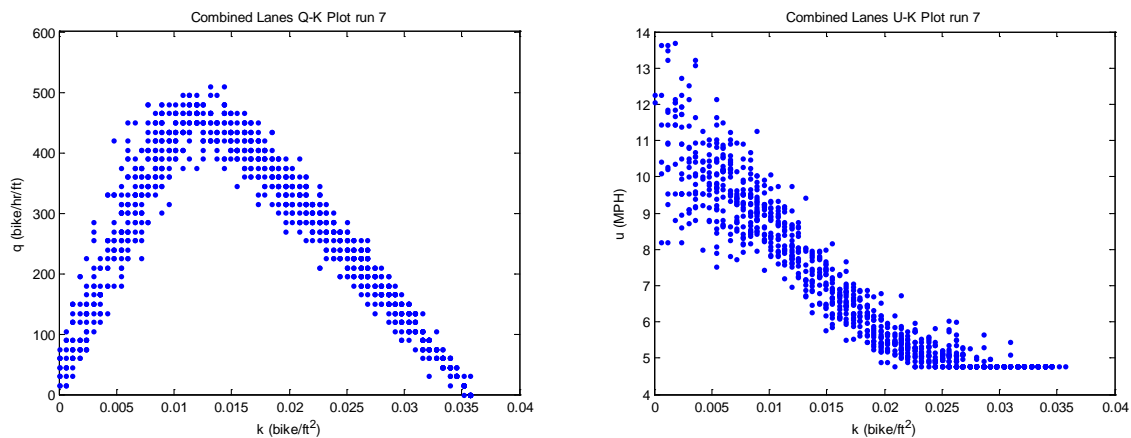


Figure 18: One cell look-back

In addition to looking at the usual traffic measures, we also look at lane changing behavior. Ideally, we would measure the actual number of passing and delayed passings in order to derive LOS as defined in Hummer et al. (2006) or the 2000 Highway Capacity Manual (TRB 2000a). While the data do exist to do this from the model output, additional computational effort would be required to extract the data, which we leave to future work at this time. However, the amount of lane changing indicates how the model is performing and provides an alternative (yet closely related) measure of hindrance. The frequency with which cyclists change lanes is related to their dissatisfaction with the performance of their current lane, so a greater amount of lane changing indicates greater hindrance. Additionally, as densities increase, passing opportunities decrease and delayed lane changes occur, a second form of hindrance. While we do not directly observe delayed lane changes, we do observe the drop in the amount of lane changing as densities increase.

Lane changes are observed from a matrix which records when a lane change is made in the CA network matrix. Lane changes are observed at the cell to the left of the boundary used to observed speed and flow. Similar to the method used to observed density the lane change observations are summed and averaged over a 30 second time window. The reported number of lane changes per hour therefore represents the number of bikes engaged in a lane change at the boundary.

The plots of lane changes over density in Figure 19 show the effect of different proportions of slow moving vehicles on lane changing frequency. The plots are basically the same, indicating that the proportion of slow moving vehicles does not greatly influence the amount of lane changing. The plots in Figure 19 also show that the amount of lane changing increases with increasing density until around 0.025 bikes/ft². At this density, which corresponds to the point at which speeds are severely reduced and many vehicles are stopped (see Figure 12), the amount of lane changing drops off sharply. This is caused by the inability to switch lanes because of occupancy by another vehicle, indicating that traffic conditions are likely to be just as bad in the other lane.

