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# DESIGN AND APPLICATION OF THE PECAS LAND USE MODELLING SYSTEM

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**Abstract:** PECAS is a recently-designed generalized approach for simulating spatial economic systems.  It operates by clearing spatial submarkets for various goods, services and factors in a short run equilibrium, with floorspace supply handled separately based on development event probabilities.  It is currently being applied in the development of land use transport interaction models for practical use in several contexts in North America.  This paper describes PECAS and discusses its application.

**Keywords:** Land Use Transport Interaction Modelling, Spatial Input-Output Modelling; Land Development Modelling

## 1. INTRODUCTION

PECAS stands for Production, Exchange and Consumption Allocation System.  It uses an aggregate, equilibrium structure with separate flows of exchanges (including goods, services, labour and space) going from production to consumption based on variable technical coefficients and market clearing with exchange prices.  Flows of exchanges from production to exchange zones and from exchange zones to consumption are allocated using nested logit models according to exchange prices and transport disutilities.  These flows are converted to transport demands that are loaded to networks in order to determine congested travel disutilities.  Exchange prices determined for space inform the calculation of changes in space thereby simulating developer actions.  The system is run for each year being simulated, with the travel disutilities and changes in space for one year influencing the flows of exchanges in the next year.   PECAS is a generalisation of the spatial I/O modelling approach used in the MEPLAN and TRANUS land use transport modelling systems.  PECAS also includes elements of the land use components of 'connected' land use transport modelling systems such as DELTA.  As such, under certain simplifying assumptions, PECAS reduces to these other systems.

PECAS or its components are currently being applied in the development of state-wide transportation land use modelling systems for Ohio and Oregon, in the development of an urban land use model for Sacramento and in the anticipated development of urban land use models of Calgary and Edmonton in Canada.  It is also being used as the basis for a recommended design for a model of the Los Angeles Region.

This paper describes PECAS and discusses its application and the nature of its contribution in practical modelling work.

## 2. PECAS STRUCTURE

### 2.1 Overview

PECAS has two component modules. One is the space development module, which represents the actions of developers in the provision of space (land and floorspace) where activities can locate, including the new development, demolition and re-development that occurs from one point in time to the next. The other is the activity allocation module, which represents how activities locate within the space provided by developers and how these activities interact with each other at a given point in time.

### 2.2 Treatment of Space

The area covered by PECAS is organized into a set of land use zones. Activities locate in these zones and interact between them. Ideally these zones are the same as the transport zones used in the transport model being run with PECAS, or at least are aggregations of whole numbers of adjacent transport zones. The connectivity among the land use zones is then based on the representation provided by the transport model, where the transport model uses its networks to establish congested network times and costs and associated interchange (dis)utilities that PECAS uses in its consideration of the interactions between land use zones.

Currently it is recommended that a maximum of about 750 such land use zones be used because of the 2 gigabyte memory address space limitations of most current operating systems.

### 2.3 Treatment of Time

PECAS works through time in a series of discrete, fixed steps from one point in time to the next, with the activity allocation module running at each point in time and the space development module considering the period from each point in time to the next. The steps can be of any fixed duration, but 1 year time steps are recommended since they allow an appropriately quick response of land developers in the land development module to the prices established in the activity allocation module.

Figure 1 illustrates the interactions among the modules and with other models when the two PECAS modules are run as part of an integrated land use transport modelling system. The transport model used to calculate the congested travel times and disutilities is normally run for each year (after PECAS completes its run for that year) or, if travel conditions are relatively stable, the transport model can be run less often to save computation time.

## 3. ACTIVITY ALLOCATION MODULE

### 3.1 Approach

The PECAS activity allocation module is an aggregate representation based on spatially-disaggregated forms of extended make and use input-output tables, with variable technical coefficients and with households included – making the tables 'social accounting tables' rather than just input-output tables. Categories of activity produce 'commodities' as indicated in the make table and consume these 'commodities' as indicated in the use table, where these 'commodities' include goods and services, labour and space (land and/or floorspace). The movement of these commodities from where they are produced (in zones) to where they are
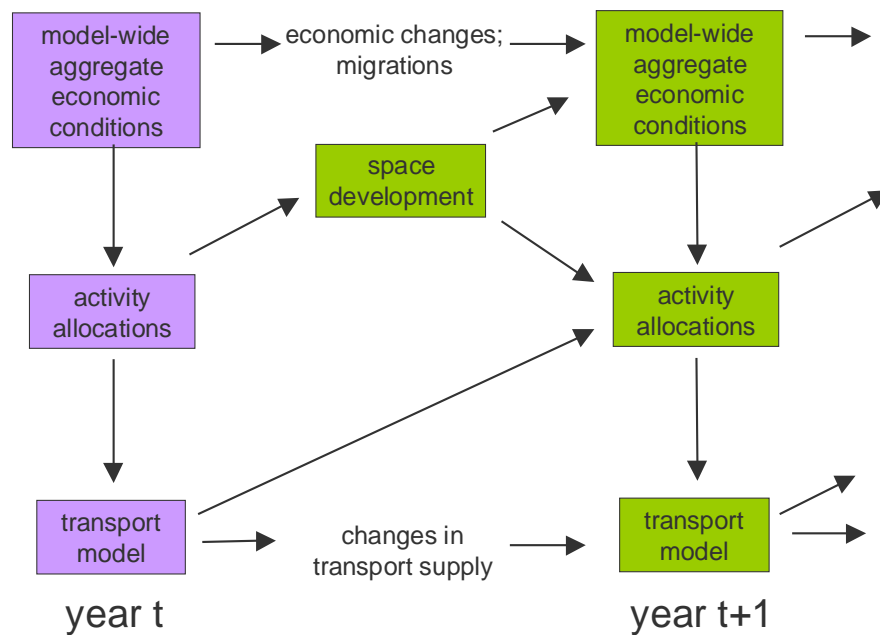
Figure 1: Interactions among modules simulating temporal dynamics

consumed (in zones) is the economic basis for travel and transport in the land use model. The travel conditions – distances, costs, times and associated (dis)utilities by mode – for the movement of these commodities is how the transportation system influences the attractiveness of zones as locations for different activities.

