

## **Spatial market representations: concepts and application to integrated planning models**

### **DRAFT**

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### **ABSTRACT**

This paper describes representations of spatial markets, and how such representations can be used as the mechanism for connecting together the various components of an integrated large-scale planning support simulation.

Over the past three decades many spatial planning and transportation planning simulation models have adopted random utility location choice theory, and integrated it into a traditional view of the economic system. These integrated simulations have been commercially successful and practical for policy analysis, but there have been a number of theoretical inconsistencies in the integration of the two theories. Recent work seems to overcome most of these inconsistencies. These simulations solve a system of simultaneous equations for a set of prices, and hence have been limited to considering equilibriums, or "point equilibriums".

Dynamic economic systems theory, with a "price update" procedure based on "excess demand", has been adopted in some more recent work, and is shown to hold substantial potential as a method for a more explicit method of representing time while still representing space. Most importantly, such methods can represent the disequilibrium that occurs in an evolving spatial system like the cities or regions that exist in the world today.

In the quest for more spatial and temporal detail, however, the traditional Walrasian theory of market behaviour (and the corresponding view of the overall economic system) that is the basis for these approaches tends to breakdown. A new approach is necessary which does not need to consider average ("market") prices and reasonably large aggregations of individuals, of time, and of space. This paper shows how a microsimulation of specific offers to purchase or consume baskets of goods and services in space can be broadly consistent with the Walrasian view, but more suitable for integrating detailed representations of the diversity of behaviour over individuals, time and space.

## INTRODUCTION

Economic systems are inherently spatial, in that location and transport costs are important. This paper discusses the representation of location and transport costs in economic simulation models. The paper focuses on the representation of spatial markets, that is the exchange of goods and services at different locations and between different actors located at different places.

The topic is approached from the perspective of government agency forecasting models for urban and regional planning. These models are designed to represent the economic system of a city or region with sufficient accuracy to allow the analysis of possible future scenarios regarding land use regulations and zoning controls, location specific and industry specific taxes and subsidies, investment in neighbourhood amenities and physical infrastructure, and the financing of (and charging for) the use of transportation infrastructure. These types of models are typically used by city or regional planning agencies to aid with transportation planning and land development regulations. The models are used to forecast the urban spatial economy, and related land use and travel, for 15 to 50 years into the future.

The paper first discusses the standard "Walrasian" view of the economy, describing some of the important assumptions and limitations from the perspective of spatial planning models. The additional assumptions associated with input-output modelling are then described. The use of random utility theory to add an element of space to these models is discussed, focusing on two models, the Martin Centre model (MEPLAN-style) which focuses on adding spatial choice, and the PECAS model which incorporates random utility theory more fully and more clearly separates long run and short run concerns.

A price-update/excess-demand framework is described that allows a more detailed consideration of dynamics.

The detailed consideration of time and space leads to a breakdown of all these approaches, however. A full transaction simulation is described, which is shown to be broadly consistent with the prior representations, but that supports infinite disaggregation of time and space. The transaction simulation is seen to be a particularly useful framework for further research and development.

Finally, conclusions are offered.

## BACKGROUND

### Walrasian view

The standard microeconomic "grand view" of an economy has the following distinguishing factors (Katzner, 1988)

1. Categories of goods and services

All goods in the same category are treated as homogeneous, and are measured in the same units.

