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Optimal Design of a Financially Sustainable HOT Network

August 2010

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1. INTRODUCTION

A number of demand management strategies are being considered to counter the rapid growth in transportation related problems such as traffic congestion, air quality and increasing operating costs. One of the most recent of these strategies directed towards congestion alleviation is the implementation of High Occupancy/ Toll (HOT) lanes. HOT lanes combine the concepts of congestion pricing and High Occupancy Vehicle (HOV) lanes by offering Single Occupant Vehicles (SOVs) priced access to the carpool (HOV) lanes. These lanes, thus, provide an opportunity to use both price and vehicle occupancy as means for managing traffic as opposed to the HOV lane where only vehicle occupancy is used as a control mechanism [1]. There are currently seven such HOT facilities operating at different locations in the United States. In addition to generating much needed revenues, these projects have been able to improve the performance of the system with respect to a number of measures such as revenue, total cost, total vehicular time and so on.

The success of the existing HOT lanes in realizing the objectives has engendered considerable interest in the concept across the country. The HOV lanes in a number of regions including California, Texas, Washington, Florida and Oregon are now being examined for upgrading them to HOT lanes [1]. In addition to the benefits mentioned above, the underutilization of the HOV lanes, evinced in the form of "empty lane syndrome" at a number of locations, has furthered the case of HOT lane implementation [2].

This interest in turn has necessitated development of methodologies for evaluating the conversion of a HOV lane into a HOT lane. As discussed in [3], there are numerous and

diverse factors influencing this decision on conversion. These factors may broadly be categorized as facility, performance and institutional considerations. The focus of this study is to investigate the potential improvements resulting from the conversion only in terms of performance. As a part of this study, an optimization model is developed in order to quantify the benefits in terms of various measures under different objectives that might, at times, be competing. The basic output of the optimization model here describes the pricing strategy to be followed, which can then be interpreted to determine the optimal operation strategy, i.e. whether to operate the existing lane as a mixed flow lane, HOV, or HOT lane.

The model developed as a part of this study is intended to act as decision support for evaluating the conversion of an HOV lane on a given corridor. A behavior model describing user behavior under pricing was estimated and embedded into the model as constraints. The model was then used to determine the impacts of converting the HOV lane into an HOT lane on a selected stretch of I-80 in the San Francisco Bay Area.

A salient feature of the model is the explicit incorporation of equity into the planning process. A set of equity constraints limiting the inequity in different dimensions were imposed and an analysis of the loss in efficiency that results from improving the equity was conducted. Another interesting analysis that was carried out involved examining the differences in a number of variables as the planning agency's objective changed. In addition to the above, the propensities of different user groups to use the managed lane were also examined. This model was then extended to construct two multistage models that solve for the optimal conversion schedules when HOV lanes on more than one corridor are to be converted.

The specific objectives of this study are:

i. To develop a methodology for quantifying the impacts of converting an HOV lane into a HOT lane by incorporating equity considerations at the planning stage itself.

ii. To analyze the impact of varying levels of equity on the efficiency of the HOT lane.

iii. To analyze the differences in pricing strategies, managed lane use propensities and other performance measures as the objective of the planning agency varies.

iv. To develop a multistage modeling approach that would solve for optimal sequences/ schedules for conversion of multiple HOV lanes.

The rest of this report is organized in the following manner. The next chapter reviews the current measures that are being adopted in order to alleviate the congestion problem. The relevant literature on HOV lanes, congestion pricing and HOT lanes is reviewed and this study is situated appropriately. Chapter three elaborates the methodology that is followed for development of the decision support model. The various objective functions and different types of constraints are described here. A case study applying the above model to a selected corridor is presented in Chapter four. The pricing strategies and impacts of conversion under different objectives are discussed. The trade off between equity and efficiency is also dealt with in this chapter. The next chapter examines the issue of optimal sequencing/ scheduling of multiple HOV to HOT conversions. Two Dynamic Programming formulations which can be solved to obtain the self-financing sequence and

the sequence that minimizes external funding are described. The last chapter includes the

inferences from this study and a few directions for future research.

2. LITERATURE REVIEW

The costs associated with the transportation-related byproducts of economic growth have been increasing at a rapid pace in the recent past. For instance, the cost of congestion[1] in 85 metropolitan areas of the nation jumped from $12.5 billion in 1982 to $63.1 billion in 2003 [4]. This may be attributed to the burgeoning growth in the demand for transportation infrastructure and as noted in [4], the supply has not been able to keep pace with the demand.

The various efforts towards alleviating road congestion, as presented in [5], may broadly be grouped into three categories:

a) *Supply-side measures:* These measures are primarily concerned with adding more capacity to the system. The additional capacity may take the form of new roads, additional lanes, new transit lines and so on. Supply side measures are the most apparent and widely used measures geared towards congestion mitigation. Conventional wisdom, however, suggests that it is not possible to build a way out of congestion and thus the scope of supply side measures is limited. This is especially so in urban areas owing to higher land costs and opposition from various groups. In addition to these, the impact of adding capacity on urban roads might not always be beneficial as it might lead to generation of more trips, i.e. an increase in demand, or lead to an increase in the travel times as exemplified in the Braess' paradox phenomenon,.

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[1] These numbers are a quantification of only the delay and extra fuel consumed due to congestion and do not include other effects such as worsening of air quality, lower reliability of travel, opportunity costs of missed activities and so on.

b) *Operational improvements:* This class of efforts towards improving road conditions may be described as "getting more out of what we have" [5]. These measures focus on improving the efficiency of the existing infrastructure by improved management of short-term demand and by mitigating effects of road incidents on traffic. Operational improvement measures include ramp metering, signal timing optimization, incident management, restrictions on lane and intersection usage, improvements in road geometries, and prominently, a number of Intelligent Transportation Systems (ITS) applications. These measures may be thought of as improving the return on the investment and as reported in [4], can have a significant impact on delay reduction. However, the benefits of these approaches are limited by the maximum possible efficiency of the existing infrastructure and as such, it will not suffice to deploy these measures on their own.

c) *Demand management strategies:* These measures involve altering the demand for the transportation facilities by inducing behavioral changes with respect to travel decisions. A wide range of strategies are grouped under this category and are directed towards improving transit usage and vehicle occupancy (HOV lanes, transit improvements, etc.), changing mode choices and time of travel (flextime, pricing, fuel taxes, bike/ transit integration, telecommuting, pedestrian improvements, etc.) and proper land use management (parking management, smart growth reforms, transit oriented design, etc) [6], [5]. The main obstacle for implementation of demand management strategies stems from the fact that the

effectiveness of these measures depends on changing the lifestyle patterns of general populace and the trends of markets.

A comprehensive taxonomy of the congestion alleviation measures can be found in [5].

2.1 HIGH OCCUPANCY VEHICLE LANES:

The concept of rationing road space for High Occupancy Vehicles (HOV) is one of the primary demand management strategies that are currently being implemented with the aim of alleviating congestion. A change in the American lifestyle towards greater individualism has contributed significantly to the increase in the percentage of Single Occupant Vehicles (SOV) over the years [7]. This phenomenon in turn resulted in consumption of more resources for transporting fewer people.
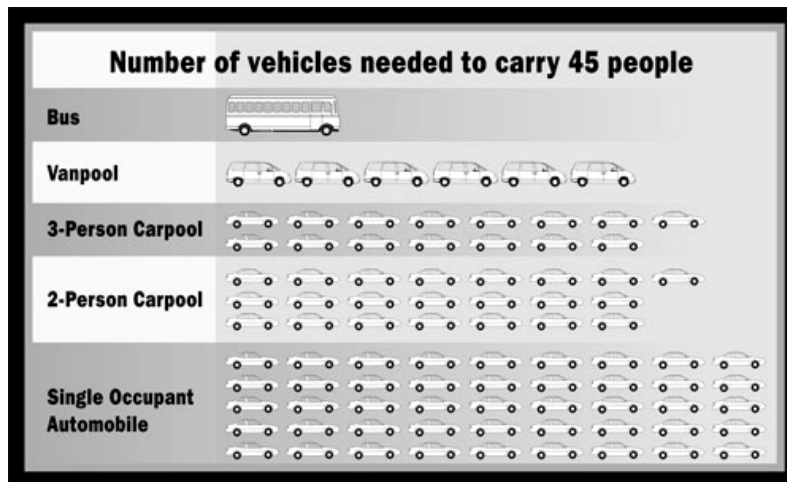

Figure 2.1: Number of Vehicles Needed to Carry 45 People[2]

HOV lanes are a type of managed lanes, wherein the access is limited to only the vehicles that meet the person occupancy criteria. The implementations of HOV lanes were an attempt at checking the drop in the HOV mode share [8] and increasing the number of

[2]Source: FHWA (http://ops.fhwa.dot.gov/publications/exemptvehicleshov/chapter2.htm)

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persons per vehicle. The primary objective here was to provide improved services to HOVs and encourage carpool formation (and transit usage) by reducing travel time and by improving trip time reliability for such vehicles. Other objectives for HOV lanes include improving overall system-wide travel times, improving the efficiency of public transit services and reducing fuel consumption [9].

The first major HOV project in the U.S. was implemented on the Shirley Highway (I-395) in northern Virginia in 1969 [10]. There has been a steady rise in the number of HOV facilities ever since and different versions of these projects have been implemented all over the United States. As of now, there are 126 facilities spread across 27 metropolitan areas in the US and more are being planned [11]. A complete inventory of HOV lane projects in the US can be found at http://www.hovworld.com/. As noted in [12] and [13], there are a number of instances wherein the HOV lanes proved to be a valuable addition by encouraging carpooling and improving the vehicle occupancy levels. However, the effectiveness of the HOV lanes has been limited in a number of other areas such as New Jersey where a lane was closed in 1998 owing to lower carpool utilization [12]. Analysis in [13] revealed that a HOV lane would be worth only in a narrow range of conditions. The results of this analysis suggested that a HOV lane would be better than a general purpose lane only when there is a high proportion of HOVs and when there is a high volume of traffic. Consequently, the higher priority accorded to HOVs has led to these lanes being underutilized giving rise to the "empty lane syndrome" occurring when a congested general lane is adjacent to a free flowing HOV lane. [14] analyzed the California HOV system, which incidentally is one of the most extensive in the nation, using empirical data from the Freeway Performance Measurement System (PeMS)

database. It was found that the HOV lanes offer few benefits and are often underutilized or suffer from degraded operations. The operation of HOV lanes has been questioned in a number of regions including New Jersey, Twin Cities (Minnesota), Long Island and Virginia. Furthermore, as quoted in [3] and [4], the issues regarding the environmental impacts and returns on other alternatives to HOV lanes are still not resolved.

On the whole, although there are a number of instances of successful HOV lane operation, there does seem to be a need for efficient utilization of the capacity offered by the HOV lanes in some of the regions.

2.2 CONGESTION PRICING:

Congestion pricing represents a widely advocated example of the Travel Demand Management strategies. The concept of road pricing, first proposed by Pigou in 1920, has long been propounded by economists in order to achieve higher efficiency in the usage of transportation infrastructure [15]. Vickrey [16], for instance, stated that "in no other major area are pricing practices so irrational, so out of date, and so conducive to waste as in urban transportation". Congestion pricing is proposed as a means for cutting down on these inefficiencies occurring in the transportation system.

It has been argued that users should be charged their external marginal costs which are given by the difference between the actual social costs imposed by the user and the individual trip cost experienced [17, 18]. The additional costs imposed by the additional user on society include higher travel times, higher wear and tear, increased emissions and so on [19]. The basic idea is to make the users cognizant of the true cost of their trips and

thus encourage only the trips whose benefits outweigh the total costs [18]. This marginal cost congestion pricing has been frequently referred to as first-best congestion pricing. There are, however, a number of problems associated with the implementation of this first-best pricing. These include difficulties in computing optimal tolls in real world scenarios, political opposition, equity issues and other technological issues [17]. In light of these obstacles, research on implementing congestion pricing has focused on second-best pricing strategies to a large extent [20].

Implementation of second-best pricing strategies can be broadly divided into two categories [17], [21]:

a) Area-wide/ Cordon Tolling: This form of pricing involves charging users to use a congested part of the city. The tolls here can be variable (time/ distance based) or fixed and are to be paid at different entry locations. This type of pricing has been implemented in practice successfully at a number of locations. Notable examples of this form of congestion pricing include Singapore's area licensing scheme (peak period pricing), London's congestion pricing to enter the downtown area and more recently Stockholm's cordon pricing for the city center.

b) Facility Tolling: This form of tolling involves priced access to a single stretch of a road/ bridge or even one or some of the lanes of a given segment. This has been the predominant type of congestion pricing that has been operational in North America. The most common form of facility pricing being implemented in the US is HOT lanes. Examples of such projects are listed in the next section.

The main advantage of congestion pricing, as encapsulated in optimizing the objective function, is the improvement in the welfare level of the society as measured by total

travel time, total cost, total emissions and so on. Individual drivers and businesses would also be benefited by lower travel delays and improved reliability of the service [21]. Transit users and operators would similarly benefit due to improved speeds, reliability and reduced costs [21]. In addition to the above, pricing also generates a stream of revenue which could be used for improving the travel infrastructure of the region and/ or for redistribution purposes.

A number of studies have focused on the mathematical modeling of congestion pricing problems in transportation networks. Models solving for prices and tolling locations that optimize some measure of social welfare have been formulated and solution methods devised. These problems are usually formulated as a bilevel problem with the upper level being optimization of the system-wide objective and the lower level being the user equilibrium problem. The structure of the problems is similar to that of the well-studied Network Design Problem. Some of the studies that present formulations and solution algorithms to the pricing problem in transportation networks include [22], [23], [20], etc.

A number of other variations of the pricing problem incorporating multiple user groups [24], variable demand [25, 26], road space rationing and pricing [27], stochastic and dynamic equilibria [26] and so on have also been formulated.

Concerns about equity have also been incorporated into these formulations, albeit for small networks, by Yang et al. in [24] and [28] and by Sumalee in [29]. The equity-related constraints limited the cost incurred by each user group to be less than a certain threshold, which is a certain percentage more than the pre-pricing cost. Yang et al. [28] also analyzed and arrived at Pareto-improving pricing schemes for a small network under equilibrium conditions.

The advances in methodological and technological aspects notwithstanding, implementation of congestion pricing has not taken place in a manner commensurate with the accepted magnitude of the traffic problems. The problem here has mainly been the political and public acceptability of the concept [30]. "The implications of status quo bias and the invisibility of the prospective gains" [31] result in the existing conditions being favored over proposed improvements, especially when the changes involve paying for something which used to be free. The political acceptability of these projects is further hindered by the associated equity issues with pricing being seen primarily as benefiting the rich [30]. The idea that pricing is always regressive, however, has been refuted in studies such as [18]. Appropriate usage of revenues plays a very important role in shaping public opinion and the opinion can be turned around over time [32]. However, [30] and [33] note that full-fledged pricing might be difficult to implement and tolled access to HOV lanes for SOVs might be a way out.

2.3 HIGH OCCUPANCY/ TOLL LANES:

The low utilization of HOV lanes in some instances coupled with the necessity to improve efficiency through pricing has led to the coining of the HOT concept by Fielding and Klein [31]. The HOT lane concept represents an effort towards combining the essence of pricing and HOV lanes (i.e. higher priority to HOVs). HOT lanes allow HOVs at a reduced or no price (depending on the occupancy requirements) and provide priced access to SOVs.

The prices and occupancy restrictions may be thought of as control mechanisms that enable the HOT lane operator to manage the amount of traffic using the lane [1]. The congestion (or utilization) level of the managed lane can, thus, be controlled better in the

case of a HOT lane. In addition to effectively using the excess capacity on the HOV lanes, HOT lane implementation can potentially lead to improvements in a number of system performance measures such as total travel time, revenue, cost and so on. However, as noted in [34], setting the tolls to balance these objectives would involve certain compromises on the part of the planners. The other benefits accorded by HOT lanes include improved reliability of travel, generation of additional revenue, more trip options for users (with a free option still in place), transit improvements, etc. [1]. Apart from these, as noted in [31], HOT lanes can also act as an intermediate step for full-fledged pricing of the highways. As noted in the previous section, HOT lanes are the dominant form of congestion pricing that is being implemented in the US. As of February 2007, there are seven places in the US where HOT lanes are operational.

Table 2.1: Details of currently operational HOT facilities in the US

Location	Name	Length	Lanes	Occupancy	Pricing
Houston, TX	Katy I-10 QuickRide	13 mi	1	HOV2 toll/free off-peak, HOV3+ free, SOV prohibited	Flat $2 toll
Houston, TX	Northwest US 290 QuickRide	13.5 mi	1	HOV2 toll/free off-peak, HOV3+ free, SOV prohibited	Flat $2 toll
Minneapolis, MN	I-394 MNPASS	11 mi	2	SOV toll, HOV2+ free	Dynamic Pricing[3]
San Diego, CA	I-15 FasTrak	8 mi	2	SOV toll, HOV2+ free	Dynamic Pricing
Orange County, CA	SR 91 Express Lanes	10 mi	4	SOV toll, HOV3+ discount/free off-peak	Variable Pricing[4]
Denver, CO	I-25 HOT Lanes	6.5 mi	2	SOV toll, HOV2+ free	Variable Pricing
Salt Lake City, UT	I-15 Express Lanes	38 mi	2	SOV toll, HOV2+/clean-fuel free	$50 /vehicle/month

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[3] Dynamic Pricing: Prices vary by the level of traffic in order to maintain speeds on the managed lane.
[4] Variable Pricing: Tolls vary according to a predefined timetable that is known to the public in advance.

The locations, physical and operational details of the current HOT projects in the US are shown in Table 2.1 [35]. In addition to these, two other projects are being constructed in Houston and Maryland. A complete list of the HOT lanes projects that are under development can be found in [35].

A number of studies have investigated and evaluated the impacts of these HOT lanes. Sullivan and Burris [36, 37] conducted a benefit-cost analysis of SR-91 and the QuickRide projects for a period of ten years. The benefits and costs considered include travel time savings, fuel costs, emissions, capital costs and operating costs. The overall benefit-cost ratio was found to be 1.5 and 1.6 for the SR 91 and QuickRide projects respectively, with significant savings in travel time observed in both the projects. However, the benefits in terms of reduced emissions were found to be negative in both the projects. The results for the fuel costs were mixed with an increase in consumption in case of SR 91 and decrease for the QuickRide projects. On the whole, the net benefits, as evinced by the benefit-cost ratio, were positive in both the cases.

Studies examining different impacts of the I-15 project have also been conducted. The traffic-related effects of the project have been found to be beneficial on a number of counts by Supernak et al. [38]. These include better utilization of the managed lane, sufficient revenues and redistribution of volumes to the peak shoulders. It was also found that there was an improvement in the reliability of travel times and free flow conditions were maintained for most of the periods [39]. An evaluation study of the MnPass [40] also revealed beneficial effects of the HOT facility. The improvements that were observed include increase in the vehicle throughput of the corridor, decrease in the travel time on the general purpose lane and no negative impacts on CO emissions.