For each zone there are two 'lowest-level' logit allocation models for each category of commodity considered in the model. The first is an allocation of the quantities purchased among various 'exchange locations' where they are sold to other activities. The second is an allocation of the quantities sold among the various exchange locations where they are bought by other activities. The utility of each alternative in these models is influenced by the price at the exchange location, and the characteristics of transporting the commodity to or from the exchange location. The composite utility values from these two 'lowest-level' logit models are called the 'buying utility' and the 'selling utility' for the commodity in the zone. They are used as the transportation-related inputs into the 'next-higher-level' logit allocation process for allocating the production and consumption of commodities by activities. From there, via the resulting utilities of production and consumption, they feed into the location utilities used at the 'highest-level' to allocate categories of industries and households to the land use zones.

The exchange locations are location-specific markets for commodities, where sellers sell commodities to buyers. Prices are established at exchange locations so that the quantity bought equals the quantity sold – thus the spatial allocation procedure assumes a short-run market equilibrium in commodities.

For simplicity and realism, commodities can be assigned to be either purchased from the exchange location in the zone they are consumed. (e.g. for labour, this occurs in the employment zone, where the labour is exchanged and the price set) or purchased in the exchange in the zone they are produced (e.g. for retail goods this is at the retail establishment, where the goods are exchanged and the price is set). In this case for each commodity in each

zone, one of the logit allocation models (either the buying or the selling) consists of only one alternative.

Imports and exports also enter the system at exchange locations. The commodity import and export quantities at each exchange location are determined as functions of the exchange price, so that as prices rise more imports are attracted and as prices fall more exports are produced. These import and export functions connect the model economy to the rest of the world.

**3.2 Quantity Definitions and Categories**

Figure 2 provides a summary of the make and use tables and the categories of activities and commodity categories used within them, showing the specific types of interactions being represented. Commodity production is represented in the make table at the top of Figure 1; commodity consumption is represented in the use table at the bottom; commodities 'flow' down the columns from where they are produced to where they are consumed. The general nature of the exchange location for each commodity category and the treatment of imports and exports are also indicated.

Table 1 provides an example listing of the activity categories that could be used in the activity allocation module. In general, standard industrial sector categories are used as the starting point for the representation of the business and government components of the economy, with these categories split according to the type of employment and associated space consumed. In general, a given industrial sector is split into blue-collar and white-collar components and associated 'shopfloor' and 'office' uses in this way so that the allocation process in the model can consider the very different behavior of these two components regarding location, labour use and space type use.

Different types of household are included as different activity categories. Labour is included as a commodity that is produced by households and consumed by economic production activities, with labour in different occupation categories treated as different commodity categories.

Different types of floorspace are also included as commodities in the use table in that they are consumed by activities, but the amount of floorspace available for consumption in each zone is fixed in any one time period being considered with the activity allocation module. It is the space development module running between time periods that adjusts the amount of floorspace available in zones, as described below. Floorspace is a 'non-transportable commodity' in that it must be exchanged and consumed in the zone where it is produced and thus located.

Such things as water, energy, and air-borne or water borne emissions can also be included as commodities in the make and/or use table structure, which allows the model to determine aggregate demands or impacts arising from activities consistent with the rest of the model simulation.

**3.3 Activity allocation**

The model-wide total quantity of production activity for each activity category is input to the activity allocation module. This amount is allocated among the land use zones using a logit allocation as follows:

$$W_{a,z} = TW_a \cdot ( \exp ( \lambda_a \cdot LU_{a,z} ) / \Sigma_{z \in Z} ( \exp ( \lambda_a \cdot LU_{a,z} ) ) ) \qquad (01)$$

with:

| commodities | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| goods | internal management services | services | education services | retail services | domestic services | labour | non-residential space | residential space |
| **producing activities (making)** | | | | | | | | |
| *industry:* goods production by industry | internal management services production by industry | services production by industry | education services production by industry | retail services production by industry | | | | |
| *government:* goods produced by government | | services production by government | education services production by government | | | | | |
| *households:* | | | | | domestic services production by households | labour production by households | | |
| **exports** yes | | yes | | yes | | yes | | |
| **exchange locations** at production | at consumption | at production | at production | at production | at consumption | at consumption | non-transportable | non-transportable |
| **imports** yes | yes | yes | | yes | | yes | | |
| **consuming activities (using)** | | | | | | | | |
| *industries:* goods consumption by industry | internal management services consumption by industry | services consumption by industry | education services consumption by industry | | | labour consumption by industry | space consumption by industry | |
| *government:* goods consumption by government | | services consumption by government | education services consumption by government | | | labour consumption by government | space consumption by government | |
| *households:* goods consumption by households - direct | | services consumption by households | education services consumption by households | goods consumption by households - via retail | domestic services consumption by households | | | space consumption by households |