2. The world is a collection of homogenous categories of households and establishments  
Firms are divided into industries based on primary output. Households are also divided into categories, usually based on income. A single production function is used for each industry in its entirety, and a single expenditure function is used for each category of households. Both firms and households are assumed to be optimizers, and will make tradeoffs in their consumption decisions.
3. A market and a single price for each category of good and service  
Goods are bought and sold at a price. At any time there is one price (and one market) for each category of good. Within a market there are a large number of small players.
4. Equilibrium assumption (short run and long run separately)  
For most goods there is an equilibrium between supply and demand, the exceptions being "capital" goods such as money capital, machinery, and land improvements. There is the option of using a long run equilibrium, where prices are a function of input costs (profit is zero) and for households, income equals expenditures, or a short run equilibrium, where short run supply curves and expenditure functions are specified as an additional inputs. (Rutherford and Paltsev, 1999)
5. Usually no need for modelling processes.  
A *tâtonnement* dynamic process is usually assumed, in which the equilibrium price is established through some process that doesn't involve trade, and then all trade occurs at that equilibrium price. The assumption is that any *nontâtonnement* dynamics that do occur do not significantly contribute to model error, and hence the process itself does not need to be considered.
6. Modelling of processes are driven by price-updates.  
If the *tâtonnement* processes assumption is considered inadequate, or if there is some other reason to consider shorter term dynamics, then the dynamics are based on the following assumptions: firms and consumers make decisions at instants, decisions interact simultaneously through markets, markets are guided by rules of price adjustment, and the price adjustment rules beget dynamic behaviour (Katzner, 1988)
7. There is no cost to entering a market.
8. The same information is available to all participants.

This standard microeconomic view is the basis for computational general equilibrium (CGE) models, which are a system of equations describing the economy based on the flows of goods between firms and between firms and households and the production and expenditure functions of firms and households. A CGE model can be solved to determine the equilibrium state that is consistent with a set of input assumptions.

### **Input Output**

An input-output model is consistent with the predominant Walrasian view of economics, but emphasizes the quantity of transactions. It generally adds the following restrictive assumptions:

1. constant (Leontief-style) production functions,

2. a fixed ratio of production for exports to production for internal consumption, and
3. assumes long-run equilibrium.

An input-output model traditionally collapses out the role of goods, using a technical coefficient to directly relate each industry or household category to the other industries or household categories it requires, giving a "square" input-output table. For instance, a unit of manufacturing production is said to require a certain number of units of service production, and a certain number of units of households, without decomposing these requirements based on the types of services required, or the amounts of different types of land, labour and capital required from the households. The fixed production functions remove the assumptions of optimizing behaviour.

Many modern implementations of input-output models do not collapse the goods, but instead use rectangular make and use tables of coefficients to represent the fixed production and consumption functions.

### Random Utility Location Choice

The traditional microeconomic paradigm is aspatial – the only spatial component is the separation of the model region from the external economy. In addition, the traditional paradigm assumes homogeneity in categories. Random utility theory can be used to relax both of these assumptions.

Random Utility Theory assumes heterogeneous actors and/or heterogeneous goods, and so is fundamentally different than the dominant economic paradigm described above.

#### *Random utility theory overview*

In random utility theory, each actor is assumed to maximize its utility. Its utility is assumed to consist of two components – a deterministic component that can be calculated based on model equations, and a stochastic component reflecting the uniqueness of individuals and situations that varies according to a distribution.

The dominant application of random utility theory in spatial modelling is through *discrete choice* models, representing the choice of one alternative over others in a set of alternatives. In the *logit model* the stochastic component of the utility is assumed to vary with an independent Weibull distribution, which leads to the following closed-form equation for the probability of choosing one option  $i$  out of a set of options  $J$ :

$$P_i = \frac{e^{\lambda U_i}}{\sum_{j \in J} e^{\lambda U_j}} \quad (1)$$

where  $U_i$  is the deterministic component of the utility of alternative  $i$ , and  $\lambda$  is inversely related to the variance of the stochastic component.

A larger stochastic term in the utility function represents more variation in the utility of the individual options. Random utility theory assumes that actors choose the best of the available options, and so a larger variation leads to an expectation of an actor finding a better option. This is reflected in the expected utility of choosing one option from a set of alternatives:

$$U = \frac{1}{\lambda} \ln \left( \sum_{j \in J} e^{\lambda U_j} \right) \quad (2)$$

Note the composite utility is inversely proportional to  $\lambda$  and so is proportional to the variance in the stochastic term in the utility of the options.