The public response to the HOT lanes has been positive in general and it was noted that people would be willing to pay in order to bypass congestion at times [1, 41, 40]. The various factors impacting the usage and acceptance of the HOT concepts have also been empirically studied. Li [42] analyzed data from the SR 91 project and inferred that income, commute trip, vehicle occupancy and age play a significant role in the user decision regarding usage of the HOT lane. Sullivan also conducted a study into the factors impacting SR 91 express lanes usage [41]. An analysis of the QuickRide programs' users behavior was conducted [43] and it was found that the carpool formation disutility acted as a major deterrent to the facilities' usage. Furthermore, it was found that perception of higher travel time savings, longer trips, college education and sharing of the tolls were found to increase the usage propensity. In addition to these, a FHWA report [1] on development of HOT lanes lists the following variables as factors impacting HOT lane use: toll, pricing structure, travel time on HOT lane, Value of Time (VOT), perceived HOT lane operating cost, costs associated with alternate means and routes, trip purpose and frequency, vehicle occupancy, risk profile, income and other demographic characteristics. Of all these variables, income, carpool formation cost, travel time savings, toll, operating cost and trip type have been incorporated into the behavior model of the current study, whose construction is described in the next chapter.

The benefits in efficiency notwithstanding, the equity issues of pricing have persisted in the case of HOT lanes and they have been disparaged as "Lexus lanes" [44]. HOT lanes have been perceived as elitist and imposing an additional burden on the poor. Equity in transportation projects is usually considered to be of two types: Horizontal equity, which deals with equal treatment to similar groups, and vertical equity, which requires the

policies to be skewed towards the needy and disabled [45]. The main equity issue which occurs in the context of HOT lanes is vertical equity [46]. User analysis studies of different HOT projects [36], [37], [39] revealed that though all income groups use the tolled facility, individuals from higher income groups are more likely to use them than those from lower income groups [47],[41],[48]. Interestingly, the HOT concept received approval from all of the income groups though. However, as noted in [46], redistribution of revenues would play a very important role in determining the equitability of the project. Thus, as noted in [46], a comprehensive approach that includes equity from planning to implementation needs to be adopted.

Another issue with the HOT lanes has been that the conversion into HOT lanes might increase the traffic on the managed lane thus leading to deterioration of conditions for HOV users [49]. This issue can easily be mitigated by controlling the price to influence the number of vehicles that will enter the managed lane in such a way that reasonable speeds are maintained [39].

The numerous positives from the current HOT projects have spawned significant interest in the concept and consequently, a number of agencies are considering conversion of HOV lanes into HOT lanes at different locations. A number of corridors in California, Texas, Florida, Oregon and Washington are being examined and feasibility studies conducted for implementation of the HOT lanes [9]. An up-to-date listing of the projects being developed can be found in [35].

This interest in HOT projects has in turn resulted in the creation of frameworks to aid in the conversion process. A wide range of information about the development of the HOT lanes can be found in [1]. Recognizing the nascent nature of the studies addressing the

conversion, a comprehensive sketch-planning tool has been developed by [3] in order to support the assessment of the conversion of HOV lanes. The various factors that need to be examined during the planning stage of the conversion were grouped into three different categories:

a) Facility considerations: This category includes factors related to facility cross section, lane separation, facility access, ease of enforcement, incident management and so on.

b) Performance considerations: Factors in this category include managed lane utilization, travel time savings/ reliability, societal benefits, environmental impacts and so on.

c) Institutional considerations: This includes factors such as political and public acceptance, revenue use, media relations and so on.

Each of the factors is then assigned weights and is scored based on the characteristics of the corridor with respect to the corresponding factor. The interactions between the factors were then quantified and a score assigned to each of the categories. These scores can then be used to make the decision regarding the conversion.

[50] is another study catering to the conversion's managerial aspect where a route map for conversion of the HOV lanes into HOT lanes is presented.

Given this framework, the current study may be positioned in the performance component of the conversion. As mentioned in the previous chapter, the focus of this study would be to evaluate the conversion in terms of the benefits and costs resulting from the conversion.

Other studies addressing the quantitative aspect of the planning process include [51], [52], [53]. Kim [51] studied the conversion of an HOV lane in a single corridor by assuming the tolls to be minimizing the system delay and concluded that HOT lanes are more beneficial to the system when compared to HOV lanes or general purpose lanes. Murray et al. [52] evaluated the impact of HOT lanes in a network by incorporating a logit model predicting mode choice into the DYNASMART model. A study using this methodology revealed that the system can be made more efficient by creation of HOT lanes. The tolls charged were obtained by scaling up the link density with a multiplicative factor. A sensitivity analysis of the improvement in travel time to various factors was also conducted. McDonald and Noland [53] used a simulation model (with a nested logit model) of a hypothetical corridor to analyze the effects of HOT lanes. The analysis here was conducted using a flat toll and the results suggested that HOT lanes provide the greatest mobility benefits among general, HOV and HOT lanes. Safirova et al. [49] studied the impacts of converting HOV lanes in Northern Virginia into HOT lanes by incorporating a toll of 20 cents per mile into the demand model (Washington-START mode) and concluded that all income groups gain from the conversion with higher income groups gaining more. These inferences are along the lines of the results obtained in this study. A number of planning agencies also conducted feasibility studies for converting the HOV lanes in their area into HOT lanes [50], [54].

In spite of useful insights from these studies, two important aspects that need to be addressed at the planning stage include:

1) Equity: As mentioned above, this has been a major impediment for implementation of the HOT concept. None of the above studies addresses this issue in the planning stage of the HOT development.

2) Pricing objectives and strategies: The potential benefits accrued from the conversion are very much a function of the price. However, this price has been chosen in a non-optimal manner (except in [51]) in most of the studies. Furthermore, the optimal pricing strategy itself is a function of the performance measure that needs to be optimized and thus, this issue warrants careful consideration.

The gaps related to managed lanes' planning literature have been discussed in [55] and the above two aspects were alluded to in the planning and policy research discussion. The methodology taken towards incorporating these issues in the current study is elucidated in the following chapter. The first distinguishing feature of this study is the incorporation of equity right at the planning stage. Equity, unlike in other studies, was considered explicitly along two dimensions - vertical and temporal equity. The incorporation of equity in the form of constraints here gives the planners an extra handle in limiting the inequity along different dimensions and according to different measures. This treatment also enables investigation of the relationship between equity and various measures of efficiency.

As noted above, the pricing strategy depends on the performance measure that the planning agency seeks to optimize. As a part of this study, the optimal pricing strategies corresponding to different objectives were computed for a chosen corridor. This enabled comparison of the trends in tolls and benefits across different objectives. Another

important contribution of this study is the development of a methodology that would solve for an optimal approach to converting the given HOV infrastructure into HOT lanes in a self-financing manner.

2.4 CHAPTER SUMMARY:

The relevant literature in the areas of HOV lanes and congestion pricing was discussed along with the pros and cons of these two demand management strategies. The concept and development of HOT lanes as a combination of HOV lanes and congestion pricing were then presented. Next, existing studies examining the benefits/ costs and the user characteristics of the HOT lanes currently operational in the US were reviewed. Studies dealing with the HOV to HOT lane conversion were discussed and this study positioned appropriately. The main contributions of this study to the literature in this area are the incorporation of equity in the planning stage, the analysis of equity versus efficiency and the analysis of multiple objectives for conversion.

3. MODELING METHODOLOGY

The decision about converting a HOV lane to HOT lane is rooted in wide ranging considerations such as potential benefits, public acceptance for the pricing concept, social equity, readiness of the operating agencies, lane geometry and other operational and policy factors. The scope of our study, however, is limited to quantifying the potential improvements in terms of system performance that could be brought about by such a conversion. Thus, the models here are intended to assist decision makers by giving them feedback on the pricing policies that correspond to optimal system performance measures. The models here are constructed for the situation where conversion to HOT lanes of the HOV lanes on a *single corridor* is being considered.

As mentioned earlier, the benefits and costs that accompany the conversion are a function of the choices that different users make when faced with a certain toll for using the carpool lane as a two-person carpool (HOV2) or as a single occupant vehicle (SOV). Thus, the effects of conversion to a HOT lane must be quantified according to the pricing strategy that would be adopted. As a part of this study, the prices to be set are treated as decision variables and a program that optimizes the planning agency's objective is solved. The problem here has a convex objective function and nonlinear (convex) constraints. A simplistic version of the problem formulation is shown below:

Optimize   *Objective Function*

Subject to,   *a) Constraints on lane travel times*

                 b) Constraints describing the Behavior Model

                 c) Equity related constraints

                 d) Constraints on tolls

Each of the above elements of this program is described in detail under the following sections. The above described optimization program was solved for the scenario where there is one HOT lane on which three-person carpools (HOV3) travel for free while HOV2s pay a reduced toll. The benefits/ costs in terms of a number of performance measures were then quantified and compared against the base case scenario where there is only a HOV lane. Such a scenario-based analysis was performed for a chosen corridor in the next chapter.

Note that the pricing strategy would also give the definition of the carpool that needs to be adopted. For instance, an output of a very high toll for two-person carpools would imply that the HOT lane needs to be operated as a HOV3 carpool lane excluding SOVs and HOV2s. Similarly, a low toll for HOV2s and a very high toll for SOVs would imply that SOVs are not to be allowed to buy into the managed lane while HOV2s may be allowed at a price.

3.1 OBJECTIVE FUNCTIONS:

As far as the objective functions are concerned, different operating agencies might have different measures of performance for operating managed lanes. It is also possible that the same agency might have multiple objectives such as to minimize the total travel time, maximize the revenue or to minimize the emissions and so on. The agency would, thus, need to implement different pricing strategies. The pricing strategies corresponding to the following objectives were considered for evaluating the benefits/ costs of the conversion:

   a) Minimize Total vehicular travel time: This is a common objective function that is
      used in a number of planning studies and can be obtained by summing up the

travel time experienced by all the vehicles on the road segment under consideration.

b) Minimize Total passenger time: This can be computed by summing up the travel time experienced by each individual user.

c) Maximize Total revenue: The total revenue is given by the total toll money that can be collected using a certain pricing strategy.

d) Minimize Total number of vehicles: One potential objective for the agency might be to reduce the total number of vehicles that are on the corridor. Minimizing the total number of vehicles here is the same as minimizing the total VMT on a corridor, another commonly used objective. However, this function was found to behave very much like the total vehicular time for the case study that will be described in chapter four.

e) Minimize Total user cost: The cost for a user here is the equivalent expected dollar cost that can be obtained from the probabilities and the costs of different alternatives in dollars. This cost here is an aggregate of a variety of costs such as carpool formation costs, travel time costs, tolls and so on. Further details about the costs are included in the behavior model discussion.

The implementation of the optimization model with each of the above objectives will be referred to as a program from here on. For instance, the above model with revenue maximization as the objective will be referred to as revenue maximization program.

Another transportation system performance measure that was considered for experiments here was the total vehicular emissions. However, this measure could not be approximated with a convex objective and had to be left out of the objective functions set. Instead, the

emissions were estimated under different programs in order to help decision makers evaluate the impacts on air quality. A brief discussion on the quantification of emissions under each scenario is presented at the end of this chapter.

3.2 CONSTRAINTS ON LANE TRAVEL TIMES:

These constraints describe the following elements of the model:

a) The relationship between the volume and the capacity on the general purpose as well as the HOT lane. For the purpose of this study, the BPR function was assumed to capture this relationship.

$$t = t_0(1 + a(\frac{v}{c})^b),$$

where t = travel time on the lane,

t_0 = free flow travel time on the lane (travel time at speed limit),

v = volume on the given lane,

c = capacity of the lane (assumed to be 1600 vph [51]), and

a, b = BPR parameters (obtained from PEMS[5] for the given segment).

b) The quality of service (travel time) on the HOT lane. The idea behind the service constraint is to maintain a certain level of service on the managed lanes. In other words, it assures the users that their travel time on the HOT lane would be less than a certain threshold. The threshold for this study was set at the corridor travel time corresponding to 50mph. These constraints also ensure that the travel time on the HOT lane is always no greater than that of the general purpose lanes.

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[5] PEMS web address: https://pems.eecs.berkeley.edu/

3.3 CONSTRAINTS DESCRIBING THE BEHAVIOR MODEL:

A behavior model is embedded in the optimization model as constraints to predict the choices of different classes of users over regular and managed lanes upon implementing a certain pricing regime on the managed lane. Given the attributes of an individual and those of the alternatives, the behavior model gives the probability of an individual choosing the given alternative. In order to reflect the heterogeneity of the corridor's users, the users were categorized into different classes based on the following attributes:

a) *Income:* Individuals were categorized into four different quartiles according to their hourly wage rate. The categorization was necessitated by the well documented higher Value of Time (VOT) for individuals with higher incomes, which might translate into a preference for reducing travel time through paying tolls. The income distribution of all the corridors here was assumed to be the same as the income distribution of the study region – the San Francisco Bay Area. The values for the 10th, 25th, 50th, 75th and 90th percentiles of the incomes in this area were obtained from the Bureau of Labor Statistics website [56] and an income distribution curve was fitted with an R^2 of 0.997.
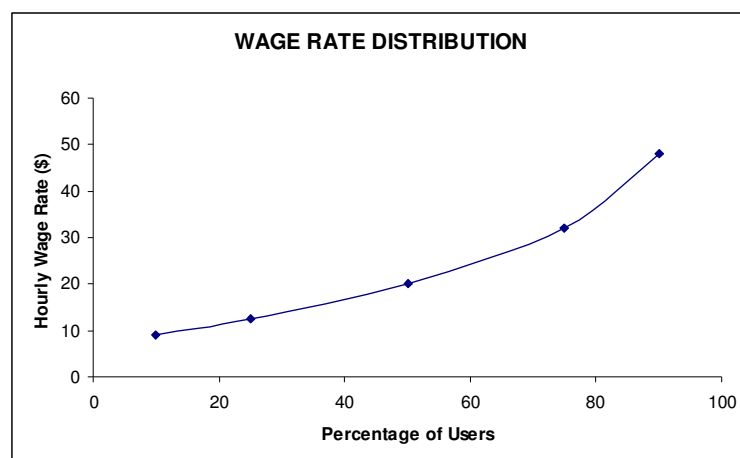

WAGE RATE DISTRIBUTION

Figure 3.1 Income Distribution (2006) curve in Bay Area

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The curve shown in Figure 3.1 was then used to divide the users into four quartiles based on their hourly wage rates - $43.49 per hour, $26.11 per hour, $15.68 per hour and $9.42 per hour.

b) *Trip type:* Corridor users were further classified into four classes based on the type of their trip. The rationale behind this classification was the difference in the VOT attached by the same user to different kinds of trips. For instance, an individual making a work trip is much more likely to pay for the better service on HOT lanes when compared to the same individual on a shopping trip. The distribution of trip types for the Bay Area that was obtained from the BAYCAST-90 summary [57] and it was assumed the traffic on all the corridors of the study area was similar to the following composition.

Table 3.1: Distribution and Value of Time for different trip types

Trip Type	% of traffic	VOT (as % of hourly wage)
Work	40.37	46.40
Shopping and Social	29.33	23.00
School	12.40	2.00
Other	17.90	5.20

The average VOT resulting from the above trip type distribution was found to be 26.67% of the wage rate.

c) *Carpool formation cost:* Users were further classified into four different categories based on the carpool formation cost, which corresponds to the extra amount of time an individual needs to spend in order to form the carpool. Such time would include

time spent on pick up and drop-off of the rideshare partner(s). As mentioned earlier, this cost plays an important role in the carpooling tendencies of the individual and can vary significantly from individual to individual. The average carpool formation time for a HOV2 is about 7.2 minutes and for a HOV3 is 11 minutes according to a survey conducted in the Bay Area [58]. The exact distribution of this cost for Bay Area users was not available. Hence, this distribution was estimated using the data from a similar survey conducted in Texas. A distribution of this cost was reported for Houston in [59], with an average value of about 6.18 minutes. The distribution in [59] was then scaled up accordingly and the users were divided into four quartiles. A similar procedure was followed for obtaining the distribution of three-person carpool formation time.

Note that the above carpool formation times were reported by carpool users alone and, thus, do not represent the inconvenience costs for current SOV users to form carpool. In order to account for the costs to SOV users, the above values need to be scaled up. The carpool formation time for Los Angeles (LA) users was assumed to be eight minutes on average in [60]. The average for LA users including SOV users was assumed to be 15 minutes in [32]. The ratio of these two values was used to scale the distribution shown proportionately. The final carpool formation costs incurred by SOV users for each type of carpool are shown in table 3.2.

Table 3.2: Distribution of carpool formation costs

Quartile	HOV2 cost	HOV3 cost
I	0	0
II	0.047 hr	0.073 hr
III	0.212 hr	0.323 hr
IV	0.664 hr	1.014 hr

Thus, the total number of classes into which the corridor users have been classified is 4×4×4 = 64. Distinguishing users in different categories allows for determining the losers and winners under each of the scenarios. For instance, this treatment would allow us to quantify, on average, the travel times experienced by the rich and the poor under each of the scenarios and thus aid in analyzing the important vertical equity issues. In addition to the above, as opposed to most of the other studies where a single average VOT is assumed, this study incorporates a distribution for VOT by allowing for heterogeneity in users' incomes and trip types.

In the absence of a full fledged stated preference data set for assessing the user response to different pricing regimes, a logit-based behavior model was constructed by enumerating the costs that an individual attaches to various alternatives. Accordingly, the probability that an individual belonging to class i chooses alternative j is given by:

$$P(i, j) = \frac{\exp(-\beta C_{ij})}{\sum_j \exp(-\beta C_{ij})},$$

where C_{ij} is the equivalent dollar cost of alternative j for user class *i*, and

$\quad\quad \beta$ is a scaling coefficient that needs to be estimated.

The above model may be thought of as a logit model in which the only variable is the total dollar cost of an alternative for an individual. The total cost experienced by the user for different choices is constituted by the following elements:

i) *Travel time cost:* This is simply the cost of travel time corresponding to the particular alternative, converted into monetary units based on the income group and the importance of the trip.

ii) *Toll cost:* This cost consists of the toll the individual pays in the tolled options and is zero for the non-toll options. It is assumed here that the members of the carpool share the toll costs, if any.

iii) *Carpool cost:* The carpool formation cost, as discussed above, is simply the extra time needed to form a carpool and would thus depend on the number of persons forming the carpool.

iv) *Time shift cost:* The cost incurred by users who shift their trip times from their desired time to a different period is quantified using this. This cost is assumed to be 100% of the hourly wage rate for shifting an hour of travel time. This estimate was obtained from [61]. However, preliminary model runs using this cost indicated it is highly improbable for users to shift their time even by half an hour. This is because of the fact that this cost dominates all the other costs and consequently, the impedance attached to the corresponding alternatives is much larger in magnitude. Thus, for the rest of the study, the users were assumed to be traveling at the same departure times across all the scenarios. This, however, may only be partly true in a number of situations. For instance, [41] suggests that while there was a change in the magnitude and length of the PM peak period, there was very little shifting during the AM peak. Another difficulty in quantifying this cost stems from the large variation in the estimates of this cost, which ranged from 2-3% to 300% of the hourly wage rate [61], [62], [63].

v) Operating costs: These constitute the costs associated with operating a vehicle for the trip distance including fuel cost and parking. This cost is assumed to be shared by the members of the carpool. However, lack of data on trip distances, parking cost distribution and other hidden costs necessitates treating this cost as a parameter to be estimated. The procedure for estimating the scaling coefficient β and the operating costs is presented below.