Figure 2: Summary of categories of activities and commodities in make and use tables

Table 1: Potential activity categories for activity allocation module

## Industry

Agriculture in Office Floorspace
Agriculture on Agricultural Land
Light Industry in Office Floorspace
Light Industry in Light Industrial Floorspace
Heavy Industry in Office Floorspace
Heavy Industry on Heavy Industrial Land
Wholesale in Office Floorspace
Wholesale in Warehouse Floorspace
Retail in Office Floorspace
Retail in Retail Floorspace
Transportation Handling in Office Floorspace
Transportation Handling in Depot Floorspace
Other Services in Office Floorspace
Other Services in Light Industrial Floorspace
Other Services in Retail Floorspace
Gradeschool Education in Office Floorspace
Gradeschool Education on Gradeschool Land
Post-Secondary Education in Office Floorspace
Post-Secondary Education on Insititutional Land

## Government

Government in Office Space
Government in Government Support Land

## Households

Low Income Households
Medium Income Households
High Income Households

$$LU_{a,z} = \alpha_{size} \cdot Size_{a,z} + \alpha_{inertia} \cdot PrevW_{a,z} + Constant_{a,z} + \Sigma_{v \in V} (\alpha_{a,v} \cdot X_{v,z}) \quad (02)$$

$$+ \alpha_{prod} \cdot CUProd_{a,z} + \alpha_{cons} \cdot CUCons_{a,z}$$

where:

| | | |
|---|---|---|
| $z$ | = | index representing land use zones; |
| $Z$ | = | the set of all land use zones; |
| $a$ | = | index representing activity categories; |
| $c$ | = | index representing commodity categories; |
| $W_{a,z}$ | = | quantity of activity a in zone z; |
| $TW_a$ | = | model-wide total quantity of activity a; |
| $LU_{a,z}$ | = | location utility for a unit of activity a in zone z; |
| $Size_{a,z}$ | = | representation of relative size of zone z for activity a, indicating the *a priori* expected share of activity a for zone z; |
| $PrevW_{a,z}$ | = | the proportion of model-wide quantity of activity a in zone z in the previous time period; |
| $v$ | = | index representing 'other' zonal attributes; |
| $V$ | = | the set of all 'other' zonal attributes; |
| $X_{v,z}$ | = | one of the 'other' zonal attributes; |
| $CUProd_{a,z}$ | = | composite utility associated with production by activity a in zone z, defined in equation 07 and discussed below; |
| $CUCons_{a,z}$ | = | composite utility associated with consumption by activity a in zone z, defined in equation 08 and discussed below; |
| $\alpha_{size}$ | = | utility function coefficient for the sensitivity to size; |
| $\alpha_{inertia}$ | = | utility function coefficient for the sensitivity to the previous proportion of activity a in zone z, representing inertia in allocation of activity a; |
| $\alpha_{a,v}$ | = | utility function coefficient for the sensitivity to the 'other' zonal attribute $X_{v,z}$; |
| $Constant_{a,z}$ | = | utility function alternative specific constant for zone z for allocation of activity a; |
| $\alpha_{prod}$ | = | utility function coefficient for the sensitivity to composite utility associated with production; |
| $\alpha_{cons}$ | = | utility function coefficient for the sensitivity to composite utility associated with consumption; |
| $\lambda_a$ | = | utility function dispersion parameter for allocation of activity a. |

The 'other' zonal attributes vary depending on the production activity being allocated. For residential activities in particular, these 'other' attributes include representations of various amenities relevant to housing location choice, including school quality, general noise levels, air quality, open space density, municipal taxation levels and possibly zonal-level income distributions and racial compositions.

### 3.3 Production and consumption allocation

The make and use technical coefficients indicating the rates at which activities produce and consume commodities are elastic, representing opportunities for substitution (between inputs in consumption and between by-products in production), as follows:

$$M_{c,a,z} = MMin_{c,a} + MDisc_{c,a} \cdot (\exp(\lambda_{m,a} \cdot UIProd_{c,a,z}) / \Sigma_{c \in C} \exp(\lambda_{m,a} \cdot UIProd_{c,a,z})) \quad (03)$$

with:

$$UIProd_{c,a,z} = MDisc_{c,a} \cdot CUSell_{c,a,z} \quad (04)$$

and

$$U_{c,a,z} = UMin_{c,a} + UDisc_{c,a} \cdot (\exp (\lambda_{u,a} \cdot UICons_{c,a,z}) / \Sigma_{c \in C} \exp (\lambda_{u,a} \cdot UICons_{c,a,z})) \qquad (05)$$

with:

$$UICons_{c,a,z} = UDisc_{c,a} \cdot CUBuy_{c,a,z} \qquad (06)$$

where:

$M_{c,a,z}$ = make technical coefficient for production of commodity c by activity a in zone z, indicating the quantity of commodity c produced per unit of activity a

$U_{c,a,z}$ = use technical coefficient for consumption of commodity c by activity a in zone z, indicating the quantity of commodity c consumed per unit of activity a;

$UIProd_{c,a,z}$ = utility of producing the potential discretionary amount of commodity c associated with a unit of activity a in zone z

$UICons_{c,a,z}$ = utility of consuming the potential discretionary amount of commodity c associated with a unit of activity a in zone z

$MMin_{c,a}$ = minimum amount of commodity c made per unit of activity a

$MDisc_{c,a}$ = potential discretionary amount of commodity c made per unit of activity a

$UMin_{c,a}$ = minimum amount of commodity c used per unit of activity a

$UDisc_{c,a}$ = potential discretionary amount of commodity c used per unit of activity a

$CUSell_{c,a,z}$ = composite utility associated with selling a unit of commodity c produced by activity a in zone z;

$CUBuy_{c,a,z}$ = composite utility associated with buying a unit of commodity c consumed by activity a in zone z;

$\lambda_{m,a}$ = utility function dispersion parameter for allocation of by-product substitutes made by activity a;

$\lambda_{u,a}$ = utility function dispersion parameter for allocation of input substitutes used by activity a.