### *Spatial choice*

In spatial economic representations, random utility is used to represent spatial choice. This can represent, for example, an establishment's choice of where to purchase inputs, a household's choice of home location, or an individual's choice of where to go to school. The model region is divided into polygons called "zones", and each zone is an element of the set of alternatives in the logit model.

### *Advantages of Random Utility Theory and the Logit Model*

The adoption of random utility theory provides substantial advantages to planning models, including:

1. **Continuous response to policy** – an infinitesimal change in the utility of any of the  $U_j$  leads to an infinitesimal change in the probabilities  $P_i$
2. **Easily estimated** – observed choices can be compared to the calculated probabilities, and utility function coefficients adjusted to maximize the probability that the model would reproduced observed choices.
3. **Estimates from incomplete data** – the formula for a choice from among a set of alternatives does not depend on whether the set of alternatives is the full set of available alternatives. This allows model parameters to be estimated based on observations of an observed choice and a non-exhaustive set of available but unchosen options.
4. **Makes explicit the perceived benefit of attributes** – the estimation of a logit model involves establishing coefficients in the utility function based on observed behaviour, and so establishes the partial utility associated with a particular attribute used to describe an option. This leads to a decomposed view of preferences, which can aid welfare analysis.
5. **Eases understanding** – the choice of one option from amongst a set of options is an easily understood behavioural concept.

## **RANDOM UTILITY TRADITIONAL I/O**

### **MEPLAN style spatial IO**

For over 20 years now, a form of spatial modelling called "spatial input-output" modelling, or the "Martin Centre Model" (Hunt and Simmonds, 1993), has been operationalized in land-use/transport models for urban planners and transportation planners. These models use the input-output framework, but introduce an element of spatial choice using logit models. These have been encapsulated into the commercial software packages TRANUS and MEPLAN.

In these models, the logit model equation 1 and the law of large numbers is used to apportion inputs amongst zones. For industry, the factors and intermediate demand required by the industry located in one zone is apportioned to be produced amongst all zones, assuming that the portions

are equal to the calculated probabilities. For households, the final demand for goods and services by households in one zone is allocated to be produced in other zones based, again, on the probabilities in equation 1.

These models use the traditional (square) input-output model, where the role of categories of goods is not explicitly represented. The ownership of land and capital by households is not generally represented – households are assumed to supply labour only, and land and capital are represented separately, as fixed inputs (vertical supply curves).

The price of each sectors' output is allowed to vary by zone, and that price is a component of the utility function for that zone. Costs are assumed to be the weighted average of the prices plus transport costs for each of the zones that are used for inputs. A "long range" equilibrium is assumed – with prices equal to the sum of these weighted average costs.

The framework is usually made "quasi-dynamic" by computing a point equilibrium for a series of time steps, and

1. assuming floorspace quantities are fixed in each zone at the end of each time step, and adjusting the floorspace quantities between time steps based on the market-clearing floorspace rents that are established in the equilibrium,
2. using fixed transport costs and transport disutilities in computing the equilibrium, then calculating a matrix of zone-to-zone trips based on the spatial flows from the economic model, and assigning those trips to routes and travel modes in a travel model by assuming that route and mode choices are in equilibrium with congested travel times. These newly calculated congested travel times are used in the next time step, and
3. sometimes adding an "inertia" term to the utility of producing in any one zone, with that inertia term proportional to the log of the amount of activity in that zone in the previous time step.

The Martin Centre model was an important development in bringing economic modelling to transportation planning and urban planning, and for extending input-output modelling theory to space. But it does have some theoretical inconsistencies (Abraham, 1988):

1. It uses the logit model formula for allocating space to zones, but does not follow through with adopting random utility theory. In particular:
  - a) it uses the law of large numbers (assuming portions are equal to probabilities) even when categories are small, and
  - b) it uses weighted average costs, instead of the expected maximum utilities derived from integrating over the range of possibilities, as the basis for calculating long-run equilibrium prices.
2. It adopts the equations associated with a long-run equilibrium, yet uses a short-run calculation of transport costs and of floorspace supply.