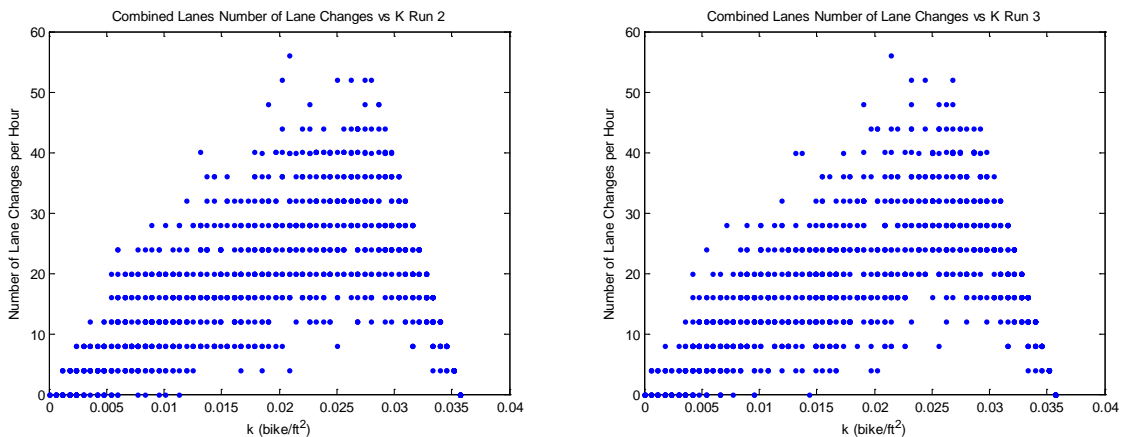


Figure 19: Lane changing with 25 (left) and 75 (right) percent slow moving bikes

Figure 20 shows the impact of varying the probability of lane change parameter. As would be expected, the amount of lane changing increases when this parameter is increased. With a 100 percent probability of lane changing (every time a lane change would be beneficial, a lane change is made) a maximum lane changing rate of almost 80 lane changes per hour is observed.

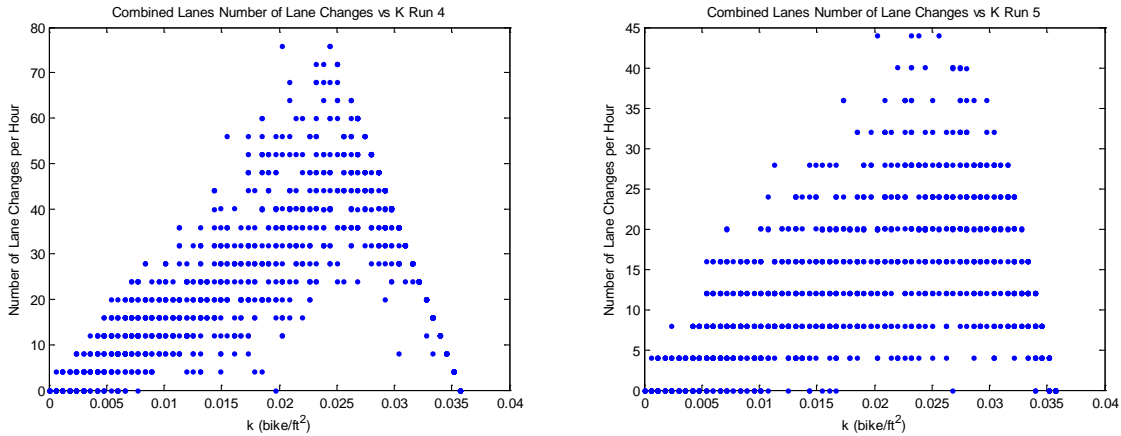


Figure 20: Lane changing with 100 percent (left) and 70 percent (right) probability of lane change

When a one cell look-back is used, capacity and free-flow speeds are increased due to more prudent lane changing. Figure 21 shows that with a one cell look back, far fewer lane changes are made. Additionally, the maximum lane changing rate occurs at a much lower density than in the zero look-back scenarios—at about 0.008 bikes/ft² compared to 0.025 bikes/ft², respectively. This is sensible since at higher densities there will be fewer times when the gap is great enough to change lanes.

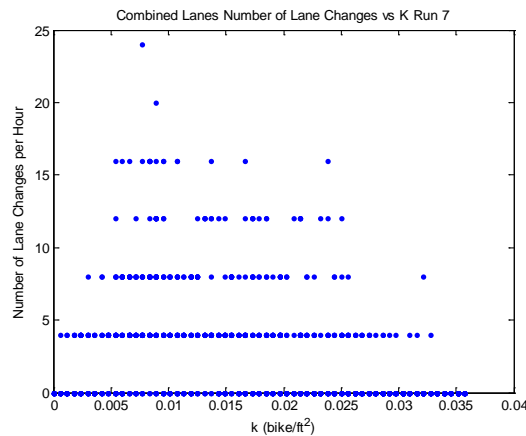


Figure 21: Lane changing with one cell look-back

Comparison to observed data

A CA model was created based on some basic assumptions, known properties, and the observation of cyclists. The model results presented above behave as we would expect and generally appear to be able to simulate some operational aspects of bicycle paths. We compare

two scenarios from the CA model which produced the two most distinct fundamental diagrams, Run 1 and Run 7, with plots of our field data. These are shown in Figure 22.