Given that only HOV3 vehicles can use the HOT lane for free, the various alternatives a traveler faces have been enumerated and the applicable costs are shown in Table 3.3.

Table 3.3: Details of alternatives and costs after conversion

Alternative	Travel time costs	Toll costs	Operating costs	Carpool costs
With toll as SOV on HOT lane	t_1	T_1	OC	0
Without toll as SOV on regular lane	t_2	0	OC	0
With toll as HOV2 on HOT lane	t_1	$T_2/2$	$OC/2$	$CC2$
Without toll as HOV2 on regular lane	t_2	0	$OC/2$	$CC2$
Without toll as HOV3 on HOT lane	t_1	0	$OC/3$	$CC3$

where t_1 = travel time on HOT lane,

t_2 = travel time on general lane,

T_1 = toll imposed on SOVs,

T_2 = toll imposed on HOV2s,

OC – operating cost,

CC2 – two-person carpool formation time, and

CC3 – three-person carpool formation time.

3.3.1 Estimation of Scaling Coefficient (β) and Operating Costs (OC):

The coefficient β along with the vehicle operating cost (OC in Table 1) will be estimated using the pre-conversion choice data (i.e. data from the HOV lane scenario). The absence of disaggregate data on the vehicle occupancy choice of different individuals necessitated using aggregate data. The parameters here were estimated using the overall mode split between carpools and SOVs during the peak period. The modal split was used as a proxy for revealed choices of a "representative" individual, i.e., the modal shares were assumed to be the probability with which the representative individual would choose each of the alternatives. The alternatives that exist for this individual before effecting the conversion are SOV, HOV2 or HOV3.

At the first step, the costs associated with each of the modes will be computed for this user. The carpool costs will be computed using average values for VOT and carpool costs. In order to compute the travel costs, the total vehicular demand can first be obtained from the PEMS database for a particular segment. The modal split on this segment in conjunction with the BPR function can be used to compute the travel times on the general purpose and carpool lanes. The toll costs before conversion are zero. Note that computation of OC would need data on the trip distances and lack of this data necessitates estimation from the revealed choice data. The following table shows the alternatives and the corresponding costs in the pre-conversion scenario.

Table 3.4: Details of alternatives and costs before conversion

Alternative	Travel time costs	Toll costs	Operating costs	Carpool costs
SOV on general lane	t_2	0	OC	0
HOV2 on general lane*	t_2	0	$OC/2$	$CC2$
HOV3 on carpool lane	t_1	0	$OC/3$	$CC3$

(* Assuming HOV2s are not allowed on the carpool lane)

The estimates for β and OC can then be obtained by solving two equations that set the probability of choosing each alternative to be the existing market shares of these alternatives:

$$e^{-\beta(C_1-C_2)} = \frac{x_1}{x_2} \quad \text{and} \quad e^{-\beta(C_1-C_3)} = \frac{x_1}{x_3},$$

where β = scaling coefficient to be estimated,

C_i = cost (in $) of choosing alternative i (vehicle occupancy i), and

x_i = modal share of vehicle with occupancy i ($x_1 + x_2 + x_3 = 1$).

The operating cost OC is embedded in the cost corresponding to each alternative and is obtained, along with the scaling coefficient β, as a solution to the above two equations. An instance of the above described procedure has been constructed in chapter four (case study) and the estimate for β was found to be comparable to the β in one of the models in the literature [64].

3.3.2 Additional Notes on the Behavior Model:

a) The travel costs on other parts of the trip beyond the studied corridor are assumed to be the same for all the alternatives. In other words, there is no special treatment given to any of the alternatives in the rest of the user's trip which might lead to additional cost

components. This assumption would enable leaving out the costs corresponding to the other parts of trips since these costs would cancel out. Hence, the individual's choice would depend only on the impedances of the alternatives on the corridor being examined. One such situation where this assumption would not hold is the case where there are carpool lanes elsewhere in the journey. The costs associated with carpool alternatives would then be lower than the SOV alternative's cost. The impact on the estimation of ignoring the effect of these carpool lanes is that the estimate for parameter OC would be lower than the case when there are no carpool lanes. The OC term now would be required to incorporate the lower costs associated with the carpool alternatives as well, thus pushing the estimate downwards.

b) An important assumption here is that users on the corridor continue to use the same route even after the conversion. This assumption might hold reasonably well in situations where the alternative routes involve a significant amount of impedance of any kind.

c) The above described model is an attempt at capturing the essential elements of a choice model that should be obtained from a Stated Preference survey and only a model based on survey data would provide a basis for drawing robust conclusions.

3.4 EQUITY RELATED CONSTRAINTS:

One of the significant criticisms levelled against the HOT lane concept is the idea that they favor the rich. Equity constraints are introduced to place bounds on the potential inequities of welfares between different income groups. The welfare of each income group for the purpose of equity is quantified using two measures:

a) Average travel time: The weighted average of travel times experienced by all 16 groups in each of the four income groups is used as one of the measures to quantify the welfare of the four income groups.

$$T_i = \frac{\sum_j \sum_k \sum_l n_{ijk} P_{ijk}(l) t(l)}{\sum_j \sum_k n_{ijk}},$$

where T_i = average travel time for income group i,

$P_{ijk}(l)$ = probability that a user of income group i, on trip type j and belonging to carpool cost group k will choose alternative l,

$t(l)$ = travel time experienced when alternative l is chosen, and

n_{ijk} = number of users belonging to the group defined by income group i, trip type j and carpool cost group k.

b) Average travel cost: This measure quantifies the expected impedance experienced by an income group on average. The expected travel costs experienced by all 16 groups in each of the income groups are computed first. The dollar cost thus obtained is then converted into its time equivalent for that particular individual by scaling it down using the VOT of that group:

$$C_i = \frac{\sum_j \sum_k \sum_l (n_{ijk} P_{ijk}(l) c_{ijk}(l)) / VOT_{ij}}{\sum_j \sum_k n_{ijk}},$$

where C_i = average travel cost (in time units) for income group i after conversion,

VOT_{ij} = value of time for individual belonging to income group i and on trip type j,

$c_{ijk}(l) =$ monetary cost associated with alternative l defined by income group i, trip type j and carpool cost group k, and all of the other variables are as defined above.

The conversion into time units is performed in order to provide a uniform measure of costs according to which all the groups' welfare can be judged. The dollar costs incurred by higher income individuals will be higher than those for the lower income users because of the higher VOTs. The dollar costs, if directly used, would wrongly indicate that the rich are experiencing higher costs and thus, the policies directed at reducing the costs would be skewed in favour of the rich. Costs in terms of time units, on the other hand, would provide a more uniform measure to quantify the welfare of each income group. Note that the travel time costs are only a part of the average cost and there are other costs (toll, carpool formation cost, operating costs) which influence this variable.

Equity constraints in the model attempt to limit the inequities with respect to the above two measures. Equity constraints can further be classified into two types based on the dimension they address – Temporal or Vertical equity.

The conversion project here is said to be *temporally equitable* if the conversion results in the situation where the future users belonging to each income group are at least as well off as they were before the conversion. In other words, perfect temporal equity refers to the condition where the average costs of each income group are non-increasing with time. Temporal equity thus involves comparing the welfare of individuals belonging to each income group before and after the conversion. Constraints corresponding to temporal equity specify that all of the income groups should be better off when compared to their

states of welfare before the conversion. Using the same notation as defined above, the general form of temporal equity constraints is shown below:

$$C_i \leq C_i^0, \forall \text{ income groups i,}$$

where C_i^0 = average travel cost (in time units) for income group i before conversion.

Similar temporal equity constraints could also be imposed with respect to travel time. A relaxed version of the above constraint can be given in the following manner. Further analysis with this relaxed form will be carried out in Chapter four for a particular corridor.

$$C_i \leq (1 + x/100)C_i^0, \forall \text{ income groups i.}$$

Vertical equity, on the other hand, is concerned with the welfare of the users only during the post-conversion period. The principles of vertical equity require the policies to favour individuals who are at a disadvantage such as individuals with low incomes, minorities, disabled and so on [45]. In the context of this study, vertical equity takes form as the difference in the benefits/ costs incurred by each income group. Constraints corresponding to vertical equity limit the average benefits/ costs across different income groups and are intended to reduce the spread in these benefits/ costs. The general form of vertical equity constraints is:

$$C_i \leq (1 + \theta)\overline{C}, \forall i \text{ and } C_i \geq (1 - \theta)\overline{C}, \forall i \text{ , i in income groups,}$$

where θ = parameter to be specified by the planners beforehand, and

$\overline{C}$ = mean travel cost across all the income groups.

The above constraints limit the average travel costs of each group to be within a certain percentage of the overall mean. This treatment may be thought of as constraining the maximum difference across groups to be less than a certain fraction of the overall mean.

The impact of changing θ on the efficiency loss has been studied for a specific corridor in chapter four.

Note that the unit of analysis here is the income group and thus temporal equity does not imply that all the users (belonging to all 64 groups) gain from the conversion. It is possible that the average measures corresponding to each income group improves but there are both losers and winners within each income group.

A simpler way of incorporating equity concerns into the model is to use a weighted objective that attaches appropriate weights to the terms associated with different groups in the objective function. For instance, a modified revenue function can be defined a lower weight can be assigned to the revenue from the lower income groups while a higher weight is attached to the higher income groups. This 'weighted revenue' function can then be maximized instead of the regular revenue function to obtain a toll regime that is more equitable to the lower income groups.

The advantage of addressing equity in the form of constraints rather than as weights in the objective is the control achieved by directly imposing limits on the extent of benefits/ costs' distribution. The actual benefits/ costs of each group in the weighted objective approach may not exactly reflect the desired distribution.

3.5 CONSTRAINTS ON TOLLS:

The constraints imposed on the tolls to be set include the non-negativity bounds on each of the two tolls. Additionally, the toll for HOV2s is constrained to be less than or equal to the toll for SOVs. Another potential constraint that can be imposed on the tolls, if necessary, could set upper limits for each of these tolls.

In addition to all the above constraints, a set of constraints that describe the current conditions were placed. These constraints were imposed only for computation of the initial conditions and do not affect price setting as such. The whole of this model was coded in AMPL (code shown in Appendix A) and the optimization solvers available on the NEOS website were used to solve for the optimal tolls. Note that the problem here is a convex (nonlinear) optimization program.

It is acknowledged that the model described in this chapter is a rather simplistic one and does not account for network effects, elastic demand, route and time shifting. However, as discussed in the following chapters, a number of useful insights can be obtained from this model.

3.7 DETERMINING THE IMPACT OF CONVERSION ON EMISSIONS:

The emissions corresponding to the before and after conversion scenarios were computed using the MOBILE6 software. MOBILE6 is an emission factor model and computes the amount of pollutant per unit of travel (grams per mile traveled). The impact of conversion on the emissions of Volatile Organic Compounds (VOC), Carbon monoxide (CO) and NO_X was studied using a simple model.

The composite emission factor or the amount released per mile of travel for each type of emission depends on a number of variables which include vehicle speed distribution by hour and type, VMT distribution by vehicle type and roadway type, diesel sales fractions by vehicle type and age and other site specific characteristics such as altitude, humidity, etc. The complete list of variables impacting these factors can be found in the MOBILE6

manual [65]. As a part of this study, the values for all the input parameters except for speed and facility type were set at the national default values provided in the model. The facility type was set as freeway.

All three emission factors were then estimated at different speeds using the MOBILE6 model software. It was assumed here that the traffic is composed of only one stream with all the vehicles moving at the same speed. The following graphs (Figure 3.1) show the relationship between speed and emission factors for NO_X, VOC and CO.
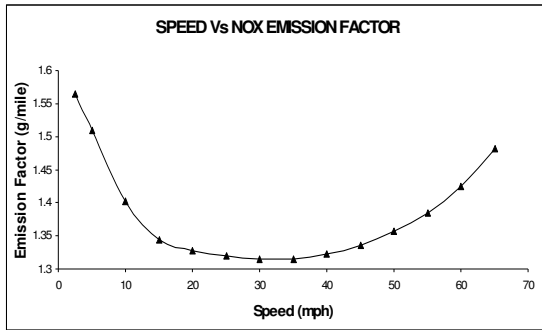
Figure 3.1 (a)   Figure 3.1 (b)
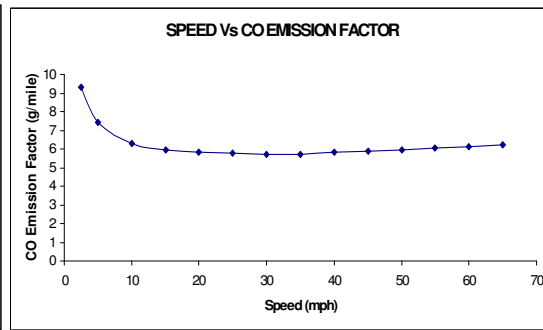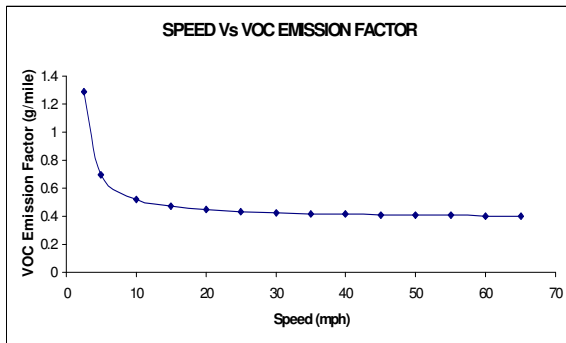
3.1(a) Plot of Speed Vs NO_X Emission Factor
3.1(b) Plot of Speed Vs CO Emission Factor
3.1(c) Plot of Speed Vs VOC Emission Factor

Figure 3.1 (c)

While the VOC emissions per mile travel decrease first and then essentially level off with an increase in speed, the NO_X emissions first decrease and then increase with speed. The lowest rate of NO_X emissions seem to be occurring at 37.5 mph. The CO emissions per mile first decrease and increase at a rate very small compared to that of the NO_X

emissions. The lowest emissions seem to be occurring at 35 mph. These patterns in are in accordance with those in [66]. The above relationship between speed and emissions was then used to compute the quantities of emissions both before and after conversion in the following manner.

$$\text{Quantity of emission X (in kg)} = \eta_X(s_1).VMT_1 + \eta_X(s_2).VMT_2$$

where, $\eta_X(s)$ - emission factor (of X) at speed s,

s_1 and s_2 – speeds on managed and general lanes respectively, and

VMT_1 and VMT_2 – Vehicle Miles Traveled on managed and general lanes.

The speeds and VMTs (number of vehicles) on the lanes change once the conversion has been effected and the above expression can be used to compute the emissions under the base scenario and also for different programs.

The rather simplistic nature of the emissions model here implies that the estimates for each scenario's emissions may not be accurate, but they are likely to provide insights into the ordinal ranking of scenarios/ objectives based on vehicle emissions.

3.8 CHAPTER SUMMARY:

The core model that can be used to analyze the potential benefits and costs of converting an HOV lane into an HOT lane was presented in this chapter. The model consists of an optimization program that recommends the most effective pricing setting on the managed lanes for a specific agency-defined objective. This approach would, thus, provide insights into the optimal benefits that could be achieved by means of tolling. The heterogeneity in road users was accounted for by classifying them into different categories based on hourly wage rate, trip type, and the carpool formation cost. A logit-like model was

constructed for describing the behavior of different types of users and was embedded into the optimization model as constraints. The estimation of necessary parameters using aggregate data was also discussed. Inclusion of equity constraints in the model allows for direct handling of equity concerns at the early planning stage. Equity here was considered along two dimensions (Temporal and Vertical) and using two welfare measures (travel cost and travel time). Other types of constraints imposed include constraints describing travel times, tolls and current conditions.

4. CASE STUDY

The impacts of converting an HOV lane to an HOT lane on a particular corridor are examined in this chapter. The corridor being considered belongs to the Interstate 80 freeway in Contra Costa county and is five miles long (Buchanan to the I-880 split on 1-80W). This particular corridor has been identified as a high priority project for implementation of the HOT lane because of the high growth rate of carpools on this corridor [54]. It is expected that the HOV lane here will become crowded by the year 2020. Figure 4.1 shows a map indicating the extent of the corridor.

Figure 4.1: Extent and location of study corridor

There are five lanes on this stretch with one of the lanes serving as a carpool lane during the AM (0500 – 1000 hours) and PM (1500 – 1900 hours) peak periods. The HOV lane operates by allowing only carpools with three or more people to use it during the peak periods. Data from the PEMS database indicates significant levels of congestion (v/c ratio ~ 88% during peak periods) on this stretch. Apart from utilizing the excess capacity on

HOV lanes, the HOT project on this stretch can also aid significantly in reducing travel times and optimizing various performance measures.

The benefits/ costs associated with the conversion here were computed for a demand level corresponding to the mean peak hour volume. It was further assumed that the HOT lanes will be tolled only during the peak period. This is because the HOV lanes currently are being operated only during the peak period and thus any extension to a 24 hour period might encounter higher public resistance for this concept. A public opinion survey conducted by the Metropolitan Transportation Commission (MTC) revealed opposition to SOVs buying into the HOV lanes with 64% of the respondents answering "no" to the concept. A majority (61%), however, did agree that carpool lanes are currently being underutilized in the Bay Area [54]. Other studies have, however, found that the HOT concept finds more acceptance when the usage of revenues is explicitly mentioned and with progress of time [67].

A behavior model was first estimated for this stretch and the tolls that optimized various performance measures were computed. The different objectives that were considered here include maximizing revenue, minimizing total vehicular time, total cost, total passenger time and total VMT. The behavior model was then embedded as constraints into the optimization model and the following aspects were studied for the conversion policies generated from various programs:

i)   Changes in various performance measures before and after conversion.

ii)   Usage of managed (HOT) lanes by income, trip type and carpool formation costs.

iii)   Vertical and Temporal equity related issues.

In addition to the above, some of the key questions answered in this chapter include: impact of conversion on number of vehicles and carpools, pricing and operating strategies under different programs, impact of conversion on emissions, welfare of income groups before and after conversion, losses in efficiency due to equity and so on. The estimation of the choice model parameters is presented in the next section. Impacts of conversion under different optimization programs are discussed in subsequent sections. The last section in this chapter then deals with the variation in the performance measures and other impacts across different programs.