### **PECAS spatial IO**

The PECAS model is a more complete and consistent integration of Walrasian economic principles with random utility location choice. PECAS is described more fully in a companion paper (Hunt and Abraham, 2002).

PECAS uses the rectangular make and use matrices to represent production and expenditure functions more fully. Thus there is an explicit representation of the transaction of goods and services from supplier to demander. PECAS uses a logit model to allocate the interactions in space. The allocations are done to 'exchanges', which are submarkets for a particular good within the regional market. The price at the exchange is a term in the utility function, with a positive coefficient for sellers and a negative coefficient for buyers. For some commodities either buyers or sellers are restricted to a single market (for example, employers can be restricted to buying labour at their workplace location if one is willing to assume that no-one works at their home for an employer.).

The prices of goods at exchanges are adjusted to clear the market at each exchange – leading to a price landscape for each commodity.

PECAS uses equation 2 to measure the attractiveness of selling (or buying) a good to (or from) the set of available exchanges. These attractiveness functions for all relevant goods can influence the production functions or expenditure functions, causing firms or households to consume or produce more of those goods that are easier to come by or sell, respectively.

The set of attractiveness functions are combined in a way consistent with the theory of the production or expenditure function, to measure the overall attractiveness of a location as a business or residence location.

The overall quantity of activity in a given industry or household category is allocated to zones using equation 1 based on the attractiveness of each location.

If the production functions and expenditure functions are given a logit form (Anderson, de Palma and Thisse, 1992; Brown and Walker, 1989), then PECAS can be considered a nested logit model (a nested logit model has separate independent stochastic error terms in the utility, corresponding to different facets of a choice set, see Ben Akiva and Lerman, 1985) with three levels:

1. The choice of where to locate,
2. The choice of the quantities of production or consumption, conditional upon the chosen location of the establishment or household, and
3. The choice of where to buy or sell (or otherwise acquire or divest) various inputs and outputs, conditional upon both the location of the establishment or household and on the quantities produced or consumed.

The full use of equation 2 to ensure consistency between these three levels in PECAS means that PECAS adopts random utility theory more completely than the Martin Centre models. PECAS is based on the assumption that aggregate flows and stocks are made up of individual actors and individual transactions, each unique. This is critically important in this type of modelling because the primary advantage of urbanization is the wide range of goods and services available to those living and doing business in the city (Jacobs, 1969). The logit model equation 2 shows that the composite utility increases when there is a wider range of options available.

In PECAS, the amount of each industry or household category is specified, and that quantity of activity is allocated to zones, and production functions, and flows between locations, using the nested logit formulation. The prices in each exchange zone are adjusted to clear the markets for commodities consistently with the nested logit allocation process. Floorspace (and, possibly,

other capital items) are given vertical supply curves in each zone. Thus the PECAS equilibrium is a short run equilibrium, since there is no assumption that prices must equal costs, that profit must equal zero, nor that income must equal expenditures.

PECAS is a quasi-dynamic model, and uses four strategies to move through time. The first three strategies are the same ones described above for the Martin Centre models (floorspace development as a separate dynamic model; stepping through time jointly with a transportation route and mode equilibrium model; and using inertia terms.) PECAS also requires a dynamic representation of the change in industry size. The log sum (equation 2) from the top level location choice model is a measure of the overall welfare or attractiveness of the region to an industry or household category. This top level index is a measure of region wide consumer or producer welfare (see, for example, Ben-Akiva and Lerman, 1986) and can be used to guide a long term dynamic model of migration and industry growth and decline.

PECAS is an aggregate model, and so uses the law of large number to allocate quantities using equation 1.

## **DYNAMIC ECONOMIC SYSTEM**

### **Dynamic approach with timeless behaviour**

The PECAS model is a consistent integration of random utility theory for transaction/production/location choice with a general equilibrium model of an economy. The system of equations in PECAS is solved by adjusting the prices at exchange locations until each local submarket clears.