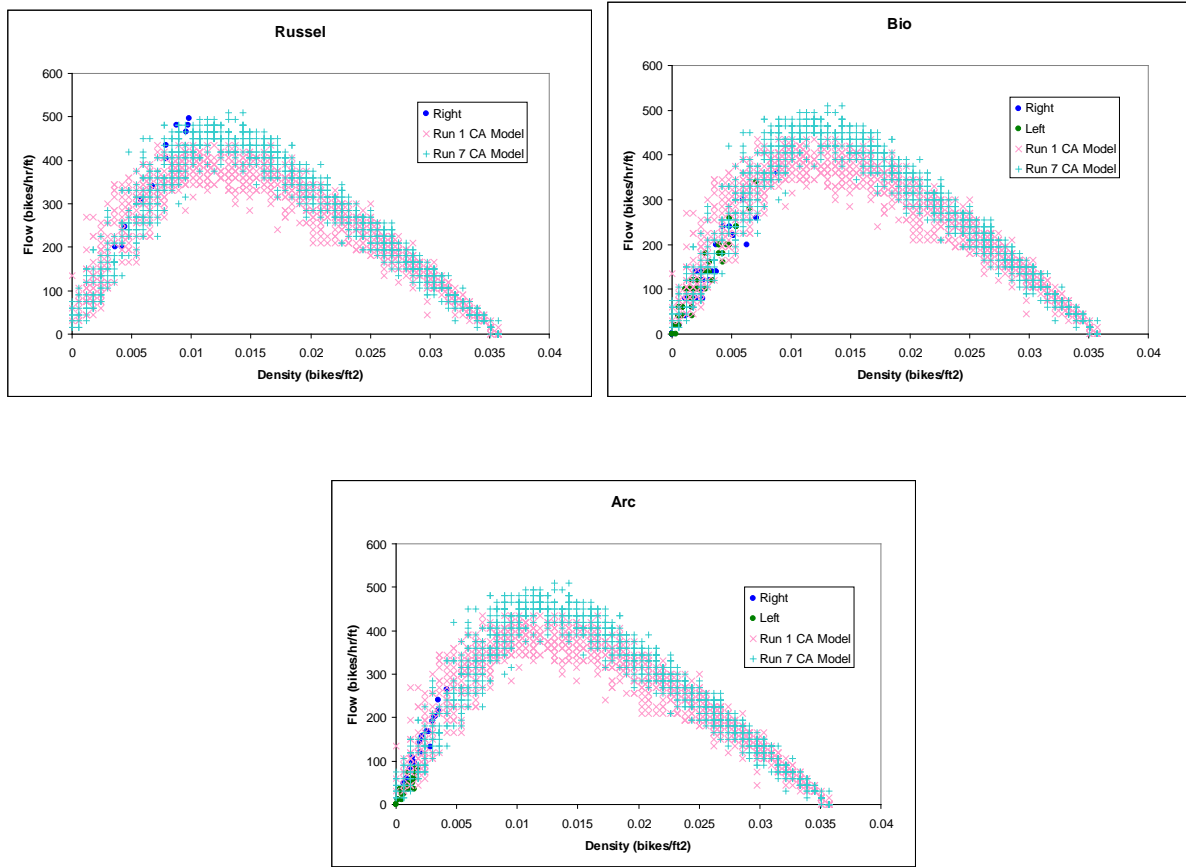


Figure 22: Field data and simulation fundamental diagram comparison

The field data closely matches both simulation results, completely falling within the range of values produced by the simulation. The Russell and Arc site data appear to more closely match the Run 7 scenario while the Bio site data matches more closely to the Run 1 scenario. The modeled capacities (maximum flow) also fall within the lower range of bike path capacities reported in the literature which were previously mentioned. While these results do support our argument that CA models can be used to model cyclist behavior, they do not offer concrete proof. To validate and calibrate our model will require more field data over a larger range of densities, and collection of additional types of data such as speed distribution and lane changing (or passing). Limitations presented by the time frame of this project and available equipment to reduce the labor intensity of the data collection effort precluded us from collecting these data.