Then the composite utilities associated with production and consumption by activity a in zone z, as used in equation 02 above, are as follows:

$$CUProd_{a,z} = (\Sigma_{c \in C} MMin_{c,a} \cdot CUSell_{c,a,z}) + (1 / \lambda_{m,a}) \cdot \Sigma_{c \in C} \exp (\lambda_{m,a} \cdot UIProd_{c,a,z}) \qquad (07)$$

and

$$CUCons_{a,z} = (\Sigma_{c \in C} UMin_{c,a} \cdot CUBuy_{c,a,z}) + (1 / \lambda_{u,a}) \cdot \Sigma_{c \in C} \exp (\lambda_{u,a} \cdot UICons_{c,a,z}). \qquad (08)$$

These composite utilities combine the accessibilities for buying and selling individual commodities consistently with the production and consumption options for the activity. The location utility (determined in equation 02) of an activity that consumes a large amount of a commodity will be strongly influenced by the composite utility of buying that commodity, and the location utility of an activity that produces a large amount will be strongly influenced by the composite utility of selling. As such, the last two terms in equation 02 provide overall indications of the accessibility of the zone for the activity consistent with the technology of the production process for the activity.

To allow consistency with earlier spatial input-output representations in MEPLAN (Hunt and Simmonds, 1993) and TRANUS (de la Barra, 1998), a form of elastic consumption and production using independent exponential functions for each commodity is also supported by the PECAS software. This alternate form does not have the same consistency with random utility theory as the form described here.

### 3.4 Buying and selling allocation

The total quantities of commodities produced and consumed by each activity in each zone are calculated using the technical coefficients as follows:

$$TP_{c,z} = \Sigma_{a \in A}\, M_{c,a,z} \cdot W_{a,z} \tag{09}$$

and

$$TC_{c,z} = \Sigma_{a \in A}\, U_{c,a,z} \cdot W_{a,z} \tag{10}$$

where:

$TP_{c,z}$ = total quantity of commodity c produced in zone z;
$TC_{c,z}$ = total quantity of commodity c consumed in zone z.

The total quantity of each commodity produced by production activities in a given zone is allocated among exchange locations (where it is sold) as follows:

$$S_{c,z,k} = TP_{c,z} \cdot (\, exp\,(\,\lambda_{s,c} \cdot SU_{c,z,k}\,) \,/\, \Sigma_{k \in K}\, exp\,(\,\lambda_{s,c} \cdot SU_{c,z,k}\,)\,) \tag{11}$$

with:

$$SU_{c,z,k} = \delta_{size,s} \cdot Size_{s,c,k} + \delta_{price,s} \cdot Price_{c,k} + \delta_{tran} \cdot Tran_{c,z,k} \tag{12}$$

where:

$k$ = index representing exchange locations (land use zones);
$K$ = the set of all exchange locations;
$S_{c,z,k}$ = quantity of commodity c produced in zone z that is allocated to (go to) exchange location k;
$SU_{c,z,k}$ = utility for selling to exchange location k a unit of commodity c produced in zone z;
$Size_{s,c,k}$ = representation of relative size of exchange location k for selling commodity c, indicating the *a priori* expected share of commodity c sold in exchange location k;
$Price_{c,k}$ = the unit exchange price for commodity c in exchange location k;
$Tran_{c,z,k}$ = the utility for transporting a unit of commodity c from zone z to exchange location k;
$\delta_{size,s}$ = utility function coefficient for the sensitivity to size;
$\delta_{price,s}$ = utility function coefficient for the sensitivity to price when selling;
$\delta_{tran}$ = utility function coefficient for the sensitivity to transport utility;
$\lambda_{s,c}$ = utility function dispersion parameter for allocation of selling of commodity c.

The selling composite utility for commodity c (independent of the producing activity) is determined consistent with equations 11 and 12, as follows:

$$CUSell_{c,z} = (1/\lambda_{s,c}) \cdot ln\,(\,\Sigma_{k \in K}\, exp\,(\,\lambda_{s,c} \cdot SU_{c,z,k}\,)\,) \tag{13}$$

The selling composite utility for commodity c when produced by activity a in particular, as used in equations 04 and 07, is then established as follows:

$$CUSell_{c,a,z} = \phi_{s,c,a} \cdot CUSell_{c,z} + USellRef_{c,a} \tag{14}$$

where:

$\phi_{s,c,a}$ = factor adjustment to the composite utility of selling commodity c by activity a
$USellRef_{c,a}$ = offset adjustment to the composite utility of selling commodity c by activity a.

The scaling and offset adjustments $\phi_{s,c,a}$ and $\text{USellRef}_{c,a}$ have been found useful in calibration to adjust (usually remove by setting $\phi_{s,c,a}$ to zero) the effect of individual buying and selling composite utilities on production utilities and thus on location utilities.