A dynamic model similar to PECAS can be implemented by:

1. solving the PECAS allocation equations given a set of initial prices at the exchanges,
2. calculating the *excess demand* (shortage or surplus) at each exchange, and
3. adjusting the price at each exchange over time based on the excess demand at that exchange.

This is consistent with the dynamic approach to Walrasian market theory in which an equilibrium system can be converted to a dynamic system by adding only a formal specification of how prices vary (see Katzner, 1988, p 258). Thus, the "complete dynamic system, then, contains two parts – the economic, market excess demand equations; and the dynamic or Walrasian adjustment rule" (p. 264). The Walrasian approach assumes that behaviour is timeless or instantaneous, but reacts to prices that change over time. PECAS could be operationalised as a Walrasian dynamic model by replacing the price search algorithm with a price update algorithm, and appropriately considering the stability of the resulting system.

To operationalize such a model as a computer simulation requires that time is discretized. A set of initial prices are assumed, the economic equations of PECAS are calculated to allocate quantities and flows, the excess demand is calculated, prices are adjusted based on a time-discretized version of the price update rule, and the time counter is updated to the next time step.



**Dynamic approach with time behaviour and agent based modelling.**

The Walrasian notion that behaviour is timeless, while price updates take time, is clearly abstract. Individuals take time to react, and the prices themselves are set by people who are making a decision, in time, to transact with one another in markets. An alternative approach is to model the processes by which the suppliers and demanders adjust their decisions in response to changing conditions.

As facets of actual time series behaviour are introduced, the treatment of categories of industry or households in aggregate becomes increasingly difficult conceptually. An agent based approach is appealing, where individual decision making units are simulated making decisions through time. This can be considered to be a Monte-Carlo approach: rather than calculating the expected values of complex joint distributions, some (or all) of the distributions reflecting behaviour are sampled, with each sample representing the plausible behaviour of an actual agent in a population of synthetic agents generated to match aggregate totals.

Consider also that PECAS can be viewed as a matrix algorithm, with one element for each unique combination of:

- industry/household category,
- good category,
- location zone for the establishment or household,
- exchange zone, and
- transaction direction (i.e. "buying" or "selling")

The PECAS implementation in the State of Oregon in the USA has 59 industry and household categories, 79 goods categories, 750 zones for locating, 750 exchange zones, and 2 directions. Thus it is essentially a nested logit model with 88 million transaction possibilities for each of the 59 industry or household categories. Given that the population of the state of Oregon is 3.471 million, it is apparent that implementing PECAS at this scale can lead to more elements in a matrix than there are transactions in a typical day in the model region. The use of the law of large numbers becomes questionable. An agent based approach, with dynamic behaviour, can overcome some of these limitations.

This approach is demonstrated in a companion paper (Khan, Abraham and Hunt, 2002) where the location and trading behaviour of individual Business Establishments (BE's) is modelled. An aggregate approach (with shares equal to probabilities) is still used to choose the production function for each BE, and to allocate trading activity amongst exchange zones. Time is divided into time steps of fractions of a year, and all BE's are considered once per year (a fraction of them in each time step). Each BE's relocation choice (and location, if relocating) are sampled from logit model probability functions. The lower two levels of the PECAS 3-level nested model are then applied to each BE to calculate production functions and exchange locations and quantities for each good. The excess demand at each exchanged is totaled at the end of each time step, and the prices adjusted.

## TRANSACTION SIMULATION

Any theory that relies on a "price update" function still requires an average price for a good, and relatively smooth responses for the excess demand function. As models move to more spatial accuracy (more zones, and more temporal accuracy (more time steps) inevitably smaller and smaller submarkets will be represented. The "market" will no longer be representable by a landscape of average prices, because any such averages will be over such small areas of space and time that most will contain only 1 or 0 transactions. It is mathematically possible to apply the "law of large numbers" to give fractionally stable numbers of transactions, with an average price for that fraction -- but that is a flagrant violation of the etymology of the law. Instead one must think of the market as a large number of individual transactions, each unique in time and space as well as in any stochastic error term associated with unobserved heterogeneity in random utility theory.