4.1 ESTIMATION OF CHOICE MODEL:

The first step towards estimating a choice model was to obtain an estimate of the passenger demand for the stretch during the peak hour. The PEMS database was used to obtain the average flow (per hour) during the peak hours on the detector closest to the entry of the stretch in the West bound direction. Peak period flows on each Monday though Thursday during June 2007 were obtained and their average value was used as the vehicular demand for the stretch. In order to convert this vehicular demand into the passenger demand, the mode shares of SOV, HOV2 and HOV3 were needed. The modal split for the Bay Area during the peak period was then computed using the modal splits for each trip type during 2006 from [68] and the breakdown of peak period traffic according to trip type as given in [57].

The average values for carpool formation cost and the value of time for a 'representative' individual were then obtained by appropriately weighting the numbers using modal splits and traffic composition (based on trip types) from [57] and [68] respectively. The Bureau of Public Roads (BPR) function was used to compute the travel times on each of the lanes

and the necessary coefficients were obtained from the PEMS database: a = 0.506 and b = 5 (rounded off to the nearest integer due to solver limitations). The capacity per lane, as given in [54], was assumed to be 1600 vehicles per lane per hour. The travel times thus computed were then used to obtain the travel costs for the representative user using an average VOT. Thus, the total cost (excluding OC) corresponding to each alternative for the representative user was computed in the HOV scenario. Note that there are no toll costs in the pre-conversion scenario.

Next, assuming the share of each mode to represent the probability of the mode being chosen, the parameters β (scaling coefficient) and OC (operating costs) were estimated using the procedure described in the previous chapter to be 0.5782 and 74.65 cents respectively. This value for coefficient β translates into 0.0864 when the impedances (costs) for each of the alternatives are expressed in terms of time units instead of dollars. This value falls within the range of the coefficients estimated in a similar mode choice model, which is from 0.05 to 0.085 [64].

Assuming the gasoline operating cost in the Bay Area to be 9.89 cents/mile (in 2006 dollars) [69], the estimate of 74.65 cents for OC translates into 7.55 miles of trip distance on average. After subtracting out the actual average trip distance in the Bay Area in 2006 (6.7 miles [57]) from the OC estimate, the rest of the operating cost (~7.8 cents) is somewhat low to be considered as other costs such as the parking costs. Thus, the model here seems to underestimate the exact operating costs. However, as explained in Chapter 3, this might partly be because of the preferential treatment given to carpools elsewhere in the trip or due to the exclusion of other types of costs that are incurred by users.

This behavior model was then used in solving for the tolls under different optimization programs, whose results are presented in the following sections.

4.2 REVENUE MAXIMIZATION:

The tolls that maximize total expected revenue were computed and the toll for both SOVs and HOV2s was found to be $5.46 per trip. This toll, though somewhat on the higher side, is still within the reasonable range (The SR 91 toll schedule, for instance, contains tolls that are at times even higher). Such reasonable toll for both types of vehicles suggests that the optimal policy here would be to operate the lanes as HOT lanes with free access only to HOV3s. This result also implies that a revenue of $2824.78 per hour could be obtained by operating HOT lanes under revenue maximization, which translates into an annual revenue of about $6.36 million, assuming operation only on weekdays.

These toll revenues from the HOT lane, if used entirely for repayment, will be sufficient to recover the capital and operating costs in just over three years. The capital cost estimate from MTC was put at $3.7 million per mile for upgrading the HOV lane on this corridor [54]. Thus, the total capital cost here amounts to $18.5 million. The operating and maintenance costs, on the other hand, were estimated to be $0.35 million per year.

This toll policy, however, has led to a worsening of travel time on the managed lane. The average travel time on the HOT lane was found to be six minutes as opposed to 4.75 minutes in the pre-conversion case. The travel time on the other lanes, however, improved from 8.11 minutes to 6.54 minutes. The conversion, thus, seems to reduce the difference between the travel times on the lanes. An interesting observation here is that the service quality constraint on the managed lane was binding. This suggests that there is

still scope for increase in the revenue if the service (i.e. travel speed) on the HOT lane were to be lowered from 50mph. This has been confirmed with a numerical experiment which reduced the quality on the HOT lane and found an increase in the revenue generated. As expected, the utilization level of the HOV lane improved by almost 28 percentage points while the utilization level of the general lanes fell by eight percentage points.

4.2.1 Impact on Performance Measures:

Table 4.1 shows the changes in the various performance measures caused by the conversion, under the revenue maximization program. The measures shown here are for one hour of operation.

Table 4.1: Comparison of performance measures under revenue maximization

Performance Measure (for one hour of operation)	Before Conversion	After Conversion	Difference	% Change
Revenue	$ 0	$ 2824.74	+ $ 2824.74	-
Total Vehicular Time	952.58 hrs	815.46 hrs	-137.12 hrs	-14.39%
Total Passenger Time	1333.38 hrs	1236.45 hrs	-96.93 hrs	-7.27%
Total Cost	$ 17924.2	$ 19692.6	+ $ 1768.4	+9.87%
VMT	37536.95 miles	38021.85 miles	+ 484.9 miles	+1.29%

It can be seen from the above table that there is a significant amount of benefit in terms of revenue, total vehicular and passenger times by converting the HOV lane into the HOT lane and operating under the revenue maximization program. The largest improvement, in terms of percentage change, was in the total vehicular travel time which decreased by 14.39%. On the flip side, there was also an increase in the total cost (impedance) that is

incurred by the users, on the order of $1768.4. This cost may be considered a loss in social welfare if there is no redistribution of the revenues. However, if it were possible to return the toll revenues perfectly to the users, it would still be possible to make a "profit" worth $1056.38 and ensure that the total cost does not deteriorate after the conversion. There is also an increase in the total VMT which is a direct consequence of the increase in the number of vehicles using this stretch. The volume here increased from 7507 vehicles per hr to 7604 vehicles per hr.

Table 4.2: Comparison of modal shares before and after conversion

Mode	Before Conversion		After Conversion	
	# Users	Proportion	# Users	Proportion
SOV	4882	43.43%	4900.58	43.60%
HOV2	3028	26.94%	3542.24	31.51%
HOV3	3332	29.64%	2798	24.89%

As shown above, while the modal share of SOVs and HOV2s increased by 0.17 and 4.57 percentage points respectively, the share of three-person carpools decreased by 4.75 percentage points. These observations suggest that the negative effect on volumes here is brought about by the dominance of the latter effect - three-person carpools breaking-up into SOVs and HOV2s – over the former effect i.e., increase in two-person carpools. This greater dissolution of the three-person carpools seems to indicate that the savings in carpool formation time brought about by switching to the two-person carpool outweigh extra costs associated with the HOV2 alternatives under the HOT lane scenario.

Note that despite the increase in the number of vehicles, there is a drop in the total vehicular travel time which is brought about by the decrease in the travel time experienced by a number of vehicles on the general lane.

The impact of this conversion on emissions was estimated for three different types of emissions – VOC, CO, NO_X using the model described in Chapter 3. Table 4.3 shows the amounts of each of these (per hour of operation) released under both before and after conversion scenarios.

Table 4.3: Comparison of Emissions before and after the conversion

Emission Type ↓	Before (kg/hr)	After(kg/hr)
VOC	15.47	15.59
CO	220.38	224.39
NO_X	50.46	51.05

The above results seem to suggest that the conversion here has a slightly detrimental impact on air quality, with all of the emissions predicted to be higher after conversion. The values for all the emissions, however, are quite close to the initial values and given the simplistic nature of the model, the strength of this inference is not exactly known at this stage. The increase in all the three emissions here might be due to increase in the number of vehicles (increase in VMT).

4.2.2 Users of Managed Lanes:

The probability of an individual choosing to use the HOT lane (as a HOV or SOV) is examined through segmentation of users by income, trip type and carpool formation costs. The following table shows the variation in the probability of individuals belonging to different income groups choosing to use the HOT lane.

Table 4.4: Comparison of managed lane use propensity by income groups

Income Groups↓	Before	After
Quartile 1 (Low)	0.322	0.361
Quartile 2	0.303	0.336
Quartile 3	0.286	0.309
Quartile 4 (High)	0.274	0.284

The results here suggest that lower income people have a higher probability of choosing the HOT lane when compared to higher income individuals. This might seem counterintuitive at first glance. However, it should be noted that lower income individuals, even prior to the conversion, are more likely to carpool and thus have a higher probability of using the managed lane as carpool on HOT lanes. Empirical evidence for lower income people carpooling more than higher income people may be found in [68]. This might be because of the fact that lower income individuals might be willing to carpool more in order to save more on the (fixed) operating costs that are uniform for all of the income groups in this model.

Although there is an increase in the propensity to choose the managed lane, the reason for such a choice differs across the groups. While higher income individuals use the lane more by paying a toll either as a SOV or HOV2, lower income individuals gain access by carpooling more either as a HOV2 or HOV3. This is because higher income individuals, who are more willing to pay money for savings in time, are more likely to choose the toll option once the conversion has been made.

The results for the probabilities of travelers belonging to different trip types choosing the HOT lane (Table 4.5(a)) are similar to the results for income based segmentation. Users on trips with lower values of time are more likely to choose the managed lane. This is

predominantly due to the higher carpooling tendencies that are associated with lower value trips in a manner similar to the behavior of the lower income groups. Empirical evidence for higher carpooling rates in school, other and shopping trips can be found in [68].

Table 4.5: Comparison of managed lane use propensity by trip types and by carpool formation costs

Trip Type ↓	Probability
Work	0.272
Shop	0.316
Other	0.389
School	0.409

(a)

Group #	Probability
Group 1	0.429
Group 2	0.393
Group 3	0.291
Group 4	0.178

(b)

The results for the segmentation along the carpool costs (Table 4.5(b)) are along expected lines. Individuals who have a higher carpooling cost are less likely to use the HOT lane when compared to individuals with lower carpooling costs.

Note that all of the above results are obtained without imposing the equity constraints.

4.2.3 User Equity Analysis:

As mentioned in the previous chapter, the two variables that are used to measure the welfare of the income groups here are average travel time and average travel costs (in units of time). Equity here is analyzed along two dimensions –Temporal and Vertical. Discussion about the results of the experiments and the relationships between equity and efficiency under this revenue maximization program follows.

4.2.3.1 Travel Cost Equity:

a) Temporal Equity: The average costs (in units of time) for different users, segmented by income, before (HOV) and after the conversion (HOT) are shown in Table 4.6.

Table 4.6: Average travel costs of users in each income group before and after conversion

(unconstrained case)

Income	Quartile 1 (Lo)	Quartile 2	Quartile 3	Quartile 4 (Hi)
HOV case	0.819 hrs	0.574 hrs	0.418 hrs	0.318 hrs
HOT case	1.112 hrs	0.746 hrs	0.515 hrs	0.371 hrs

As shown in the table, the conversion, when evaluated in terms of the average cost, seems to have a negative effect on all the groups with an increase in cost observed across all the income groups. It can also be seen that this increase is largest in the case of lower income groups. This project, when operated under the pure revenue maximization program, thus seems to have a detrimental effect on temporal equity by making future users worse off when compared to the current users.

It was then attempted to arrive at a pricing strategy which would ensure that each income group of the current users on an HOV lane are not made worse off in the future if the HOV lane is converted to an HOT lane. This was done by imposing constraints which ensured that the average cost for each income group was non-increasing. These additional constraints on the optimization problem narrow down the solution search space thus leading to a loss in efficiency. The "cost" corresponding to achieving temporal equity in this case may be thought of as the decrease in the optimal revenue from the original (unconstrained) problem to this new constrained problem. The optimal revenue, on solving the constrained optimization problem, was found to be $84.42 per hour, which is

$2740.35 less than that of the original problem. This difference represents in the loss in efficiency due to temporal equity and the policy makers would, thus, need to strike a balance between equity and efficiency.

Further analysis was carried out in order to ensure that policy makers get a higher amount of flexibility in this seemingly binary decision on efficiency vs. equity. This was done in the following manner: Instead of constraining the average costs of each income group to be strictly less than 100% of original costs, constraints specifying that the new average costs can be less than (100+x)% times the original costs were imposed.

Average Cost (i) ≤ (1+x/100) × Initial Average Cost (i),

where i is the index denoting income group.

The variable of x in the above inequality was then increased and the objective value observed. Figure 4.2 shows the increase in efficiency as the extent of temporal equity is decreased i.e. as x is increased.
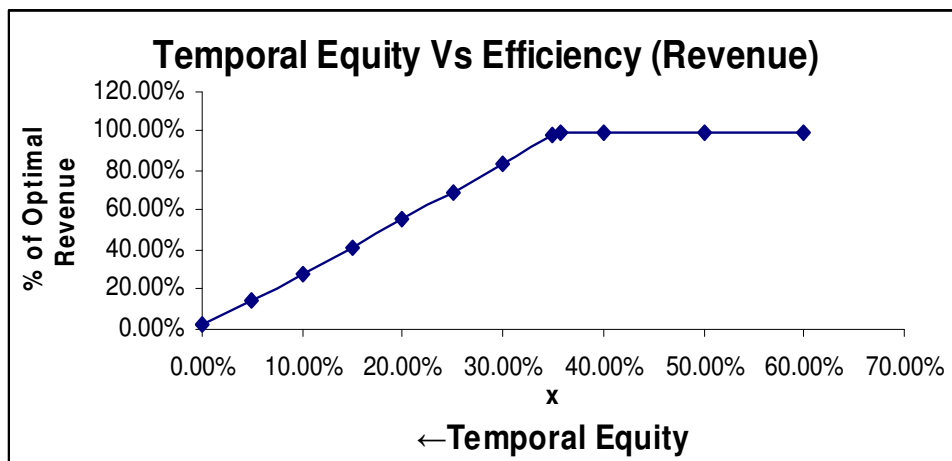

Figure 4.2: Relationship between Temporal Equity and Revenue

The y-axis of the plot shown above gives the ratio (percentage) of the optimal revenue in the constrained case to the optimal revenue in unconstrained case and thus, is a measure of efficiency in this case. Thus, as seen from the plot, the efficiency here increases linearly with a decrease in temporal equity i.e. as x is increased from 0%. However, once a certain threshold value for x is reached, there would be no further increase in the efficiency. This threshold value here corresponds to x being 35.7%. In other words, once x hits a value of 35.7%, the constrained problem becomes equivalent to the original unconstrained version and no loss in efficiency is observed for higher values of x.

Note that whole of the above analysis corresponds to the case when there is no possibility of compensating any of the losing groups. However, in cases where it is possible to perfectly redistribute the revenues obtained, equity can be achieved at a lower loss of efficiency, i.e. at a lower cost by simply reimbursing the users who have lost because of this conversion. The cost of achieving total temporal equity in this case, as noted in 4.2.1, would be $1768.4 (the difference in total cost between the HOV and HOT scenarios). Note that this value is much lower than the loss in efficiency resulting from the constrained optimization problem where the value of x is set to 0. However, this is only for theoretical purposes since perfect redistribution would not be possible in reality. Therefore, depending on the effectiveness of the available redistribution mechanisms, policy makers would then need to decide upon a combination of the appropriate constrained problem (i.e. x) and redistribution package.

b) *Vertical Equity:* Table 4.7 shows the average travel costs (in units of time) of different income groups before and after the conversion.

Table 4.7: Average travel costs of users in each income group before and after conversion

Income	Before Conversion	After Conversion
Quartile 1 (Low)	0.819 hrs	1.112 hrs
Quartile 2	0.574 hrs	0.746 hrs
Quartile 3	0.418 hrs	0.515 hrs
Quartile 4 (High)	0.318 hrs	0.371 hrs

Focusing on the post-conversion scenario alone, it can be seen that the travel costs of lower income individuals are a lot higher than those of higher income individuals. Experiments that involved constraining the maximum difference between the average costs of each groups under the HOT lane scenario were conducted. This was done by constraining the cost of each of the groups to lie within a certain percentage (θ) of the overall average travel cost. The variation in efficiency (measured by revenue here) was then examined by varying the value of this percentage θ.
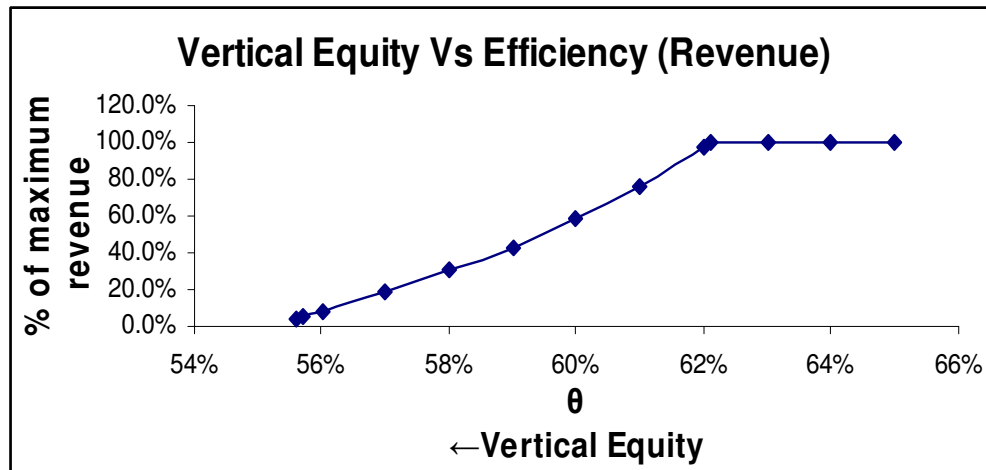

Figure 4.3: Relationship between Vertical Equity and Revenue

As shown in Figure 4.3, the relationship between equity and efficiency here is characterized by two thresholds. The lower of the two thresholds corresponds to the value

below which it is not possible to reduce θ without making the problem infeasible. An implication of the existence of this lower threshold is that it is impossible to ensure that all of the groups experience exactly the same cost. Decreasing θ value below this threshold would simply make the problem infeasible. The value of this lower threshold was found to be 55.6%.

The graph shown above also indicates that there would not be any loss in the performance measure (revenue) when the allowable percentage deviation (θ) from the mean is more than a certain upper threshold (62.1%). The problem, on crossing this thresholds, once again becomes equivalent to the original unconstrained problem. However, once θ goes below this threshold, the efficiency gradually decreases until the predetermined deviation value (θ) hits the lower threshold (55.6%) below which the problem simply becomes infeasible.

On the whole, there seems to be a (linearly) decreasing relationship between vertical equity and efficiency. Thus, vertical equity (θ) would be one of the parameters that policy makers need to fix at the planning stage itself, with all the political considerations in mind.

Now, turning to the vertical equity situation before the conversion, note that the travel costs of lower income individuals are much higher than those of the higher income groups even before the conversion (Table 4.7). This is again because of the higher prevalence of carpooling among lower income individuals. The maximum deviation (θ) from the mean in the pre-conversion scenario is 53.8%, which is lesser than the minimum θ that could be achieved by imposing vertical equity-related constraints on the model. Thus, unlike temporal equity, there *will* be some loss of equity (in vertical equity sense)

that takes place upon implementing the conversion under the revenue maximizing price regime.