Given the lack of validation, the CA model with lane changing still appears suited to the task. By varying the look-back distance we can observe a change in bike path capacity. We observe from the field that many cyclists (at least at UC Davis) do not look back, cutting off other cyclists and thus creating slow downs. The model gives us a way to explore how a cyclist education program could help improve bike path operation by increasing look-back. Additionally, by increasing the proportion of fast cyclists we can observe that capacity is reached at a lower density. This feature is important to model how bicycle paths should be designed for different types of users. These are just two examples, but they demonstrate how the model is sensitive to parameters that are known to effect bicycle path operation.

Future work

The CA model presented in this paper has shown good agreement with observed data. However, it models a critical density which is at the low end of those observed in the literature (see Allen, Roupail et al. 1998 and above discussion). This is likely due to the model update rules. Recall: if the space between the current vehicle and the next is greater than the current vehicle's speed, it decelerates to the distance between vehicles minus one. This means that closely packed vehicles all traveling at equal speeds are not possible given the current model configuration. While this rule applies to automobile flow, it may not adequately capture bicyclist behavior at high densities. Future simulations will include varied update rules allowing for the formation of tightly packed, high speed platoons. This will push up the critical density and thus maximum flow. It will also lend more confidence to the capacity predictions output by the model which are undoubtedly already a vast improvement over fitting a polynomial expression to data with a dubious fit.

The effect of lane width will also have to be further investigated. As the model currently functions, there is no influence of lane width—in calculating densities on an area basis, we have simply assumed that we more closely modeled a narrow path (Bio) since on that path, bicycles traveled in mostly a two-lane configuration and could not extend far beyond that. However, the model outputs could simply be scaled by an arbitrary lane width. A more realistic approach would introduce friction factors that increase as lane width narrows. These would introduce *lateral* interactions in addition to parallel ones within the CA model structure. Under this

framework, a passing cyclist could slow down the cyclist that was passed which could have the effect of causing slowdowns.

An additional extension closely related to the above would be the addition of multiple lanes to allow for the modeling of wider bike paths. As noted above, for the case of automobiles, this has been attempted by Daoudia and Moussa (2003) and could be relatively easily applied to the present case of bicycles. The influence of cross traffic could also be considered in a bidirectional framework such as that proposed for vehicular traffic by Simon and Gutowitz (1998). While they only considered the case of a two-lane highway, we would necessarily have to consider more complex interactions since bicycle flows necessarily in multiple 'lanes' in one direction.

Finally, the technological methods of our empirical data collection could be vastly improved. Manually counting vehicles was tedious. Automated detectors similar to those used for automobile traffic would be an improvement, as would methods of collecting speeds using a radar gun, for example. This would allow validation of the flow and density measurements which could be extracted from detectors.

Conclusions

We have presented the first empirically validated, multilane, inhomogeneous CA model for bicycle flow in the literature. It was found to accurately model the fundamental diagram of bicycle flow while possibly underestimating bicycle lane capacity due to the CA update rules. With additional research, this could become an invaluable planning tool for use by city and regional planners in concert with enhanced bicycle trip forecasting methods. With the national and global issues of climate change, air pollution and urban congestion combined with crumbling national infrastructure, the time for planning for adequate bicycle facilities is now.