A method of representing markets as individual transactions has been developed. It was first described in Abraham and Hunt, 2001.

### Basic theory

There are categories of goods and services, as in the Walrasian view, but all goods are considered heterogeneous.

The world is divided into a heterogeneous population of households and business establishments (called Economic Units.) Such a population can be generated using an iterative proportional fitting procedure from aggregate and sample data, such as the one described for generating synthetic households in Beckman *et al*, 1996. A representative production or expenditure function might be available for each industry or each household category, but they are taken to be measures of central tendency for the production functions of each Economic Unit within the category. Establishments and households are assumed to be optimizers, and will make tradeoffs in their consumption decisions.

Production and expenditure functions are in the form of lists of needs and wants (applying to both sellers and buyers – sellers "want" and "need" to sell things.) This can be consistent with Maslow's hierarchy of needs – allowing a more psychological basis for expenditure functions. Needs and wants are described as a quantity (a "parcel") of a certain category of good, along with a number of other integer or real number attributes describing the specifics of the parcel. These attributes can correspond to measured heterogeneity in observed data regarding a category of good, or they can correspond to unmeasured heterogeneity. A full set of parcels can be described, or a sample of the full set can be used (for instance, if 1/365 of the actual transactions are included, the model only simulates a "typical day's" worth of transactions in each year of simulation time.)

Needs and wants are assumed to be considered repeatedly over time. Each time a need or want is considered the Economic Unit goes through a process attempting to meet that want or need through either accepting an offer or making an offer. The process of arranging a transaction between a buyer and a seller is assumed to have the following steps:

1. One party publishes an open offer to transact a specific parcel of a category of good, in a particular location, at a particular price.
2. Another party considers the list of open offers while considering one of its wants or needs, and makes a decision to accept an offer (rather than publish an offer of its own) and to accept the one particular outstanding offer out of the list.
3. The transaction actually occurs, with the necessary travel or shipping.

A certain number of additional constraints are imposed on the system, at least initially, to make the system as close as possible to the aggregate market representations discussed above:

4. Economic units adjust (over time) the prices and quantities associated with each of their wants and needs in response to their ability to meet those needs. This represents variable production functions and expenditure functions.
5. The consideration of offers, and the additional alternative of publishing an offer, is done using a logit model. The deterministic component of the utility of a considered offer is a linear-in-terms function:

$$V_{co} = \sum_i f_i(X_i^{CO}, X_i^{EP}) - TC \quad (3)$$

where  $X_i^{CO}$  is the value for attribute  $i$  for the Considered Offer,  
 $X_i^{EP}$  is the value for attribute  $i$  for the want or need, and  
 $f_i()$  is a functional form for evaluating the attribute  $i$  of the Considered Offer and comparing it, if necessary, to the attribute of the want or need.  
 $TC$  is a measure of the disutility of travelling between the location of the offerer and the location of the acceptor.

The stochastic component of the utility is sampled from a Weibull distribution.

6. Not all outstanding offers are considered in the logit model, but every offer of the right good and direction has a non-zero probability of being in the list of considered offers.
7. Over time, the number of economic units adjusts in response to their ability to meet needs – representing outmigration, immigration, births and deaths/bankruptcy.

Demand for transport is considered differently than the demand for other goods and services. Road space and transport is a "derived" demand, in that individuals consume it in order to achieve their other needs, not because of any essential need to travel. Thus the demand for transport is not listed as a want or need in the list of wants and needs associated with an establishment or household. Instead, the establishment or household considers the transport cost as part of the deterministic utility of an offer under consideration in equation 3.