It should be noted that the above vertical equity is only for the case where there is no redistribution. Now suppose that the operating agency is in a position to distribute the toll revenue to any of the user groups perfectly, i.e., there is no wastage associated with redistribution. The only way to reduce the extent of vertical inequity once the trips have been made is to compensate the losing groups in such a manner that their average benefits move as close as possible to those of the winning groups. Transfer of money from winning groups to losing groups is not possible once the trips have taken place. The winning group here is the high income quartile. Thus, the amount of revenue needed to achieve total vertical equity can be calculated by taking the difference (in average costs) between the corresponding groups of the lower income and high income quartile and then converting them into monetary units. This exercise has been performed for the unconstrained revenue maximization problem and the following amounts (table 4.8) were to be paid to each of the income groups in order to ensure that all four groups had the same average cost (that of the high income group).

Table 4.8: Money to be paid to users in each group in order to ensure perfect vertical equity

(Unconstrained problem)

Income Group	Money to be paid
Quartile 1 (Low)	$ 2160.94
Quartile 2	$ 1863.05
Quartile 3	$ 1199.22
Quartile 4 (High)	$ 0
Total	$ 5223.21

As expected, the money to be paid decreases as we move from lower income quartiles to higher income ones. The total amount to be paid out as compensation here is \$2398.43 more than the amount generated in revenues. A similar exercise was carried out for different values of θ in the feasible region and the results are summarized in Table 4.8.

Table 4.9: Deficit in revenue that would be needed to ensure perfect vertical equity

(Perfect redistribution case)

θ	Revenue (\$)	Money to be paid (\$)	Deficit (\$)
63.00%	2824.78	5223.21	2398.43
62.10%	2806.73	5213.67	2406.94
62.00%	2744.81	5181.52	2436.71
61.00%	2164.56	4905.39	2740.83
60.00%	1659.54	4679.55	3020.01
59.00%	1223.97	4484.76	3260.79
58.00%	847.70	4312.64	3464.94
57.00%	520.18	4158.17	3637.99
56.00%	231.59	4017.83	3786.24
55.60%	124.96	3965.02	3840.06

Thus, in case full redistribution is a possibility, implementing the pricing regime from the unconstrained problem would be the most efficient way of ensuring vertical equity. However, in cases where full redistribution is not possible, operating under the unconstrained revenue maximization may not lead to the most efficient way to equity. Suppose that only 40% of the revenues could be used for redistribution. In this case, the total amount to be paid out as compensation changes for different values of θ. It can be

seen from Table 4.10 that setting $\theta = 55.6\%$ would yield the least cost way to achieving full vertical equity.

Table 4.10: Deficit in revenue that would be needed to ensure perfect vertical equity

(Imperfect redistribution case – 40% efficiency of redistribution)

θ	Deficit ($)
63.00%	4093.30
62.10%	4090.98
62.00%	4083.60
61.00%	4039.57
60.00%	4015.73
59.00%	3995.17
58.00%	3973.56
57.00%	3950.10
56.00%	3925.20
55.60%	3915.04

The choice of the equity level at the planning stage would, thus, depend on the effectiveness of redistribution that can be achieved using different mechanisms.

On the whole, there seems to be a decreasing relationship between temporal as well as vertical equity and efficiency, when equity in travel costs is considered. Furthermore, the relationship is linear, i.e. as equity increases, revenue decreases at a linear rate within the interval described by the two thresholds. However, the decrease in revenue per unit change in the equity measures (x and θ) is a lot higher in the case of vertical equity.

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<u>4.2.3.2 Travel Time Equity:</u>

a) Temporal Equity: Table 4.11 shows the average travel times before and after conversion across different income groups.

Table 4.11: Average travel times of users in each income group before and after conversion

Income	Quartile 1 (Lo)	Quartile 2	Quartile 3	Quartile 4 (Hi)
HOV case	0.1171	0.1182	0.1192	0.1198
HOT case	0.1051	0.1059	0.1062	0.1064

Conversion of the HOV lane into a HOT lane seems to benefit all the groups in terms of travel time. There is no loss of efficiency (i.e. reduction in the objective - revenue) that occurs in order to ensure that the conversion is temporally equitable. In other words, the increase in the revenue generation here does not translate into worsening of travel time for future users when compared to the travel times of current users.

b) Vertical Equity: The travel times of the lower income individuals, on an average, are lower than those of the higher income individuals both before and after the conversion. This, as explained above, is due to the higher prevalence of carpooling among lower income individuals. Higher income groups, however, seem to be benefiting at a higher rate in terms of the percentage decrease in the travel times. In light of the closeness of the travel times experienced by all the income groups, it can be conjectured that the loss in efficiency is very little with the increase in equity. The analysis described above (section 4.2.3.1) reveals that the two thresholds on the efficiency vs. equity curve were very close to the 100% mark. This means that the loss in efficiency due to imposing the vertical

equity constraints does not begin to take place till a point that is quite close to the perfect vertical equity.

Furthermore, the maximum (θ) deviation before the conversion was 1.2% and this value reduced to 0.3% after the conversion suggesting that the conversion here improves vertical equity in terms of travel times. This is because of the fact that the conversion drives the travel times on both the lanes to be closer than they were before. Note that there is no real issue with vertical equity here since the weaker groups have lower costs compared to the stronger ones. Thus, there is no strong necessity to study the equity in the travel time sense in this program.

The above results highlight the fact that there is no single definition for quantifying the benefits and costs of different groups. For instance, in this case study it was not sufficient to use travel time as the measure in all of the instances in order to examine the equitability of a project and formulate fair strategies. Thus, a comprehensive equity analysis needs to include quantification of benefits/ costs with respect to different measures that are deemed appropriate.

4.3 MINIMIZATION OF TOTAL VEHICULAR TRAVEL TIME:

The minimization of total vehicular time (TVT) resulted in a very high toll for SOVs, suggesting that they should not be allowed onto the HOT lane. HOV2s, on the other hand, should be allowed on the HOT lane at a toll of $3.26. The least possible TVT here was 786.54 hours of travel time per hour, which is 17.43% less than the HOV lane scenario.

The revenue that can be obtained from the above tolling regime is $1824.2 per hour, which translates into an annual revenue of $4.1 million. These revenues, if used entirely towards recovering the costs, would break even with the capital and operating costs in a period of about five years. This is about two years more than the time required in the revenue maximization program.

As observed in the previous section, while the travel time on the managed lane deteriorated there was an improvement of conditions on the general lanes. The travel time on the managed lane was six minutes (26.2% more than the pre-conversion scenario) while the travel time on the general lane was 6.37 minutes (21.46% less than the pre-conversion scenario). Once again, the quality constraint was found to be binding, the travel time on the managed lane is thus same as the one observed under revenue maximization. So, a relaxation of the service constraint led to an improvement in the TVT.

4.3.1 Impact on Performance Measures:

Table 4.12 shows the changes in the various performance measures before and after the conversion, under the TVT minimization program.

Table 4.12: Comparison of performance measures under TVT minimization

Performance Measure (for one hr of operation)	Before Conversion	After Conversion	Difference	% Change
Revenue	$ 0	$ 1824.19	+ $1824.19	-
TVT	952.58 hrs	786.53 hrs	-166.05 hrs	-17.43%
Total Passenger Time	1333.38 hrs	1170.26 hrs	-163.12 hrs	-12.23%
Total Cost	$ 17924.2	$ 18688.3	*+ $764.1*	*+ 4.26%*
VMT	37536.95 miles	37465.65 miles	-71.3 miles	-0.19%

The results here indicate an improvement in all the performance measures except with respect to the total user cost. However, in the case of perfect redistribution, it would still be possible to raise $ 1060.09 in revenue, even after ensuring that nobody loses out due to the conversion. Interestingly, this value is similar to the one obtained during revenue maximization. There is also a significant improvement in the total passenger time from the pre-conversion scenario on the order of 12.23%.

In contrast to the revenue maximization program, there is a marginal improvement in the total VMT due to a decrease in the total number of vehicles from 7507 to 7493. Table 4.13 shows the variation in the total number of users across the three modes before and after the conversion.

Table 4.13: Comparison of modal shares before and after conversion

Mode	Before Conversion		After Conversion	
	# Users	Proportion	# Users	Proportion
SOV	4882	43.43%	4635	41.22%
HOV2	3028	26.94%	3938	35.02%
HOV3	3332	29.64%	2669	23.73%

As observed under the revenue maximization regime, there is a drop in the modal shares of SOVs and HOV3s. There is, however, an increase in the share of two-person carpools on the order of 8.08 percentage points. Thus, the decrease in the number of vehicles here seems to be a result of increase in the number of people choosing HOV2. Note that the formation rate of two-person carpools dominates the contrary effect – dissolution of three-person carpools. Thus, the results here seem to suggest that the two-person carpool

becomes the preferred alternative now and this is due to both the breaking up of some of the three-person carpools and SOV users forming carpools.

The impact of this conversion on the three types of emission is shown Table 4.14.

Table 4.14: Comparison of Emissions before and after the conversion

Emission Type ↓	Before (Kg/hr)	After (Kg/hr)
VOC	15.47	15.34
CO	220.38	221.69
NOX	50.46	50.46

The results here seem to be more encouraging than the previous case with a negative impact on emissions of CO alone. The values for all the emissions, however, are again very close to their initial values.

4.3.2 Users of Managed Lanes:

The likelihood of individuals belonging to different income groups choosing the managed lane before and after the conversion is shown in Table 4.15.

Table 4.15: Comparison of managed lane use propensity by income groups

Income Groups↓	Before	After
Quartile 1 (Low)	0.322	0.384
Quartile 2	0.303	0.354
Quartile 3	0.286	0.322
Quartile 4 (High)	0.274	0.289

There is a clear increase in the propensity to use the managed lane after the conversion. Furthermore, the propensity to use the HOT lane increases from higher income to lower

income individuals for reasons discussed previously. Note that all the post-conversion probabilities in this program are higher than those under the revenue maximization program. The following tables show the probabilities of using the managed lane for users segmented according to their trip types (Table 4.16(a)) and carpool formation cost groups (Table 4.16(b)).

Table 4.16: Comparison of managed lane use propensity by trip type and carpool formation cost

Trip Type ↓	Probability
Work	0.277
Shop	0.331
Other	0.415
School	0.435

(a)

Group #	Probability
Group 1	0.453
Group 2	0.417
Group 3	0.306
Group 4	0.172

(b)

The results above are in line with the reasoning that there is a higher likelihood of carpooling in the contexts of lower value trips and lower carpooling costs. Once again, the propensity to use the managed lane here is higher than the corresponding propensity observed for the revenue maximization regime, across all of the income groups. The higher probabilities (of choosing managed lane) observed in this program are a reflection of the increase in the extent of carpooling that is happening here.

4.3.3 User Equity Analysis:

As discussed above, Temporal and Vertical equity issues here are analyzed for travel time and travel cost.

4.3.3.1 Travel Cost Equity:

a) Temporal Equity: The average costs (in units of time) corresponding to different income quartiles before (HOV) and after the conversion (HOT) are shown in Table 4.17.

Table 4.17: Average travel costs of users in each income group before and after conversion

Income	Quartile 1 (Lo)	Quartile 2	Quartile 3	Quartile 4 (Hi)
HOV case (hrs)	0.819 hrs	0.574 hrs	0.418 hrs	0.318 hrs
HOT case (hrs)	1.041	0.700	0.486	0.351

The conversion here seems to increase the travel costs experienced by individuals belonging to all of the income groups, as observed under the revenue maximization program. In order to ensure that all of the income groups on average experience lower travel costs after the conversion, a constrained version of the optimization model was implemented. The objective then was found to be 857.22 hours of vehicular travel time compared 786.53 hours for the unconstrained case suggesting that the loss in efficiency here, in order to render the conversion temporally equitable for all income groups, is 70.69 hours of vehicular travel time.

Further analysis was then carried out by gradually relaxing the temporal equity constraints in a manner similar to the procedure described in Section 4.2.3.1. The limit on the average cost of each group was set to be a certain percentage (x) more than the average cost experienced in the pre-conversion state. The following graph (Figure 4.4) shows how the efficiency (in terms of vehicular travel time) changes with the change in the extent of temporal equity.
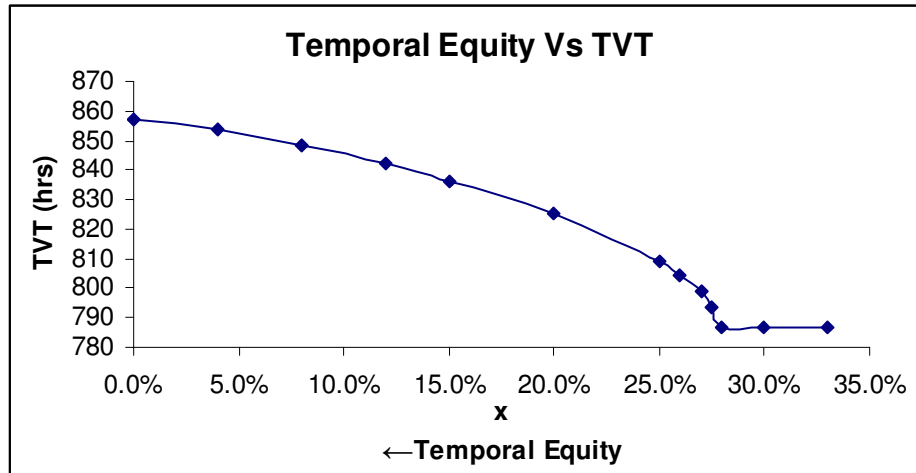
Figure 4.4: Relationship between Temporal Equity and TVT

Once again, the efficiency decreases with the increase in temporal equity till a certain threshold is reached. The threshold here is at x equals 27% and beyond this, there is no loss in efficiency with an increase in temporal equity. The relationship between x and efficiency in the region below the threshold appears to be quadratic in nature, unlike in the revenue maximization case where it was linear. Note that the quadratic relationship here results in higher losses in efficiency for unit increase in equity near the threshold when compared to losses when x equals 0. In other words, the loss in efficiency occurs at a higher rate near the threshold when compared to the loss rate under conditions closer to perfect temporal equity.

As noted in 4.3.1, in situations where perfect redistribution would be possible, the revenue from the unconstrained model would suffice to achieve total temporal equity. The problem of designing the right redistribution package, however, becomes more complex now and would require a triangular tradeoff involving revenue, vehicular travel time and equity.

b) Vertical Equity: The following table (table 4.18) shows the average travel costs (in units of time) of different income groups in the post-conversion scenario.

Table 4.18: Average travel costs of users in each income group after conversion

Income	Avg Cost (hrs)
Quartile 1 (Low)	1.041
Quartile 2	0.700
Quartile 3	0.486
Quartile 4 (High)	0.351

As expected, the time equivalent of travel cost experienced by the lower income individuals is much higher than that of higher income individuals under the HOT scenario. The differences in average costs experienced were then constrained by limiting the cost of each group to be within a certain percentage (θ) of the overall mean. Figure 4.5 shows how the efficiency decreases (i.e. TVT increases) as this θ is decreased. Note that relationship is characterized by two thresholds once again.
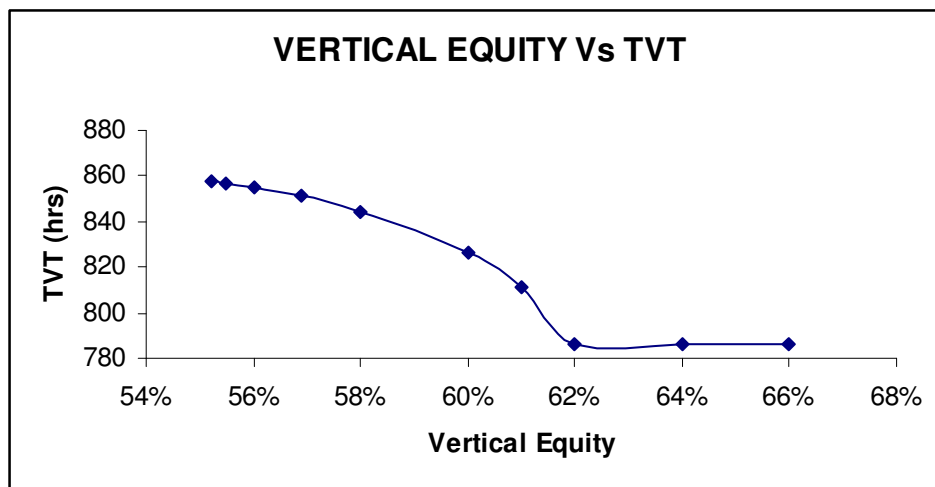

Figure 4.5: Relationship between Vertical Equity and TVT

The two thresholds under TVT minimization, as shown in the above graph, occur at θ equals 55.2% and 61%. In other words, there would not be any loss in efficiency when

the θ value is over 61% and the problem becomes infeasible when θ is reduced to a value less than 55.2%. Thus, the most vertically equitable situation in the post conversion scenario corresponds to θ equals 55.2%. Note that the relationship between the two thresholds is quadratic as in the case of temporal equity.

This θ value in the pre-conversion scenario happens to be 53.8%. Thus, there will again necessarily be a reduction in vertical equity due to the conversion. This, however, is true only if there is no redistribution of revenues. The revenue deficit under perfect redistribution of revenues was found to be the least for the unconstrained problem. The decision about the θ would, however, need to be based on the weights attached to TVT, revenue and vertical equity. Inferences similar to those in section 4.2.3.1 could be drawn for different levels of redistribution packages.

4.3.3.2 Travel Time Equity:

a) Temporal Equity: Table 4.19 shows the average travel times before and after conversion across different income groups.

Table 4.19: Average travel times of users in each income group before and after conversion

Income	Quartile 1 (Lo)	Quartile 2	Quartile 3	Quartile 4 (Hi)
HOV case (hrs)	0.1171	0.1182	0.1192	0.1198
HOT case (hrs)	0.1032	0.1036	0.104	0.1044

As expected, there is an improvement in the average travel times of all the income groups. Thus, there is no loss in efficiency, i.e. increase in TVT, with the imposition of temporal equity in terms of travel time. Also, the improvement in the travel times of all

the groups is higher under TVT minimization than was the improvement under revenue maximization.

b) *Vertical Equity:* The average travel times of the lower income individuals are lower than those of the higher income individuals both before and after the conversion because of, as explained previously, higher carpooling among lower income individuals. Since the disadvantaged individuals are already better off, the need to impose vertical equity constraints is obviated (see 4.2.3.2).

4.4 MINIMIZATION OF TOTAL PASSENGER TIME (TPT):

The optimal toll for SOVs under TPT minimization seems to suggest that the SOVs are not to be allowed on the HOT lane. The toll for HOV2s, on the other hand, was $4.10. Thus, except for the minor difference in the HOV2 toll, this program is very similar to the TVT minimization (HOV2 toll = $3.26) program. The toll revenue under the TPT regime was $1870 per hour and the corresponding annual revenue was $4.2 million.