The allocation of quantities consumed to the exchange zones where they are purchased is analogous to the allocation of quantities produced as described above. Specifically, the total quantity of each commodity consumed by a production activity in a given zone is allocated among exchange locations (where it is bought) as follows:

$$B_{c,z,k} = TC_{c,z} \cdot ( \exp ( \lambda_{b,c} \cdot BU_{c,z,k} ) / \Sigma_{k \in K} \exp ( \lambda_{b,c} \cdot BU_{c,z,k} ) ) \tag{15}$$

with:

$$BU_{c,z,k} = \delta_{size,b} \cdot Size_{b,c,k} + \delta_{price,b} \cdot Price_{c,k} + \delta_{tran} \cdot Tran_{c,k,z} \tag{16}$$

where:

| | | |
|---|---|---|
| $B_{c,z,k}$ | = | quantity of commodity c consumed in zone z that is allocated to (come from) exchange location k; |
| $BU_{c,z,k}$ | = | utility for buying from exchange location k a unit of commodity c consumed in zone z; |
| $Size_{s,c,k}$ | = | representation of relative size of exchange location k for buying commodity c, indicating the *a priori* expected share of commodity c bought in exchange location k; |
| $Tran_{c,k,z}$ | = | the utility for transporting a unit of commodity c from exchange location k to zone z; |
| $\delta_{size,b}$ | = | utility function coefficient for the sensitivity to size; |
| $\delta_{price,b}$ | = | utility function coefficient for the sensitivity to price when buying; |
| $\delta_{tran}$ | = | utility function coefficient for the sensitivity to transport utility; |
| $\lambda_{b,c}$ | = | utility function dispersion parameter for allocation of buying of commodity c. |

The buying composite utility for commodity c (independent of the consuming activity) is determined consistent with equations 11 and 12, as follows:

$$CUBuy_{c,z} = (1/\lambda_{b,c}) \cdot \ln ( \Sigma_{k \in K} \exp ( \lambda_{b,c} \cdot BU_{c,z,k} ) ) \tag{17}$$

The buying composite utility for commodity c when consumed by activity a in particular, as used in equations 06 and 08, is then established as follows:

$$CUBuy_{c,a,z} = \phi_{b,c,a} \cdot CUBuy_{c,z} + UBuyRef_{c,a} \tag{18}$$

where:

$\phi_{b,c,a}$ = factor adjustment to the composite utility of buying commodity c by activity a
$UBuyRef_{c,a}$ = offset adjustment to the composite utility of buying commodity c by activity a

### 3.5 Imports and exports

The quantities of imports and exports for a given commodity in a given exchange zones are determined using:

$$Q_{c,i,k} = QRef_{c,i} + \gamma_{c,i} \cdot ( [G_i-1]/[G_i+1] ) + \mu_{c,i} \cdot ( Price_{c,k} - PriceRef_{c,i} ) \tag{16}$$

and

$$Q_{c,e,k} = QRef_{c,e} + \gamma_{c,e} \cdot ( [G_e-1]/[G_e+1] ) + \mu_{c,e} \cdot ( Price_{c,k} - PriceRef_{c,e} ) \tag{19}$$

with:

$$G_i = \exp ( \lambda_{c,i} \cdot ( Price_{c,k} - PriceRef_{c,i} ) ) \qquad (20)$$

and

$$G_e = \exp ( \lambda_{c,e} \cdot ( Price_{c,k} - PriceRef_{c,e} ) ) \qquad (21)$$

where:

| | | |
|---|---|---|
| $Q_{c,i,k}$ | = | quantity of commodity c imported to exchange location k; |
| $Q_{c,e,k}$ | = | quantity of commodity c exported from exchange location k; |
| $QRef_{c,i}$ | = | quantity of commodity c imported to exchange location when the unit exchange price for commodity c in exchange zone k is at its import reference level $PriceRef_{c,i}$; |
| $PriceRef_{c,i}$ | = | reference price per unit for import of commodity c; |
| $QRef_{c,e}$ | = | quantity of commodity c exported from exchange location when the unit exchange price for commodity c in exchange zone k is at its export reference level $PriceRef_{c,e}$; |
| $PriceRef_{c,e}$ | = | reference price per unit for export of commodity c; |
| $\gamma_{c,i}$ | = | function coefficient for the rate of increase in imports of commodity c for exponent term; |
| $\mu_{c,i}$ | = | function coefficient for the rate of increase in imports of commodity c for linear term; |
| $\lambda_{c,i}$ | = | function coefficient for sensitivity to difference in exchange price for commodity c concerning increase in imports of commodity c for exponent term. |
| $\gamma_{c,e}$ | = | function coefficient for the rate of increase in exports of commodity c for exponent term; |
| $\mu_{c,e}$ | = | function coefficient for the rate of increase in exports of commodity c for linear term; |
| $\lambda_{c,e}$ | = | function coefficient for sensitivity to difference in exchange price for commodity c concerning increase in exports of commodity c for exponent term. |

## 3.6 Mathematical solution

The exchange zones simulate markets where the aggregate supply (which is the sum of the selling allocations together with the quantity of imports, both elastic with respect to the exchange price) meets the aggregate demand (which is the sum of the buying allocations together with the quantity of exports). The module solves for the equilibrium solution, adjusting the exchange prices in the exchange locations until all the markets clear.