### **Advantages and disadvantages of transaction simulation**

Random utility theory can be implemented directly in a transaction simulation representation of markets: the deterministic utility can be calculated, and the stochastic utility sampled from a distribution. There is no need for equation 1; the maximum utility alternative is directly available. Equation 2 is also unnecessary (although it can sometimes be useful to save computation time if there are a large number of actors with identical deterministic utility functions facing the same set of alternatives.) Since these closed form equations are not necessary, other distributions for the stochastic error term can be explored. The Independent

Weibull assumption and the resulting use of the logit model is less necessary than in aggregate models.

The assumptions in the transaction simulation framework are much less restrictive than the assumptions in the Walrasian economic view. The framework is much more realistic. Yet that realism does not come without a cost: the conditions under which such a system will possess an equilibrium state have not been shown, nor whether the equilibrium state is stable locally or globally. The reaction responses of the various actors in the system could lead to a chaotic or divergent system. (Although real systems could be divergent or chaotic, it is an immense simplification of modelling to work with a mathematical representation that is convergent.)

The assumptions are still different than reality, however. For instance, for many goods, real offers are vague, and a period of bi-lateral negotiation occurs between a potential buyer and seller. In oligopolistic markets players may be involved in some game-theory approach to interacting and setting offer prices. The framework implies that participants' only knowledge of the market is the list of current and previous offers and transactions, and that an offer is a public offer that can be accepted by anyone. Any proof of stability is likely to assume that there are a large number of small players.

It is important to note that there is no need for any averaging in the simulation. Individual actors could use some averaging in their decision making, if there is some behavioural reason to believe that individuals consult or calculate average conditions before making decisions – but such averaging is a behavioural enhancement, not an essential part of the framework. Thus, model inputs can be provided using any geography and time scale, and model outputs can be summarized using any geography and time scale. Zones and time steps become conveniences for arranging inputs and outputs – and are no longer integral to the modelling itself.

### **Current status of transaction simulation**

A software implementation of the transaction simulation framework has recently been constructed. It has been designed as an extensible framework for agent simulation with agent interactions consistent with market theory.

A test case has been set up with one industry, two household categories, and 3 goods: labour, final goods, and intermediate goods. The system will be run to examine the stability of the system in practice, and to experiment with different formulations for the elements of the model. The sensitivity of the model to the values of different parameters will be examined. The transport cost parameter in the utility function for a considered parcel will be varied to control average transaction distance, to investigate the type of calibration process explored in Abraham and Hunt, 1998.

### **Future work in transaction simulation**

The software implementation has been designed to be flexible. In particular, it has been designed to handle the following enhancements:

*Roadspace dynamic representation*

The transaction simulation is inherently dynamic, with infinite time resolution. Properly extending this dynamic representation to the transportation network requires a detailed representation of transport supply.

In the current demonstration system, transport supply between any two locations is represented as a horizontal supply curve – the cost is considered constant. Short run supply dynamics can be represented by adjusting the costs in response to modelled congestion. In particular, a government transport agency can "offer" a schedule of specific travel times on each link in a transport network at different times of the day or week. These offers can be queued into the list of offers, where travelers can then accept them. The government agency will list more offers than the road can physically handle (since no-one is turned away from a "full" road) but will adjust the schedule of travel times as the simulation runs to match those observed from the history of congested conditions.

The point-equilibrium representation of transport (as in PECAS and in the Martin Centre Models) follows by assuming that the road agency adjusts the schedule of times on their offer list periodically (say, annually) using the full demand over the past year in their calculation of congestion.

*Developer/location/floorspace*

The renting of floorspace by landowners can easily be included in the transaction simulation.

The development of floorspace by developers can be included in the transaction simulation framework, but since a development is a unique one-time occurrence at a location, developers can not rely on their past experience in that location to guide their decision. It seems essential that developers would need to consult the list of outstanding offers to determine average vacancy rates and prices over a range of similar properties before deciding to invest in a certain development in a certain location.