The travel times on the managed and general lanes were 5.73 minutes and 6.49 minutes respectively. These values differ by +20.54% and -19.98% from the previous travel times on managed and general lanes respectively. Unlike the cases for revenue maximization and TVT minimization, the quality constraint here is not binding, implying that there would not be any loss by maintaining the LOS on the HOT lane.

4.4.1 Impact on Performance Measures:

Table 4.20 shows the changes in the various performance measures before and after the conversion, under the TPT minimization program.

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Table 4.20: Comparison of performance measures under TPT minimization

Performance Measure (for one hr of operation)	Before Conversion	After Conversion	Difference	% Change
Revenue	$ 0	$ 1870.41	+ $1870.41	-
TVT	952.58 hrs	793.25 hrs	-159.33 hrs	-16.73%
TPT	1333.38 hrs	1167.26 hrs	-166.12 hrs	-12.46%
Total Cost	$ 17924.2	$ 18719.9	+ $795.7	+ 4.44%
VMT	37536.95 miles	37518.9 miles	-18.05 miles	-0.05%

The tolling regime under this program seems to improve all of the performance measures except for the total cost, whose increase can be mitigated by redistribution of toll revenues. Perfect redistribution here yields a 'profit' of $1074.71, which is the highest of all the programs by a narrow margin.

The number of vehicles and the total VMT remained almost constant when compared to the pre-conversion scenario. There is, however, certain switching between the different modes as shown in the following table (table 4.21).

Table 4.21: Comparison of modal shares before and after conversion

Mode	Before Conversion		After Conversion	
	# Users	Proportion	# Users	Proportion
SOV	4882	43.43%	4691	41.73%
HOV2	3028	26.94%	3779	33.62%
HOV3	3332	29.64%	2772	24.65%

As observed in the previous two programs, HOV2 seems to have become more attractive after the conversion. In spite of the constant total number of vehicles, there seem to be some users who are switching from HOV3 to HOV2. However, this breaking up of

carpools is balanced by the formation of new carpools from the SOV user base leading to little change in the total.

The impacts of this conversion on emissions are shown in Table 4.22.

Table 4.22: Comparison of Emissions before and after the conversion

Emission Type ↓	Before (kg/hr)	After (kg/hr)
VOC	15.47	15.37
CO	220.38	221.82
NO_X	50.46	50.51

The results here suggest slight negative impacts on CO and NO_X emissions. Once again, the values for all the emissions here are very close to their initial values and thus, these inferences would need to be verified using better models.

4.4.2 Users of Managed Lanes:

The average probabilities of individuals belonging to different income groups choosing the managed lane before and after the conversion are shown in the Table 4.23.

Table 4.23: Comparison of managed lane use propensity by income groups

Income Groups↓	Before	After
Quartile 1 (Low)	0.322	0.371
Quartile 2	0.303	0.342
Quartile 3	0.286	0.313
Quartile 4 (High)	0.274	0.284

As observed previously, there is a clear increase in the propensity to use the managed lane after the conversion. These probabilities here are lower than the corresponding

values in the TVT minimization but are higher than those in the revenue maximization program.

The managed lane use probabilities according to trip type (Table 4.24 (a)) and carpool formation cost groups (Table 4.24 (b)) are shown below. The results here follow the same patterns as discussed above under revenue maximization and TVT minimization.

Table 4.24: Comparison of managed lane use propensity by trip type and carpool formation cost

Trip Type ↓	Probability
Work	0.272
Shop	0.321
Other	0.401
School	0.421

(a)

Group #	Probability
Group 1	0.445
Group 2	0.407
Group 3	0.295
Group 4	0.164

(b)

4.4.3 User Equity Analysis:

Temporal and Vertical equity issues are analyzed for travel time and travel cost under the TPT minimization program in this section. The relationships between equity and efficiency here were found to be very similar in nature to those in the TVT minimization program.

4.4.3.1 Travel Cost Equity:

a) Temporal Equity: The average costs (in units of time) corresponding to different income quartiles before (HOV) and after (HOT) the conversion are shown in the Table 4.25.

Table 4.25: Average travel costs of users in each income group before and after conversion

Income	Quartile 1 (Lo)	Quartile 2	Quartile 3	Quartile 4 (Hi)
HOV case (hrs)	0.819	0.574	0.418	0.318
HOT case (hrs)	1.044	0.702	0.487	0.351

As observed in the previous sections, the conversion here worsens the average travel costs for all of the income groups. The loss of efficiency associated with ensuring that none of the income groups' cost increases after conversion is 48.36 hrs of TPT per hour of operation. The relationship between temporal equity and optimal TPT was then studied in a manner similar to the other programs and the relationship here was found to be similar to the TVT program case with the threshold value for x being 27.2% (x in TVT was 27.5%). Furthermore, in the region below the threshold the relationship appears to be quadratic in nature, unlike in the revenue maximization case, where it was linear.

As noted in 4.3.1, in cases where perfect redistribution would be possible, the revenue from the unconstrained model would suffice to achieve total temporal equity. The problem again becomes more complex and is a question of tradeoffs between revenue, passenger travel time and equity.

b) Vertical Equity: Table 4.26 shows the average travel costs (in units of time) of different income groups in the post-conversion scenario only.

Table 4.26: Average travel costs of users in each income group after conversion

Income	Avg Cost (hrs)
Quartile 1 (Low)	1.044
Quartile 2	0.702
Quartile 3	0.487
Quartile 4 (High)	0.351

As observed earlier, the travel cost experienced by the lower income individuals is much higher than that of higher income individuals. The relationship between equity and efficiency was again found to be quadratic and very similar to that under TVT minimization. The thresholds here matched with those under the TVT minimization program to an accuracy of 0.1%.

4.4.3.2 Travel Time Equity:

a) Temporal Equity: Table 4.27 shows the average travel times before and after conversion across different income groups.

Table 4.27: Average travel times of users in each income group before and after conversion

Income	Quartile 1 (Lo)	Quartile 2	Quartile 3	Quartile 4 (Hi)
HOV case (hrs)	0.1171	0.1182	0.1192	0.1198
HOT case (hrs)	0.1032	0.1036	0.1040	0.1044

As expected, there is an improvement in the average travel times of all the income groups. Thus, there is no loss in efficiency associated with temporal equity in the travel time sense. The improvement here is highest of all the programs for all income groups, except for the richest group for which improvement under TVT was higher.

b) Vertical Equity: As shown in Table 4.27, the average travel times of the lower income individuals are lower than those of the higher income individuals under both the scenarios. Thus, vertical equity under this program is not investigated further for reasons given in 4.2.3.2.

4.5 MINIMIZATION OF TOTAL USER COST:

The minimization of total user costs resulted in very high tolls for SOVs as well as HOV2s suggesting that they should not be allowed onto the HOT lane. In other words, the total user cost is minimized under the current HOV lane scenario and any amount of toll imposed would result in an increase of total user cost. An important implication of this result is that of all 64 groups of users, there necessarily will be at least one group which would lose out in terms of cost experienced.

Thus the model suggests that in this particular case study if the objective is to minimize the total user cost, it would not be optimal to convert the HOV lane into a HOT lane.

This observation need not extend to all the other situations and might be specific to this case. One such situation where cost minimization does not imply status quo (HOV lane) is when the operating costs are low. An experiment, where the operating cost was reduced by 50%, was conducted to test this hypothesis. It was then found that minimization of total cost yielded a very high toll for SOVs and a toll of $2.78 for HOV2s. This implies that SOVs should not be allowed while HOV2s should be charged for accessing the managed lane, unlike the original program wherein only HOV3s are allowed on the HOT lane. The lower operating costs here lead to more users forming two-person carpools (both due to breaking up of HOV3s and combination of SOVs) leading to lower carpool costs (for previous HOV3 users) and lower travel times (due to improved speeds and reduced volumes). This reinforces the hypothesis that the operating the managed lane as a HOV lane need not always lead to minimum total cost.

4.6 MINIMIZATION OF TOTAL NUMBER OF VEHICLES:

The results of this program were found to coincide with those from the TVT minimization program, with HOT access to HOV3s and HOV2s (for a toll of $3.26). Note that this observation need not hold in all situations and it is possible to construct cases where these two programs yield different results.

4.7 DISCUSSION:

The various aspects of the conversion are discussed and compared across four different programs – revenue maximization, TVT minimization, TPT minimization and total cost minimization. Note that for this particular case, total cost minimization is the same as the status quo and total VMT minimization is the same as TVT minimization.

4.7.1 Pricing Strategies:

Programs with objectives as the revenue maximization, TVT minimization, or TPT minimization yielded tolls which suggested that conversion would be beneficial. The total cost minimization program results, however, suggested that the objective would be maximized under the no conversion scenario. The revenue maximization results suggested that both SOVs and HOV2s be allowed onto the HOT lane at the same price ($5.46). The TVT and TPT minimization programs' results, on the other hand, suggested that only HOV2s should be allowed onto the managed lane at a toll of $3.26 and $4.1 respectively. SOVs were not to be allowed on the HOT lane under both these programs. All of these conversion programs generate sufficient revenue to compensate for loss in the total cost to the users, and the 'profits', interestingly, in all three conversion programs

are not too different from each other. The 'profit' in the TPT program is marginally higher than that in the other two programs though. The revenue generated in all of the programs is also sufficient to pay for the maintenance and to recover the capital and operating costs in a reasonable amount of time (three to five years).

4.7.2 Impact on Performance Measures:

Table 4.28 shows percentage differences in performance measures from their respective optimal values (shaded) across various programs.

Table 4.28: Deviation in performance measures from their optimal values across programs

	Revenue Max	TVT Min	TPT Min	Total Cost Min*
Revenue	0.00%	35.42%	33.78%	100.00%
TVT	3.68%	0.00%	0.85%	21.11%
TPT	5.93%	0.26%	0.00%	14.23%
Total Cost	9.87%	4.26%	4.44%	0.00%
VMT	1.48%	0.00%	0.14%	0.19%

(* Status Quo)

As shown in the table, there seems to be significant variation in the performance measures as the decision objective is varied. The results here suggest the amount of revenue generated is the most sensitive of all the measures, with a range of 0 to 100%. As noticed in all of the programs, total user cost would always be higher than in the pre-conversion scenario. Also, this measure is farthest from its optimal under the revenue maximization program.

There seems to be very little difference between TPT and TVT under their minimization programs. However, they behave differently under other programs. Improvement in one measure does not seem to be translating proportionately into improvement in another. For

instance, while TPT increases by 5.93% under revenue maximization, TVT increases by 3.68%. In addition to this, under the total cost program, while TPT increases by 14.23%, TVT increases by a larger percentage of 21.11%. While TVT is mainly of concern to the system planners particularly due to emissions, TPT is a measure more on the user side. This difference thus needs to be further investigated to observe the movements of these functions across various values of tolls.

4.7.3 Impact on Number of Vehicles:

The variation of VMT here seems to be minor across different programs. While TVT and TPT minimization programs have a beneficial (negative) impact on the number of vehicles, revenue maximization increases the number of vehicles on the road. The modal splits under each of these programs are shown below (Table 4.29).

Table 4.29: Variation in mode shares across programs

Mode	Rev Max	TVT Min	TPT Min	Total Cost Min*
SOV	43.60%	41.22%	41.73%	43.43%
HOV2	31.51%	35.02%	33.62%	26.94%
HOV3	24.89%	23.73%	24.65%	29.64%

(* Status Quo)

The common phenomenon across the first three programs is the increase in the mode share of the two-person carpools. This seems to be happening due to two reasons in the TVT and TPT minimization programs: a) breaking up of some three-person carpools, b) formation of new carpools by SOV users. In the revenue maximization program, however, the second effect does not seem to be taking place. Here, some of the three-person carpools seem to be breaking up and forming two-person carpools or SOVs. This

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is expected since three-person carpools would not generate any revenue and thus the tolls under revenue maximization should be such that more SOVs and HOV2s are formed. This also explains the increase in the number of vehicles under the revenue maximization regime.

The above results on the mixed impact on carpools are along the lines of the inferences drawn from other studies [70].

4.7.4 Impact on Travel Times and Volumes:

Table 4.30 shows the travel times and the volumes on the managed and general lanes across various programs.

Table 4.30: Variation in travel times and volumes on lanes across programs

	Rev Max	TVT Min	TPT Min	Total Cost Min*
Travel time on general lanes	6.54 min	6.37 min	6.49 min	8.11 min
Travel time on HOT lane	6 min	6 min	5.74 min	4.75 min
Flow on general lanes	6155 vph	6044 vph	6124 vph	6396 vph
Flow on HOT lane	1450 vph	1450 vph	1380 vph	1111 vph

(* Status Quo)

While there is an improvement in the travel times on the general lanes, there is an increase in the travel times on the managed lane across all the conversion programs. In other words, the conversion here drives the travel times on both types of lanes closer to each other than they were before, thus addressing the "empty lane syndrome". Also, while the managed lane quality constraint seems to be binding under revenue maximization and TVT minimization, there does not seem to be any improvement in the TPT that could be brought about by pushing more people into the managed lane, unlike the case for the other two objectives.

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Note that the narrow range of travel times here might be due to the relatively small stretch being considered. A longer corridor might show a greater difference between the travel times.

4.7.5 Impact on Emissions:

The impacts of conversion on emissions are somewhat mixed. The best objective with respect to this seems to be TVT, with a slight increase in CO emissions alone. This increase is still less than that of other conversion programs and it should be noted that CO emissions are higher under all the conversion programs when compared to status quo. Emissions of NO_X and VOC are, however, reduced under TVT minimization. Revenue maximizing, on the other hand, seems to result in values that are greater than the initial emissions for all three types of emissions due to an increase in the VMT under this regime.

An interesting observation here is that while the emissions contribution from the vehicles on the general lanes decreased from HOV to HOT scenario, the emissions from vehicles on the managed lane increased under all the programs for all three types of emissions. This is because of the decrease in speeds on the HOT lanes and increase in the number of vehicles on the HOT lanes under all the programs. It should be noted that all the predictions for emissions are very close to their initial values and given the simplistic model used here, these inferences need to be strengthened by further studies that specify more accurate relationship between emissions and other traffic condition. Evaluation studies of HOT lane projects, however, revealed negative impacts on emissions [36], [37].

4.7.6 Impact on Managed Lane Use Propensities:

The average probability of individuals belonging to different income groups choosing to use the managed lane under different programs is shown in Table 4.31.

Table 4.31: Comparison of managed lane use propensity by income groups across programs

Income Groups↓	Rev Max	TVT Min	TPT Min	Total Cost Min*
Quartile 1 (Low)	0.361	0.384	0.371	0.322
Quartile 2	0.336	0.354	0.342	0.303
Quartile 3	0.309	0.322	0.313	0.286
Quartile 4 (High)	0.284	0.289	0.284	0.274

(* Status Quo)

It can be seen that under each of the first three regimes, the propensity to use the managed lane increases across all of the income groups, compared to the total cost minimization program (the status quo). Note that even though the numbers of vehicles on the managed lane are the same under revenue maximization and TVT minimization, the probability is higher in case of TVT. This is because more people use the managed lane (as carpools) under TVT minimization, thus increasing the overall probability of an individual choosing the lane. This reiterates the earlier inference that while TVT and TPT minimization improve the managed lane utilization by encouraging carpooling, revenue maximization might do the same at the cost of carpooling.

A similar observation can be made when comparing the managed lane use propensity by trip types. The probabilities for all of the groups showed an increase. The largest increase was under TVT minimization followed by TPT minimization and revenue maximization, in that order. The results for segmentation by carpool formation cost were along expected

lines too, with individuals under TVT minimization showing greater propensity to use the managed lane.

4.7.7 Temporal Equity:

Table 4.32 shows the average costs experienced by users belonging to different income quartiles under different programs.

Table 4.32: Comparison of average travel costs of by income groups across programs

Income Groups	Revenue Max	TVT Min	TPT Min	Total Cost Min*
Quartile 1	1.112	1.041	1.044	0.819
Quartile 2	0.746	0.700	0.702	0.574
Quartile 3	0.515	0.486	0.487	0.418
Quartile 4	0.371	0.351	0.351	0.318

(* Status Quo)

The results here indicate that conversion here leads to an increase in the costs for all the income groups with the maximum costs occurring under the revenue maximization program and least costs under the status quo. The imposition of temporal equity constraints on the various programs leads to different levels of efficiency losses. Table 4.33 shows the percentage loss in efficiency when the constraint is imposed that the average costs of all the income groups should be non-increasing with time.

Table 4.33: Loss in efficiency due to imposition of temporal equity across programs

Program	% loss
Rev Max	97.01%
TVT Min	8.99%
TPT Min	4.14%
Total Cost*	0.00%

(* Status Quo)

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It can be seen that revenue seems to be the most sensitive measure as the percentage loss was much higher under revenue maximization. Another indication of the higher sensitivity of revenue is the threshold at which the efficiency loss begins when (relaxed) temporal equity constraints are imposed. This threshold is 37.5% for revenue and is the highest of all the conversion programs.

4.7.8 Vertical Equity:

As shown in the table individuals belonging to lower incomes lose out more than individuals from higher income groups across all the programs. It was also observed that the deviation in the costs, i.e. the extent of vertical inequity, is larger after conversion when compared to the pre-conversion scenario. The results also indicate that the costs experienced by the users and the extent of vertical inequity are the largest under the revenue maximization program.

Furthermore, analysis with vertical equity-related constraints revealed that the two characterizing thresholds for vertical equity are very similar for all three conversion programs. However, TVT and TPT programs having a slightly larger interval, over which the inverse relationship between equity and efficiency can clearly be observed, when compared to revenue maximization. The percentage loss, as in the case of temporal equity, was much higher for revenue maximization.

4.7.9 Other Equity-related Comments:

a) An important observation here is that while the relationships between equity and efficiency were linear in the case of revenue maximization, they were quadratic for

the other programs. This implies that the loss in efficiency due to a unit increase in equity is the same for all levels of equity under revenue maximization. On the other hand, the loss in efficiency per unit increase of equity increases as the extent of equity decreases under TVT and TPT minimization. Thus, in addition to political considerations, the choice of equity level has to be based on the planning agency's performance measure as well since this measure will affect both the extent and nature of the loss in efficiency.

b) Another general trend across all the programs here is that while it is possible to ensure that all the income groups gain from the conversion, the distribution in benefits/ costs worsens after the conversion - i.e. the gap between income groups increases. In other words, even though temporal equity can be ensured, the conversion has a detrimental impact on vertical equity. Note that this observation is only in the context of equity with respect to average travel costs.

c) As mentioned earlier, there does not seem to be any necessity to investigate the equity issues related to travel times. Not only do the travel times of different groups improve after the conversion, but they also move closer to each other - i.e. the extent of vertical inequity is reduced. Furthermore, the travel times of the lower income groups were found to be lower than those of higher incomes, hence obviating the vertical inequity aspects.

d) In cases where full redistribution is possible, it would be best to implement the unconstrained version and compensate the losers later on. This approach leads to lower losses in revenue when compared to imposing equity related constraints under the revenue maximizing program. However, unconstrained problem might not always

be the best choice when the redistribution mechanisms are not 100% efficient and this would involve a choice of equity level that needs to be imposed as constraints. Note that these choices under the TVT and TPT minimization programs involve tradeoffs among equity, revenue and efficiency (TVT and TPT) and thus may not be as straightforward as under revenue maximization. On the whole, redistribution would play a very important role and the planning needs to be guided by the available redistribution schemes and their effectiveness.