The search algorithm calculates the partial derivatives of the total excess demand (excess of demand by buyers and exporters over supply by sellers and importers) in each exchange zone with respect to the price in that exchange zone. The derivatives with respect to other prices in other zones or for other commodities are assumed to be zero, and a price change is calculated. A step adjustment factor is applied to the step to speed and aid convergence. If a step results in a lower aggregate sum-of-squares excess demand, then the step adjustment factor is increased slightly for the next iteration. If a step results in a higher aggregate sum-of-squares excess demand, then the step is abandoned, the step adjustment factor is reduced substantially, and a new and smaller step is calculated to replace the abandoned one.

### 3.7 Model outputs

The model provides consistent activity allocations by activity category by zone, commodity flow quantities from production zone to consumption zone via exchange zone, imports and exports by exchange zone and exchange prices by commodity by exchange zone.

It also provides the basis for assessing the changes in model-wide consumer surplus by production activity. In particular, the model-wide location composite utility for a given activity category is determined consistent with equations 01 and 02 as follows:

$$CLU_a = (1/\lambda_a) \cdot \ln ( \Sigma_{z \in Z} \exp ( \lambda_a \cdot LU_{a,z} ) ) \tag{22}$$

where:

$CLU_a$ = model-wide location composite for a unit of activity a.

Changes in $CLU_a$ for a given activity category can be used to calculate measures of the changes in consumer benefit for the activity category arising with the policy actions input to the model. These measures can help assess quality of life and business attractiveness for different household and industrial categories as represented.

### 3.8 Calibration

The data to calibrate the activity allocation module include:

- input-output (social accounting) make and use tables for base year, including import and export quantities;
- information or assumptions on elasticity of technical coefficients in social-accounting table for the base year, including floorspace consumption rates;
- spatial distribution of employment by industry and occupation in base year and if possible for one or more other years after the base year;
- labourforce participation rates by household type and by occupation in the base year;
- trip length distributions by commodity in the base year;
- indications of the average price for each commodity, the overall variation in the price around the region, and some indication of where the prices are more higher and where they are lower.

The calibration of the activity allocation module is challenging because the system is so highly inter-related (much like the real world) and because the production activity quantities for the industrial sectors are not generally available as observed values by zone. A Bayesian approach is typically adopted, where many relationships are estimated statistically based on observed disaggregate conditional choice data, and then the entire modelling system is run and adjusted to match a series of targets. Some coefficients will be determined only by adjusting them in the overall calibration, others only in a lower level estimation, and others can be estimated with some degree of confidence in an estimation of conditional choice data, then adjusted in the overall calibration.

## 4. SPACE DEVELOPMENT MODULE

### 4.1 Approach

The PECAS space development module is an aggregate representation based on a set of logit allocation models. These logit models determine the changes in aggregate quantities of space (including land and floorspace) in each category in each land use zone occurring in the period

from one point in time to the next.

Each non-zero quantity of space in each zone in each of the following categories is considered separately:

- each separate category of existing floorspace and developed land;
- serviced vacant land;
- unserviced vacant land;
- derelict.

When one of these non-zero quantity of space is considered, a logit model is used to determine the proportions of the quantity to allocate to each of the following discrete alternatives (with some restrictions as appropriate):

- remaining in the same space category;
- becoming derelict;
- transforming into each of the other floorspace and developed land categories.

## 4.2 Quantity Definitions and Categories

Figure 3 provides an example listing of the space categories that could be used, together with a summary of the allowable transitions among categories. Each space type tends to map to a fairly narrow set of activity categories by setting to zero most of the use coefficients, $UMin_{c,a}$ and $UDisc_{c,a}$, for the consumption of floorspace type c by activity a.

## 4.3 Mathematical structure and processes

In a given period from one point in time 't' to the next 't+1' – between runs of the activity allocation module – in each land use zone the quantity of space in each category is allocated among allowable categories as follows:

$$R_{j,h,z} = TR_{j,z} \cdot ( \exp ( \lambda_{r,j} \cdot RU_{j,h,z} ) / \Sigma_{h \in H(j)} \exp ( \lambda_{r,j} \cdot RU_{j,h,z} ) ) \tag{23}$$

with:

$$RU_{j,h,z} = \phi_{price,r} \cdot Price_{h,z} + \phi_{price,t} \cdot TCost_{j,h,z} + Constant_{j,h} \tag{24}$$
$$+ \phi_{regprice} \cdot ( RegPrice_h - RegPRef_h ) + \phi_{cap} \cdot ( ER_{h,z} / Cap_{h,z} )^{\theta_{cap}}$$

where:

| | | |
|---|---|---|
| $j , h$ | = | indices representing space categories; |
| $H(j)$ | = | the set of all new space categories allowable for existing space category j; |
| $R_{j,h,z}$ | = | quantity of space transitioning from space category j (at time 't') to space category h (at time 't+1') in zone z; |
| $TR_{j,z}$ | = | total quantity of space category j (at time 't') in zone z; |
| $RU_{j,h,z}$ | = | transition utility for space in category j (at time 't') changing into space in category h (at time 't+1) in zone z; |
| $Price_{h,z}$ | = | unit price for space category h (at time 't') in zone z; |
| $TCost_{j,h,z}$ | = | unit cost for transforming space from category j (at time 't') to category h (at time 't+1') in zone z, including relevant demolition, clean-up, construction and servicing costs; |
| $RegPrice_h$ | = | model-wide price for space category h (at time 't'), calculated as the weighted average of zonal prices for space category h |
| $RegPRef_h$ | = | reference level for model-wide rent of space category h; |
| $ER_{h,z}$ | = | transitional total quantity of space category h in zone z; |
| $Cap_{h,z}$ | = | capacity for space category h in zone z; |

| | | = | utility function coefficient for the sensitivity to space price; |
|---|---|---|---|