The demand for floorspace by households and establishments is more complex. The advantage of a particular location for a particular establishment or households is primarily related to the transport costs associated with meeting the wants and needs from that location. Thus, one could consider location to be a derived demand, in the same way that transport is a derived demand.

The strategy for measuring the attractiveness of a location in this framework is based on two principles:

1. **imitation:** agents look to successful agents who are similar to them in some way (their peers) and consider locations that are similar (e.g. close to) their locations.
2. **trial solutions:** for each agent in the simulation there will be a number of additional "pretend agents" in different locations. This represents agents "trying out" living their lives in different locations, determining how well they can meet their needs if they were located there.

3. **selection:** agents evaluate their overall ability to meet all of their needs from the considered locations, and then make the decision to move if one location is substantially better for them than their current location.

This approach can also be used for other "lifestyle" choices: choices that cannot be considered independently of all the other wants and needs of the economic unit, such as the choice of auto ownership for households, or the choice of large expenditures on machineries for business establishments. This is being explored in the the BE microsimulation in the companion paper (Khan, *et al*, 2002) where upstart BE's pretend to exist, to evaluate market conditions. These are called "Proto BE's".

When a large number of lifestyle choices are added, the choice of lifestyle becomes a large multi-dimensional optimization problem. The strategy will begin to resemble a genetic algorithm, and may benefit from adopting other approaches from genetic algorithm, such as DNA crossover (the selection of a portion of behaviour from one successful "parent" behaviour, and the other portion from the other parent), and mutation (random changes in individual facets of behaviour.)

#### *More complex I/O relationships*

An additional test case will be developed, based on the PECAS implementation in Oregon (Hunt and Abraham, 2002), or on the MEPLAN implementation in Sacramento (Abraham and Hunt, 1999). This will test the transaction simulation paradigm with data describing an actual economy in the United States.

#### *More complex behaviour*

A wide range of additional behavioural fidelity is possible for the individual agents within the paradigm. This research is being conducted within a larger group of researchers, most of which are exploring the more detailed behaviour of specific actors. The unlimited heterogeneity of the transaction simulation paradigm is designed to accommodate the more detailed models of individual behaviour that is being explored by these researchers (Miller *et al*, 2002).

## CONCLUSIONS

This paper reviewed a number of spatial market representations from the perspective of Walrasian economics. The spatial market representations varied in their complexity and completeness:

- The MEPLAN style "Martin Centre" models combine input-output theory with a spatial choice model, giving a comparatively simple computational equilibrium model with a spatial component. The spatial choice model isn't completely consistent with random utility theory, and the use of the long-run equilibrium concept is inconsistent with the use of time steps, fixed floorspace supply, and iteration with a transport model. Nonetheless, such models have achieved a level of orthodoxy in practical planning, with stable software implementations and a number of individuals and firms experienced in building and using such models.
- The PECAS model, being similar to the Martin Centre models, but adopting Random Utility Theory more completely, removing the inappropriate long-run equilibrium assumption, and specifically representing the role of goods in the economy.

- An extension of PECAS into a dynamic model, by adopting a price-update paradigm.
- The use of an agent based microsimulation, to overcome the inappropriate use of the law of large numbers when PECAS or Martin Centre models are applied with temporal and spatial detail
- A transaction based simulation, broadly consistent with the above approaches but supporting infinite spatial and temporal detail.

As one moves down the above list, the models become:

- more theoretically consistent,
- more realistic behaviourally,
- less proven in practice and in research, and
- supporting (but not necessarily requiring) more detailed models.

Each model seems appropriate for some uses, given the current state of research.

It is hoped that, in time, it will be proven that the transaction simulation paradigm can have stable behaviour consistent with the integration of a computable general equilibrium model and random utility theory. It is also hoped that a simple demonstration model will be developed that shows the practicality of a transaction simulation model for government spatial planning simulations. Until these conditions are fulfilled, aggregate equilibrium style models such as PECAS will be necessary for practical planning operations.

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