The above analyses and discussion address several interesting questions that have been posed in the context of HOT lanes. Some of these questions and their answers are synthesized below.

Question	Answer
1. Is it advisable to convert a HOV lane under *all* the programs?	*No*. Least total cost occurs in the HOV lane scenario for this case study. Other measures are better than status quo under corresponding programs. However, it might be optimal to convert a HOV lane under all the programs for other corridors, as demonstrated using an example.
2. Does the managed lane utilization improve under the HOT scenario?	*Yes*. There is an improvement in the utilization level of the managed lane. It is also possible to maintain reasonable speeds on this lane. The propensity to use the lane increases in all the income groups.

3. Does the HOT lane promote carpooling?	*There is no fixed answer to this.* There is a possibility of breaking up three-person carpools to form SOVs or HOV2s in order to save on carpooling costs. There is also the possibility of solo users combining to form two-person carpools. The net effect on the total number of vehicles depends on the program.
4. How long did it take to recover the capital costs?	Depending on the program, it can take *three to five years* to recover all the costs.
5. Which measure is most sensitive to the choice of the objective function?	*Revenue.* Revenue ranges from $ 2825/hr (Revenue maximization) to $0/hr (Cost minimization/ Status quo). The other measures that were tested for and found to be less sensitive in this case study include TVT, TPT, Total Cost and VMT.
6. Do HOT lanes reduce emissions?	*The results for emissions are mixed.* TPT and TVT minimization result in beneficial impacts on some of the emissions. Revenue maximization, however, has a detrimental impact on all the emissions due to increase in the total VMT.
7. Do all the income groups gain due to this conversion?	*No.* The unconstrained programs lead to losses for all the groups in terms of cost. There is, however, an improvement in the travel times for all the groups.

8. Are there sufficient revenues to compensate the losing groups?	*Yes.* Sufficient revenues are also generated to ensure that no body loses due to conversion through redistribution.
9. Is it possible to ensure that no income group loses out without any redistribution?	*Yes*. It is possible to impose temporal equity of cost constraints and find an optimal solution under all the programs.
10. Is it possible to better the degree of vertical equity that exists under status quo?	*No*. All the programs suggesting conversion had a detrimental impact on vertical equity. It can, however, be brought very close to the initial situation by imposing constraints. Also, revenues are sufficient to achieve a pre-conversion level of vertical equity through redistribution.
11. Which dimension of equity has a higher impact on efficiency?	*Vertical equity*. Imposition of vertical equity leads to higher losses in efficiency when compared to temporal equity under all the programs.
12. Which objective function is most sensitive to equity considerations?	*Revenue.* The loss in efficiency is the greatest under revenue maximization when compared to other programs. Also, the extent of flexibility offered in terms of imposing the equity constraints is smaller in the case of revenue maximization.

Note that the above inferences are specific to the case study here and may not hold in all cases. Also, it is very important to note that all the results in this chapter are for this

particular choice model and it is possible to have other patterns of user behavior leading to different results.

4.8 CHAPTER SUMMARY:

The methodology described in chapter 3 was implemented for a selected corridor on I-80. A choice model was estimated and the coefficients were found to be reasonable. This model was used in various optimization regimes and their results were discussed. The impacts on performance measures, managed lane usage characteristics and equity issues were presented for all of the programs and a comparison was made. The value of this chapter lies in the demonstrative power of the approach and the results by themselves may not be directly applicable.

5. OPTIMAL SEQUENCING OF HOT LANE PROJECTS

Planning agencies in a number of metropolitan areas are considering HOT lanes at multiple locations. The Metropolitan Transportation Commission (MTC) of the San Francisco Bay Area, for instance, is considering an "Existing and Funded Network" which would be developed by converting into HOT lanes the HOV lanes that currently exist [54]. The development of HOT lanes in an area, however, is a time as well as cost intensive process (MTC estimates the capital cost of putting an HOT lane in place to be $1.4 million to $3.7 million per mile). Thus, a primary constraint for a regional planning agency, entrusted with the completion of multiple HOT projects, would be the availability of budget for building the planned projects all at once. Hence, a phased approach that implements various conversion projects in a sequential manner would be necessary.

Such a phased approach has the potential to reduce the external funding for the projects and can introduce a degree of financial sustainability. This can be done by using revenues from completed and operational HOT lanes for financing the implementation on later segments. The other major advantage of adopting a multistage approach to this problem is that it will allow the planners to assess the individual behavior and progressively refine the behavior model. Hence, an important question that needs to be answered by the planning agency here is the optimal conversion sequence/ schedule to be followed.

A methodology, that would solve for the optimal sequence/ schedule of conversion (into HOT lanes) to be followed given the set of HOV lanes, is developed in this chapter. Two pertinent questions that would be answered by the models presented in this chapter include:

a) What is the conversion sequence/ schedule that minimizes the total upgrade (conversion) time without any external funding for any of the projects?

b) What is the conversion sequence/ schedule and the cash flow that minimizes the external funding necessary for completing all of the projects within a certain time limit?

The models providing answers to the above questions are constructed using a Dynamic Programming framework. The optimal sequences (in the sense of both (a) and (b)) were arrived at for a chosen set of existing HOV lane corridors in the Bay Area by implementing the above models. Figure 5.1 shows the entire "Existing and Funded Network" of HOV lanes in the Bay Area. The chosen corridors are projected to reach 85% of the HOV lane capacity by 2020 and should, thus, be accorded a higher priority for conversion since the impending crowding would imply reduced revenue generating potential earlier for these projects when compared to others [54].

Figure 5.1: Existing and Funded HOV Network in Bay Area

The location and extent of the five HOV corridors that are to be converted into HOT lanes are shown in red and orange. Note that except for the lanes on I-80, HOV2s are allowed to use the HOV lane under the current scenario. Also, for the purpose of analysis in this chapter, the annual growth in the HOV lane usage is not considered.

The revenue generation rates of each of these projects were computed before formulating the models that solve for the least amounts of conversion time and money. The lengths, locations, capacities and the capital costs (for conversion) corresponding to each of the five corridors were obtained from [54]. The demand data for each of the segments were collected from the PEMS database and the average peak hour volume computed. The passenger demand was then arrived at by using the average modal shares during the peak period in the Bay Area. All these details for each of the five corridors are shown in the Table 5.1. These values were then substituted in the revenue maximization model (presented in the previous chapter) and the annual revenues from each facility computed.

Table 5.1: Details of the five corridors chosen for HOT implementation

HOT #	Corridor	Demand	Length	# Lanes	Capital Cost	Annual Revenue
I	I-80W	7794 vph	5 miles	5	$18.5 million	$ 6.36 million
II	I-80W	6262 vph	16 miles	4	$59.2 million	$ 6.71 million
III	I-680S	7467 vph	5.1 miles	5	$11.22 million	$ 6.51 million
IV	I-580W	4943 vph	10.6 miles	4	$23.32 million	$ 5.36 million
V	SR-85N	3269 vph	13.9 miles	3	$19.46 million	$ 3.84 million

These values were then used to arrive at the optimal sequences by solving the models described in the next two sections. In addition to the optimal sequence models, the next section also presents an analysis of the loss in efficiency that occurs due to the existing

state law that prevents spending revenues from a given segment on other corridors' development.

5.1 TOTAL CONVERSION TIME MINIMIZATION:

The possibility of the HOT lane projects raising the revenue necessary for converting all the HOV lanes under consideration is examined using the model described here. The self-financing sequence, which minimizes the total amount of time needed to convert all the lanes, is sought through the Dynamic Programming model formulated here.

The following assumptions were made in order to simplify the model and also to reduce the computational requirements:

a) Only one lane can be converted during one time period and the length of one time period here is one year. Thus, a new project can be opened only at the beginning of each decision period, i.e. each year.

b) The payment for the upgrade is done once the lane has been converted i.e. it is not necessary to have the capital costs in hand before commencing the conversion process on a given corridor.

c) Completion of one project does not affect the nature of the demand for other corridors that need to be converted.

d) No discounting factor (interest rate) is applied to the revenues obtained.

All of the above assumptions (except for c) can easily be relaxed without any change in the basic framework. In cases where two corridors can be converted simultaneously, the minimum time interval between any two projects can be reduced to a small enough value (a week, for instance). The payment-at-the-end assumption can also be relaxed by

incorporating information about the expected conversion time for each of the projects into the model. A brief analysis relaxing the assumption on no discounting factor/ interest rate is carried out and presented at the end of this section.

Note that there is a certain amount of initial investment that is necessary in order to set in motion the HOT lane revenue generation. At least one of the five HOV lanes being considered has to be converted into a HOT lane in order to generate revenues for the next conversion. The initial investment, however, can be recovered once the whole conversion process is completed. Thus, the conversion time minimizing sequence would be contingent on the initial corridor that is chosen for conversion.

The Dynamic Programming (DP) formulation for arriving at the conversion sequence and the schedule of conversions is given below.

Stage: One year

State variables: Set of n projects that are to be converted (S_n) and available budget B_n

Decision variable at each stage: Project to be converted (i) or no conversion

Recurrence relation:

$$\Phi_n\ (S_n, B_n) = \min \begin{cases} 1 + \Phi_{n+1}\ (\{S_n - i\}, B_n + R(\{S_n - i\}) - C_i)\ , \forall i \in F \\ 1 + \Phi_{n+1}\ (S_n, B_n + R(S_n)) \end{cases} , n < N$$

Boundary conditions:

$$\Phi_n(\{\}, B_n) = 0 \text{ and } \Phi_N(S_N, B_N) = 1$$

where

n = Current stage (year),

N = Planning horizon,

F = Set of projects that can be completed with the available budget (feasibility region)

Φ_n $(S_n,\ B_n)$ = Minimum conversion time needed from the beginning of year n with S_n to be converted and with available budget B_n,

C_i = Capital cost needed to convert link i,

$R(S_n)$ = Annual revenue generated from the projects finished with S_n still to be converted.

The above formulation is based on the Bellman's principle of optimality which gives the necessary condition for an optimal policy [71]. In the current context, the requirement of the optimal policy, as encapsulated in the recursive relation, can be interpreted in the following manner: given a state (the current set of projects to be repaired and the budget), the action to be taken (convert a lane or do nothing) minimizes the objective function (the total conversion time from the current state) for the rest of the planning period.

The above model has been implemented in MATLAB (code shown in Appendix B) and the optimal conversion schedules corresponding to different initial conditions were obtained. Depending on the choice of the initial project, the optimal conversion time was found to range from seven to 11 years. The optimal conversion schedules to be followed for the next thirteen years (till 2020) are depicted using timelines (Figure 5.2) for different initial conditions. These timelines also show the amount of money (in million $) the planning agency would possess at the beginning of each year.
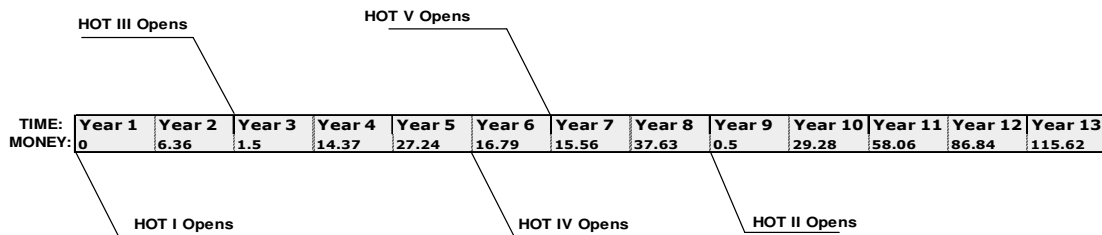
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13
TIME:													
MONEY:	0	6.36	1.5	14.37	27.24	16.79	15.56	37.63	0.5	29.28	58.06	86.84	115.62

Figure 5.2 (a): Optimal schedule when project I is undertaken first

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HOT I Opens   **HOT V Opens**

TIME:	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13
MONEY:	0	6.71	13.42	1.63	3.48	3.5	3.6	32.38	61.16	89.94	118.72	147.5	176.28

HOT II Opens   **HOT III Opens**   **HOT IV Opens**
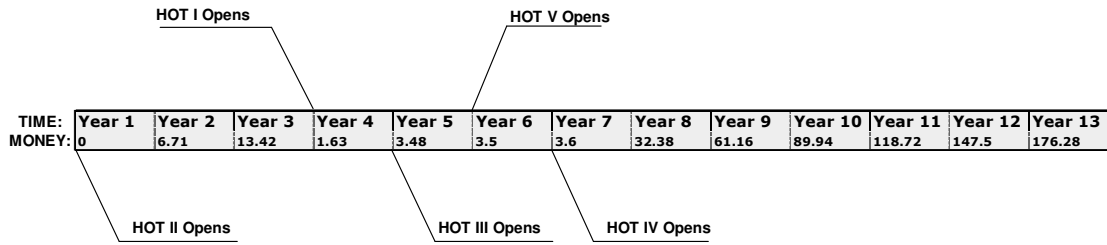
Figure 5.2 (b): Optimal schedule when project II is undertaken first

HOT I Opens   **HOT IV Opens**   **HOT II Opens**

TIME:	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13
MONEY:	0	6.51	13.02	1.03	13.9	26.77	16.32	15.09	37.16	0.03	28.81	57.59	86.37

HOT III Opens   **HOT V Opens**
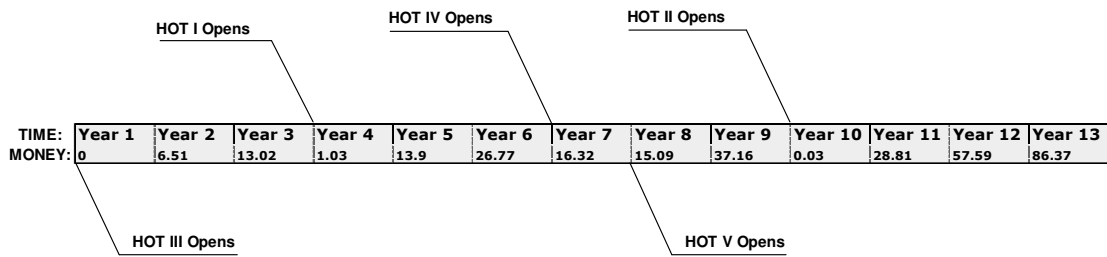
Figure 5.2 (c): Optimal schedule when project III is undertaken first

HOT I Opens   **HOT V Opens**

TIME:	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13
MONEY:	0	5.36	10.72	16.08	2.94	3.44	21.67	39.9	38.67	1.54	30.32	59.1	87.88

HOT IV Opens   **HOT III Opens**   **HOT II Opens**
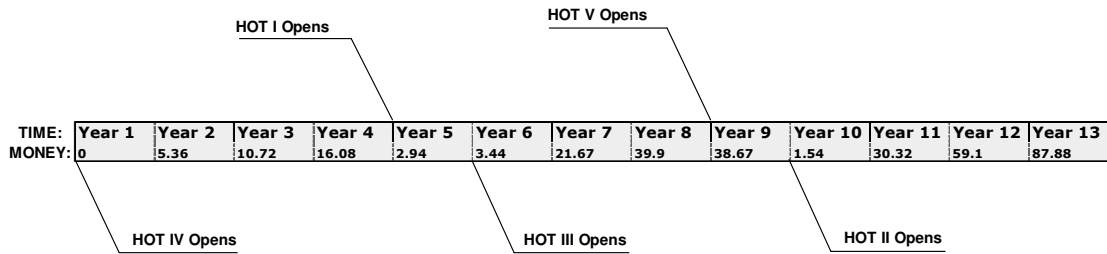
Figure 5.2 (d): Optimal schedule when project IV is undertaken first

HOT III Opens   **HOT IV Opens**

TIME:	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13
MONEY:	0	3.84	7.68	11.52	4.14	14.49	6.34	23.05	16.44	38.51	1.38	30.16	58.94

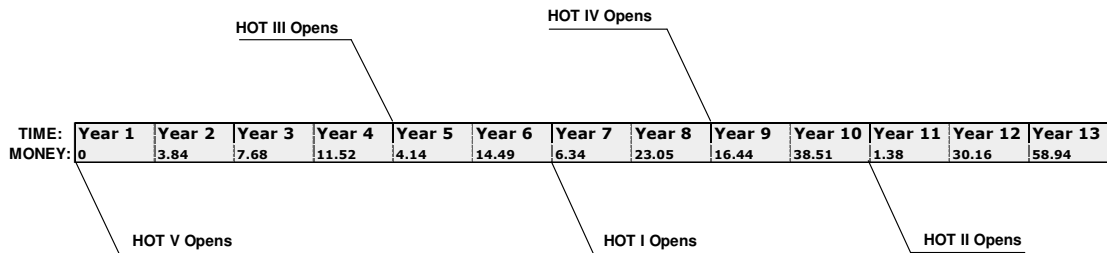**HOT V Opens**   **HOT I Opens**   **HOT II Opens**

Figure 5.2 (e): Optimal schedule when project V is undertaken first

Figure 5.2: Optimal schedules with differing initial projects

As expected, the least amount of conversion time corresponds to the case where project II, i.e. the most capital intensive project, is handled first. The total conversion time here was seven years as opposed to 11 years resulting from converting project V first.

Note that this time is the time required to complete all of the projects and does not include the time needed to recover the initial costs. Furthermore, the optimal sequences obtained in each of the above five cases are not unique. Given the initial project, there also exist other sequences which result in the same amount of conversion time. For instance, another optimal schedule which results in seven years when project II is handled first is shown in Figure 5.3.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10	Year 11	Year 12	Year 13
TIME:													
MONEY:	0	6.71	2.2	15.42	10.14	6.4	11.88	40.66	69.44	98.22	127	155.78	184.56

HOT III Opens — Year 1
HOT IV Opens — Year 6
HOT II Opens — Year 2
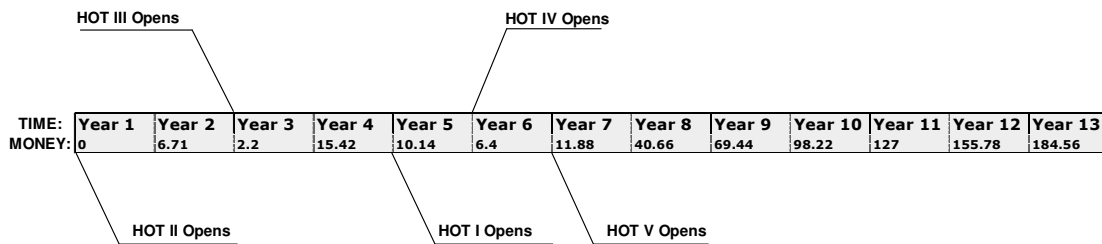HOT I Opens — Year 5
HOT V Opens — Year 7

Figure 5.3: Alternative optimal schedule when project II is undertaken first

It is clear from the above results that the HOT lane projects would be able to fund themselves and recover all of the costs in a reasonable amount of time. However, this process entails using revenues from one corridor to convert the lanes on other corridors, thus potentially benefiting other users and not the users who actually paid the toll. This might lead to concerns about spatial inequity. The current state legislation dictates that the revenues from a given corridor should be used for investments on the same corridor. Thus, the usage of revenues from one project on other projects might be problematic due to, what may loosely be called as, spatial equity in redistribution.