$\phi_{price,r}$ = utility function coefficient for the sensitivity to space price;
$\phi_{price,t}$ = utility function coefficient for the sensitivity to cost of transforming space;
$\phi_{regprice}$ = utility function coefficient for the sensitivity to region-wide price for space;
$\phi_{density}$ = utility function coefficient for the sensitivity to model-wide density of use of space relative to the relevant reference density;
$\phi_{cap}$ = utility function coefficient for the sensitivity to the available capacity term;
$\theta_{cap}$ = function power coefficient for the available capacity term;
$Constant_{j,h}$ = utility function alternative specific constant for transition of space from category j (at time 't') to category h (at time 't+1');
$\lambda_{r,j}$ = utility function dispersion parameter for transition of space category j.

| Space Category (transforming from) | vacant unserviced land | vacant serviced land | derelict | single-family detached residential | multi-family residential | office floorspace | retail floorspace | warehouse floorspace | hotel floorspace | hospital space | transportation depot floorspace | light industrial floorspace | heavy industrial land | gradeschool land | post-secondary institutional land | government support land | agricultural land |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| vacant unserviced land | X | X | | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| vacant serviced land | | X | | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| derelict | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| single-family detached residential | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| multi-family residential | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| office floorspace | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| retail floorspace | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| warehouse floorspace | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| hotel floorspace | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| hospital space | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| transportation depot floorspace | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| light industrial floorspace | | | X | X | X | X | X | X | X | X | X | X | X | X | X | X | |
| heavy industrial land | | X | | | X | X | X | X | X | X | X | X | | | | | |
| gradeschool land | | | | | | | | | | | | | | X | | X | |
| post-secondary institutional land | | | | | | | | | | | | | | | X | X | |
| government support land | | | | | | | | | | | | | | X | X | X | |
| agricultural land | | X | | X | X | X | X | X | X | X | X | X | X | X | X | X | X |
| undevelopable and protected land | | | | | | | | | | | | | | | | | |

Figure 3: Potential space categories and allowable transitions

This is done for each space category, starting with vacant serviced land, then vacant unserviced land, and then working down the list of other categories in order, such as listed in Figure 3, but considering derelict space last. As each space category j is considered, the amount for $TR_{j,z}$ at time 't' is allocated among the $R_{j,h,z}$ and then the amount for $ER_{h,z}$ for each space category h is updated so that the impacts on the available capacity are as current as possible before considering the next space category. The $ER_{h,z}$ amounts at the end of this process, after all the $TR_{j,z}$ at time 't' have been considered, become the $TR_{j,z}$ at time 't+1'.

The model-wide price term is included in order to represent the competition between zones for developers. This can provide some dampening of a more rapid response (in particular to prices) in particular zones. Thus, the capability for space development to 'overreact', resulting in boom and bust cycles in available space, as sometimes occurs in reality, can be calibrated as part of the model development.

In general there is a lot of inertia in the system over a 1-year time step, where large portions of space remain in the same category. This is implemented in PECAS using large positive values for the alternative specific constants, $Constant_{j,h}$, in the cases where $h=j$, that is, where the space is remaining in the same category.

In the Oregon and Sacramento implementations of PECAS, the total amount of space in each zone in each category is further divided into zoning schemes. A zoning scheme causes a modification to the $Constant_{j,h}$ for transitions when $j{\neq}h$. This allows the capacity information to be based on an overlay of an existing use layer in a GIS with a future permitted use GIS layer. When this mechanism is used, $\phi_{cap}$ is usually set to zero and $Cap_{h,z}$ is irrelevant because the capacity is expressed directly as the supply of land in different zoning schemes.

### 4.4 Calibration

The data to calibrate the space development module includes changes in the quantities of space by category occurring over the time from the base year to another known year (about two or three years after the base year) in at least 200 or so of the land use zones, together with indications of the zoning regulations, construction costs and quantities of unserviced and serviced vacant land in these zones in the base year

Such data concerning changes over a two-to-three year period may not be readily and immediately available from existing sources in a given application. But they can be collected for a sufficient number of zones with only a relatively modest effort by drawing from and/or coordinating with relevant sources, including municipal tax rolls, development applications records and real estate transaction information, at regular intervals over a period of about three years. It the meantime, while waiting for such data, the model can be developed with data on just construction costs, assuming that developers ignore sunk costs for land and future prices.


### 5. DISCUSSION

PECAS provides a comprehensive, generalized structure for developing land use models. It represents an ambitious attempt to include many of the relevant components of the land use system, including those related to the transport system. The intention is to provide a 'tool kit' for the development of the land use side of land use transport interaction models that can be suitably policy-responsive in practical application.

PECAS or its components are being used in a number of practical modelling applications. A statewide model of Ohio is being developed using the full PECAS structure (PB Ohio *et al*, 1999). The Generation 2 modelling work in Oregon is using many of the elements of the PECAS activity allocation module as part of a larger system where various micro-simulation treatments are applied with the results of the activity allocation acting as inputs (Hunt, 2001). The full PECAS structure has been used as the basis for a model specification for the Los Angeles Region. A land use model for the Sacramento Region that is just starting to be developed is based in part on the PECAS activity allocation module (DKS *et al*, 2001). The intention is to use the full PECAS structure in the development of urban-level land use transport interaction models of both Edmonton and Calgary in Canada.