References

- AASHTO (1999). Guide for the Development of Bicycle Facilities. Washington D.C.
- Allen, D., N. Rouphail, J. Hummer and J. Milazzo (1998). "Operational Analysis of Uninterrupted Bicycle Facilities." Transportation Research Record **1636**(-1): 29-36.
- Benjaafar, S., K. Dooley and W. Setyawan (1997). Cellular Automata for Traffic Flow Modeling. Minneapolis, MN, University of Minnesota, Department of Mechanical Engineering, CTS97-09.
- Botma, H. (1995). "METHOD TO DETERMINE LEVEL OF SERVICE FOR BICYCLE PATHS AND PEDESTRIAN-BICYCLE PATHS." Transportation Research Record(1502): 38-44.
- Botma, H. and H. Papendrecht (1991). "Traffic operation of bicycle traffic." Transportation Research Record **1320**: 65-72.
- Cervero, R. and K. Kockelman (1997). "Travel demand and the 3Ds: Density, diversity, and design." Transportation Research Part D: Transport and Environment **2**(3): 199-219.
- Daoudia, A. K. and N. Moussa (2003). "Numerical simulations of a three-lane traffic model using cellular automata." Chinese Journal of Physics **41**(6): 671-681.
- Dargay, J., D. Gately and M. Sommer (2007). "Vehicle Ownership and Income Growth, Worldwide: 1960-2030." The Energy Journal **28**(4): 143-170.
- Gartner, N., C. J. Messer and A. K. Rathi, Eds. (2001). Traffic Flow Theory: A State-of-the-Art Report, Oak Ridge National Laboratory, Transportation Research Board and US Department of Transportation.
- Homburger, W. S. (1976). Capacity of Bus Routes, and of Pedestrian and Bicycle Facilities. Institute of Transportation Studies, University of California, Berkeley.
- HRB (1965). Highway Capacity Manual 1965. Washington D.C., National Research Council.
- Hummer, J. E., N. M. Rouphail, J. L. Toole, R. S. Patten, R. J. Schneider, J. S. Green, R. G. Hughes and S. J. Fain (2006). Evaluation of Safety, Design, and Operation of Shared-Use Paths - Final Report. FHWA.
- IPCC (2007). Contribution of working group I to the fourth assessment report of the Intergovernmental Panel on Climate Change: summary for policymakers.
- Jia, B., X. G. Li, R. Jiang and Z. Y. Gao (2007). "Multi-value cellular automata model for mixed bicycle flow." European Physical Journal B **56**(3): 247-252.
- Jiang, R., B. Jia and Q. S. Wu (2004). "Stochastic multi-value cellular automata models for bicycle flow." Journal of Physics A: Mathematical and General **37**(6): 2063-2072.

- Kingham, S., J. Dickinson and S. Copsey (2001). "Travelling to work: will people move out of their cars." Transport Policy **8**(2): 151-160.
- Knospe, W., L. Santen, A. Schadschneider and M. Schreckenberg (2002). "A realistic two-lane traffic model for highway traffic." Journal of Physics A: Mathematical and General **35**(15): 3369-3388.
- Miller, R. E. and M. R. Ramey (1975). Width Requirements for Bikeways: A level of Service Approach. Department of Civil and Environmental Engineering, University of California, Davis.
- Nagel, K. and M. Schreckenberg (1992). "A cellular automaton model for freeway traffic." Journal De Physique I **2**(12): 2221-2229.
- Navin, F. P. (1994). "Bicycle traffic flow characteristics: experimental results and comparisons." ITE Journal **64**(3): 31-36.
- Porter, C., J. Suhrbier and W. Schwartz (1999). "Forecasting Bicycle and Pedestrian Travel: State of the Practice and Research Needs." Transportation Research Record **1674**: 94-101.
- Pucher, J., C. Komanoff and P. Schimek (1999). "Bicycling renaissance in North America?: Recent trends and alternative policies to promote bicycling." Transportation Research Part A: Policy and Practice **33**(7-8): 625-654.
- Replogle, M. (1992). Non-Motorized Vehicles in Asian Cities. Washington, D.C., World Bank, Technical Paper Number 162.
- Rickert, M., K. Nagel, M. Schreckenberg and A. Latour (1996). "Two lane traffic simulations using cellular automata." Physica A: Statistical and Theoretical Physics **231**(4): 534-550.
- Simon, P. M. and H. A. Gutowitz (1998). "Cellular automaton model for bidirectional traffic." Physical Review E **57**(2): 2441-2444.
- Smith, D. T. (1972). City of Davis, University of California, Bicycle Circulation and Safety Study. San Francisco, CA, De Leuw, Cather & Company.
- Smith, D. T. (1976). Safety and Location Criteria for Bicycle Facilities. Washington, D.C., Federal Highway Administration, FHWA-RD-75-112.
- Taylor, D. and W. Davis (1999). "Review of Basic Research in Bicycle Traffic Science, Traffic Operations, and Facility Design." Transportation Research Record **1674**: 102-110.
- The MathWorks Inc. (2006). Matlab: the Language of Technical Computing.
- TRB (1985). Highway Capacity Manual 1985. Washington D.C., National Research Council.

TRB (2000a). Chapter 19 - Bicycles. in Highway Capacity Manual 2000. Washington, D.C., National Research Council.

TRB (2000b). Highway Capacity Manual 2000. Washington D.C., National Research Council.

Virkler, M. and R. Balasubramanian (1998). "Flow Characteristics on Shared Hiking/Biking/Jogging Trails." Transportation Research Record **1502**.