The development of PECAS has benefited greatly from previous experience with the MEPLAN (Hunt and Simmonds, 1993) and TRANUS (de la Barra, 1998) frameworks, both of which use an 'integrated' structure based on the spatial disaggregation of input-output tables to link land use with transport. It has also benefited from consideration of connected modelling systems such as those based on DELTA (Simmonds, 1996) and UrbanSim (Waddell, 2002), which use a 'connected' structure where model-wide quantities of activities

are allocated in space according to the distribution of relevant accessibilities. PECAS is a generalization that incorporates both of these 'integrated' and 'connected' structures. If its make and use input-output tables are replaced by the corresponding reduced square table and the implied multipliers are used to determine the activity totals to be allocated incrementally, then the system becomes much like the MEPLAN and TRANUS systems - without explicit representation of either commodity flows or commodity-specific accessibilities. In PECAS, the use of accessibilities that are aggregations of composite utilities from buying and selling allocations of commodities used and made by activities, with weights based on the corresponding technology coefficients, represents a re-interpretation of the vector of accessibilities in 'connected' structures. If the buying and selling allocation processes in PECAS are removed, and the corresponding aggregations of composite utilities from these are replaced by a more ad-hoc vector of accessibilities, then the system becomes much like the 'connected' systems as used in DELTA and UrbanSim. The incorporation of these two approaches into a single framework in PECAS is seen to be a step forward that allows the benefits of each approach to be realized as appropriate.

The PECAS activity allocation module can be viewed as the aggregate outcome of a joint choice model of location, production options, consumption options, and destinations for buying and selling by households, business establishments, and other institutions. Hence it is consistent with a micro-simulation of business establishments and of households. Further, if prices are updated over time in response to excess demand (instead of adjusted to force excess demand to zero in each time period) the PECAS activity allocation module provides a dynamic representation of the economy consistent with Walrasian economics (Abraham and Hunt, 2002). Thus PECAS is seen as a practical stepping stone towards explicitly dynamic aggregate models and/or dynamic micro-simulation models.

One of the big, challenging tasks in the practical application of PECAS is model calibration. There are many coefficients whose values have to be determined, and the interactions among the model elements that influence and are influenced by these coefficients are complex. Semi-automated processes have been developed for different sets of coefficients, largely in previous work using just certain elements of PECAS. Overall, a Bayesian sequential estimation procedure is used for the entire system, where statistical methods are used with observations of conditional choice behaviour to establish some coefficients, while other coefficients are established by calibrating the system so that it matches targets. A weight sensitivity matrix can be used to explore the remaining lack-of-fit, which may lead to small changes in the details of the model design and specification (Abraham, 2000).

The other big, challenging task in the practical application of PECAS is the collection and/or development of the data required for calibration and for subsequent operation of the model. In all practical applications thus far there has had to be considerable work done in data development and synthesis. The view taken is that it is better to work from a more comprehensive modelling framework, and seek to develop data as required, rather than to limit the model form consistent with the (readily?) available data. It is also felt that a consistent treatment within a comprehensive framework is even more important when data are not so readily available. In cases where complete data are not immediately available, some form of synthetic data is used to develop a 'placeholder', so that the full system representation with PECAS is still available and the entire model is working. It is anticipated that there will be opportunities to collect and further enhance the data and adjust the model accordingly after a working model is available and, hopefully, has demonstrated its usefulness by being applied to inform policy development.

Previous experience in modelling of this sort has taught that the working versions of these models are constantly being updated and modified in order to include enhanced representation seen to be required for particular applications. The use of a consistent, generalized structure

as provided by PECAS, where further categories and representations can be added and interpreted, is very helpful in this regard.

With the PECAS framework defined as outlined above, the experiences being gained in the practical modelling that is underway can be more completely interpreted and shared, not just among the specific applications of PECAS, but also more widely among other practical land use transport modelling work elsewhere. It is anticipated that such experiences will be reported in this light in the future.

## ACKNOWLEDGEMENTS

## REFERENCES

Abraham JE and Hunt JD (2002) Spatial market representations: concepts and application to integrated planning models. Presented at the 49th annual North American Meetings of the Regional Science Association International, San Juan, Puerto Rico, November.

Abraham J (2000) **Parameter Estimation in Urban Models: Theory and Application to the MEPLAN Model of Sacramento.** PhD Thesis, University of Calgary Department of Civil Engineering, Calgary AB, Canada.

de la Barra T (1998) The mathematical and algorithmic structure of TRANUS. Unpublished paper available at http://www.modelistica.com

DKS, Mark Bradley Research and Consulting, HBA Specto and Bowman JL (2001) Land Use and Transport Modeling Design Report. Prepared for Sacramento Area Council of Governments, Sacramento CA, USA.

Hunt JD (2001) The 2nd generation Oregon statewide land use transport interaction model. **Proceedings of the 7th International Conference on Computers in Urban Planning and Urban Management,** Honolulu HI, USA, July.

Hunt JD and Simmonds DC (1993) Theory and application of an integrated land-use and transport modelling framework. **Environment and Planning, Volume 20B,** 221-244.

PB Ohio and Hunt Analytics (1999) Ohio Statewide Travel Demand Model Final Report, PID 17803. Prepared for Ohio Department of Transportation Office of Technical Services, Columbus OH, USA.

Simmonds DC (1996) DELTA Model Design. David Simmonds Consultancy, Cambridge UK.

Waddell PA (2002) UrbanSim: Modeling urban development for land use, transportation and environmental planning. **American Planning Association Journal,** Volume 68(3), 297-314.