Spatial equity is usually concerned with how HOT lanes distribute benefits across different geographical groups. Aspects of spatial equity include distribution of benefits across different travelers based on trip distances and access points to different communities [46]. Thus, the usage of revenue constitutes only a part of it. Furthermore, capturing spatial equity here is complicated by the fact that the users of each corridor might not be distinct from each other, i.e. the same individual (especially in situations like projects I and II) might be using multiple corridors in the same trip.

The current legislation requiring the revenues from a corridor to be spent on the same corridor, thus, ensures spatial equity with respect to revenue utilization. However, as observed in the previous chapter, there is a loss in efficiency associated with an increase in equity. The equity-efficiency relationship was examined here by computing the conversion time, as the percentage of revenue from a project that could be spent on the other projects is decreased (i.e. as spatial equity increased). The current legislation sets this percentage (δ) value to zero percent, while the above conversion time results (figure 5.2) assumed this value to be 100%. The δ was then varied and the loss in efficiency i.e. increase in conversion time, was recorded (Table 5.2).

Table 5.2: Variation in the total conversion time with increasing spatial equity

δ	Conversion Time
100.00%	7 years
90.00%	7 years
80.00%	9 years
70.00%	10 years
60.00%	10 years
50.00%	12 years

All of the above results correspond to the case where project II is carried out ahead of the others. It can be seen that as δ decreases (more spatial equity), the time needed to convert all the lanes increases, thus confirming the inverse relationship. Note that this relationship is sensitive to the choice of the minimum time period between two projects. For instance, the objective value remained unchanged at seven years as δ was decreased from 1 to 0.9. This, however, might not be true when the time period is decreased from one year and the conversion time corresponding to δ equals 0.9 might be higher.

It should also be noted that the above analyses correspond to the case when the revenues obtained are not acted upon by any discounting factor or interest rates. The implication of this assumption is that the money in hand currently is worth same as the money that is obtained in the future. This, however, is hardly the case in reality and it can be seen that the conversion times here can be reduced by investing the revenues elsewhere and thus growing the money. In such situations where an interest rate (γ) can be assumed, a minor modification in the recursive relation needs to be made in the following manner:

$$\Phi_n (S_n, B_n) = \min \begin{cases} 1 + \Phi_{n+1} (\{S_n - i\}, (B_n + R(\{S_n - i\}) - C_i)\gamma) &, \forall i \in F \\ 1 + \Phi_{n+1} (S_n, (B_n + R(S_n))\gamma) \end{cases}, \ n < N$$

In order to examine the extent to which this interest rate can impact the solution to the original problem, this γ was varied from 0 to 20 % per annum and the total conversion time in the case where project I was opened first observed. Experiments here revealed that there was a decrease of only one year for interest rates higher than 6.95% per annum i.e. the total conversion time decreased from nine to eight years for interest rates higher than 6.95% per annum. There was, however, no change in the objective value for interest rates lower than 6.95% per annum.

As noted in the case of δ, the variation in the objective value is very much dependent on the minimum time between the two projects. Additionally, the manner in which the compounding is performed also has an impact on the objective value. For instance, compounding revenues annually might yield different results from the situation where the money is compounded quarterly.

5.1.1 Additional Comments:

a) The optimal solution from the DP approach to this sequencing problem need not always coincide with the heuristic greedy approach solution. The greedy strategy, in this context, may be described in the following manner: Projects with higher revenue to cost ratio are given priority over others and the conversions are carried out as soon as there is enough money to do so. This approach here yielded the following sequence: III → I → IV → V → II. This strategy yielded optimal objective values for all five cases with different initial conditions (Figure 5.2).

However, situations where this prioritization scheme does not work can be constructed. One such instance is the case where only 50% of the revenues could be used on the improvement of other corridors. As shown in the table, the optimal value here was 11 years. However, the greedy strategy (with initial condition as project II open) here yielded 12 years as the objective value – an increase of one year.

b) The DP model here can easily be modified to solve for sequences that would optimize other objective functions. An example of an objective other than minimizing total conversion time would be minimization of the total vehicular time over the whole of the conversion period. An interesting aspect of solving for sequences with other objectives

would be the choice of the lower level objective. The revenue maximization objective for the lower level problem will clearly minimize the total time needed for conversion. However, the same would not hold when the upper level objective (objective for optimal sequence) changes to minimization of TVT or to something else. The appropriate lower objective then could be any of the objective functions discussed in chapter three and further analysis is needed to determine the best lower level objective to be used.

c) This model can act as a preliminary step towards further research on the development of HOT lane networks that are being considered by a number of agencies from various states including California, Florida, Texas and Virginia. The HOT networks, apart from inducing significant time savings, would aid in providing seamless connections between different road segments and have a higher likelihood of attracting more patronage [54]. The model here, as stated in the assumptions, ignores the network effects i.e. the conversion of one lane does not influence the demand patterns on any of the other corridors. A better model for computing the sequence would be to treat this problem as a variant of the multistage network design problem. Note that the lower level problem would then be an equilibrium model and consequently, the complexity of the problem would increase many fold.

5.2 MINIMIZATION OF EXTERNAL FUNDING:

The objective of this model is to minimize the total amount of external funding that would be needed to complete the conversion process by a certain deadline. In addition to the conversion schedule, the solution to the DP model here would thus give the cash flow that needs to be administered through the conversion process. The flow of external

funding would be described by the amount of money that needs to be allotted to the process every year. The formulation of the DP model for this problem is shown below.

Stage: One year

State variables: Set of n projects that are to be converted (S_n) and available budget B_n

Decision variable at each stage: Action i.e., project to be converted (i) or no action and money to be added into the process (f_n)

Recurrence relation:

$$\Phi_n \ (S_n, B_n) = \min \begin{cases} f_n + \Phi_{n+1} \ (\{S_n - i\}, B_n + R(\{S_n - i\}) - C_i + f_n) \ , \forall i \in F \\ f_n + \Phi_{n+1} \ (S_n, B_n + R(S_n) + f_n) \end{cases} \ , n<N$$

Boundary conditions:

$$\Phi_n(\{\}, B_n) = 0 \ \text{and} \ \Phi_N(S_N, B_N) = \infty$$

where

$\Phi_n \ (S_n, B_n)$ = Minimum external funding needed from year *n* with S_n to be converted and with available budget B_n,

all other variables as defined in 5.1.

The above model was implemented in MATLAB (code shown in the Appendix B) for the five segments under consideration. The least amount of external funding needed in order to complete the conversion in a period of five years was found to be $106.76 million As expected, numerical experiments here confirmed that the cumulative amounts of external funding needed for completing all the five projects decreased by relaxing the deadline for completing the projects.

5.3 CHAPTER SUMMARY:

This chapter dealt with the optimal conversion schedules when an agency is faced with multiple conversion projects. Dynamic Programming models solving for minimum conversion time and minimum external funding were formulated and solved. The minimum conversion time, depending on the initial conditions, ranged from seven to 11 years. However, this conversion time might be revised upwards because of the current legislation on spending of the revenues. The results in this context may be thought of as a quantification of the benefits that might be brought about by relaxing the current legislation. The greedy method for scheduling was found to be performing well for most, if not for all, of the cases. A DP formulation that minimized the total amount of money needed to complete the conversion process by a certain time was also presented. As expected, the objective here improved as the deadline was relaxed.

6. CONCLUSION

An optimization model that can be used to evaluate the impacts of the conversion of a HOV lane into an HOT lane was constructed in this study. The objective functions for the optimization model here include revenue, total vehicular time, total passenger time, total cost and total number of vehicles. The various constraints imposed on the problem here include travel time constraints (including service constraint), behavior model related constraints, equity constraints (temporal and vertical equity with respect to travel time and costs), constraints on toll and constraints describing the current HOV lane scenario.

A behavior model was estimated for a selected stretch on I-80 and this optimization model was applied to evaluate the conversion of the HOV lane. The impacts of the conversion on various performance factors, propensity of different users to use the managed lane, vertical and temporal equity were examined and a comparison of these aspects across different programs was made.

Next, a Dynamic Programming model that could be used to arrive at a self-financing conversion schedule, which enables completion of all the HOT lane projects in the least possible time, was constructed. The increase in the number of years needed for completing all the projects that arises due to increased usage of revenues on the revenue generating corridors themselves was also examined. In addition to these, another DP model that can be solved for the conversion sequence and the cash flow that minimizes the external funding was also constructed. The additional condition in this second model was that all of the projects are to be completed by a certain deadline. These models were then tested on five chosen corridors in the Bay Area.

The distinguishing features of the analysis here include incorporation of equity into the planning process, the relationship between different dimensions of equity and different measures of efficiency, analysis of the variation of different performance measures across programs and development of a multistage model for sequencing of HOT projects. Implementation of the various models for the case study enabled drawing several inferences, the most salient of which are listed below:

1. Conversion of a HOV lane into a HOT lane need not always improve all the system performance measures. There is, however, a beneficial effect on most of the measures under the majority of optimization programs.

2. There is a considerable variation in the measures as the objective function is changed, with the revenue generated being the most sensitive. This observation suggests that choice of an appropriate objective plays a very important role in the decision making process. The impact on emissions, in particular, seems to be mixed and depends on the program.

3. The managed lane usage propensity seems to be increasing across all the income groups after the conversion has been effected. In addition to improving the utilization of the managed lane, the case study also showed that it would be possible to maintain 50mph speed on the HOT lanes.

4. The results from the case study indicated that there might be a decrease in carpooling, i.e. an increase in the number of vehicles, under the revenue maximization program. The number of vehicles under the TPT and TVT minimization programs, however, is less than the number before conversion.

5. All the equity versus efficiency analyses here suggested that there is an inverse relationship between the two. Increase in equity leads to a loss in efficiency. The measure most sensitive to equity was found to be revenue.

6. There were also significant differences observed in the nature of the relationship between equity and efficiency across programs. While the relationship was linear under revenue maximization, a quadratic relationship was observed under TVT and TPT minimization.

7. Under unconstrained versions of the program, i.e. when no equity constraints are in place, the costs experienced by all groups of users increase after the conversion with the increase being the greatest in the case of the lower income groups.

8. Imposition of temporal equity constraints can ensure that all income groups' costs are no greater than the costs in the pre-conversion scenario. Also, revenues generated are sufficient to compensate all the groups in the case of perfect redistribution.

9. Although temporal equity can be guaranteed, the conversion does not improve on the pre-conversion vertical equity levels. In other words, the conversion improves all groups' welfare but also necessarily drives their well being farther from each other.

10. As far as the travel times are concerned, all the groups' average travel times improve when compared to the pre-conversion scenario.

11. The loss in efficiency is more sensitive to the imposition of vertical equity when compared to temporal equity. Thus, it is easier to improve the standing of all

groups when compared to the efforts needed to reduce the deviation/ spread of the standings.

12. It is possible to upgrade the current HOV infrastructure to HOTs in a self-sustaining manner using a phased approach. However, the state law in this regard might need to be revised in view of the potential benefits.

As mentioned previously, the above inferences are specific to the case study and might not be applicable elsewhere.

An important issue, in the context of planning for new projects, raised by the equity analysis in this study is as follows: Is it worth carrying out a project if it worsens the extent of deviation, i.e. increases the spread, of the benefits/ costs across income groups but does result in an improvement of everybody's welfare. In other words, the project improves the temporal equity but has a detrimental impact on the vertical equity. One implication of this phenomenon is that though such project might be able to pass independently conducted majority voting, it might be opposed by policy makers since this might not be a sustainable way forward, potentially leading to frictions in the future. Thus, policies which are marketed to be beneficial to all (Pareto-improving) would also need to be reviewed for their impact on the equity issues. It should be noted here that the efficiency-equity relationship is contingent on the definition of efficiency itself in multiple ways. Similarly, as shown in this study, the definition of equity is of paramount importance when conducting such analyses.

The results from this study also emphasize the need for further consideration and adoption multi-criteria optimization techniques in the planning process. This is especially

so given the conflicting nature of a number of objectives (such as improve travel speeds – improve safety) for agencies.

Lastly, it should be noted that the methodology presented here is for planning purposes only. The various impacts of the recommended policies are only approximations and are bound to be influenced by actual ground conditions. Thus, these policies should necessarily be revised based on the feedback from operating the projects.

Future Work:

In spite of useful insights from this study, a number of avenues exist for improving and building upon the models presented in this study:

1. A comprehensive behavior model that is estimated from real Stated Preference data would contribute significantly towards improving the confidence in the predictions and inferences drawn from this study. In particular, incorporation of issues related to fampools (carpooling by members of the same family) and improved reliability on the managed lanes might play an important role in capturing the behavior of various segments of road users.

2. The model here can be refined further by taking a network approach rather than the single corridor based approach. However, this would involve solving a bilevel problem with User Equilibrium as the lower level problem for a transportation network. This task might thus be a computationally challenging exercise.

3. Another interesting avenue would be to examine the multistage model by relaxing the assumption about no network effects. In other words, this would involve constructing a multistage model that would account for the changes in the demand

patterns of other corridors brought about by the conversion of a HOV lane on one corridor.

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DISCLAIMER

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APPENDICES

APPENDIX A

AMPL code:
```
############
set income;
set ttype;
set hovgrp;

##################
param incoeff{income};
param wage{income};
param value{ttype};
param hovcost{1..3,hovgrp}; #carpool costs
param hovbias1{1..3};
param hovbias2{1..3};
param hovbias3{1..3};
param capacity1;
param capacity2;
param altves;
param ocost{1..altves};
param coeff;
param equity;
param equityt;
param demand;
param length;
param oppcost{1..2};
```

```
param prop1;
param prop2;
param prop3;
param tprop{ttype};
param alpha;
param beta;

###########################
var probability1{income,ttype,1..altves,hovgrp};
var probability2{income,ttype,1..altves,hovgrp};
var probability3{income,ttype,1..altves,hovgrp};
var altcost1{income,ttype,1..altves,hovgrp};
var altcost2{income,ttype,1..altves,hovgrp};
var altcost3{income,ttype,1..altves,hovgrp};
var profit;
var time1;
var time2;
var sov1toll;
var hov2toll;
var totvehtime;
var totpsngrtime;
var totveh;
var totalcost;
var totcost{1..3};
var avgtime;
var demand1{income,ttype,hovgrp};
var demand2{income,ttype,hovgrp};
var demand3{income,ttype,hovgrp};
var avgeqvtimecost;
var grptime{income};
var grpcost{income};
var probability01{income,ttype,1..3,hovgrp};
var probability02{income,ttype,1..3,hovgrp};
var probability03{income,ttype,1..3,hovgrp};
var altcost01{income,ttype,1..3,hovgrp};
var altcost02{income,ttype,1..3,hovgrp};
var altcost03{income,ttype,1..3,hovgrp};
var time01;
var time02;
var prob1;
var prob2;
var prob3;
var dem1;
var dem2;
var dem3;
var grptime0{income};
```

```
var grpcost0{income};
var volume1;
var volume2;
var totvehtime0;
var totpsngrtime0;
var totveh0;
var totalcost0;
var vol{1..altves};
var probinc{income};
var probinc0{income};
var probinc01{income};
var probtyp{ttype};
var probhov{hovgrp};
var grpcostall{income, ttype, hovgrp};
var grpcostall0{income, ttype, hovgrp};
var grptimeall{income, ttype, hovgrp};
var grptimeall0{income, ttype, hovgrp};
var loss{income,ttype,hovgrp};
var lossv{income};
var totlossv;

###########################
data;
set income := inc87 inc62 inc37 inc12;
set ttype := work shop school other;
set hovgrp := grp1 grp2 grp3 grp4;

param wage := inc87 43.486 inc62 26.113 inc37 15.681 inc12 9.416;
param incoeff := inc87 1 inc62 1 inc37 1 inc12 1;
param value := work 0.464 shop 0.23 school 0.02 other 0.052;
param tprop := work 0.4037 shop 0.2933 school 0.124 other 0.179;


param hovcost:    grp1 grp2 grp3 grp4:=
            1 0   0    0    0
            2 0   0.047 0.212 0.664
            3 0   0.073 0.323 1.014;

param hovbias1 := 1 0 2 0 3 0;
param hovbias2 := 1 0 2 0 3 0;
param hovbias3 := 1 0 2 0 3 0;

param demand := 7794;
param capacity1 := 1600;
param capacity2 := 6400;
param altves := 5;
```

param coeff := 0.5782;
param ocost := 1 0.7465 2 0.3733 3 0.2488 4 0.7465 5 0.3733;
param length := 5;
param oppcost := 1 0 2 0;
param equity := 1;
param equityt := 1.5;
param prop1 := 0.6874;
param prop2 := 0.2027;
param prop3 := 0.1166;
param alpha := 0.506;
param beta := 5;

###############
minimize objective: totalcost;
#maximize objective: profit;
#minimize objective: totvehtime;
#minimize objective: totpsngrtime;
#minimize objective: totveh;

###Objective functions' definitions
###
total revenue
subject to profit_definition:
profit = sum {i in income ,j in ttype ,k in hovgrp}
(demand1[i,j,k]*(probability1[i,j,1,k]*sov1toll + 0.5*probability1[i,j,2,k]*hov2toll) +
demand2[i,j,k]*(probability2[i,j,1,k]*sov1toll + 0.5*probability2[i,j,2,k]*hov2toll) +
demand3[i,j,k]*(probability3[i,j,1,k]*sov1toll + 0.5*probability3[i,j,2,k]*hov2toll));

total vehicular time
subject to totvehtime_definition:
totvehtime = sum {i in income ,j in ttype ,k in hovgrp}
(demand1[i,j,k]*(probability1[i,j,1,k]*time1 + 0.5*probability1[i,j,2,k]*time1+
0.333*probability1[i,j,3,k]*time1+probability1[i,j,4,k]*time2+0.5*probability1[i,j,5,k]*ti
me2) + demand2[i,j,k]*(probability2[i,j,1,k]*time1 + 0.5*probability2[i,j,2,k]*time1+
0.333*probability2[i,j,3,k]*time1+probability2[i,j,4,k]*time2+0.5*probability2[i,j,5,k]*ti
me2) + demand3[i,j,k]*(probability3[i,j,1,k]*time1 + 0.5*probability3[i,j,2,k]*time1+
0.333*probability3[i,j,3,k]*time1+probability3[i,j,4,k]*time2+0.5*probability3[i,j,5,k]*ti
me2));

total passenger time
subject to totpsngrtime_definition:
totpsngrtime = sum {i in income ,j in ttype ,k in hovgrp}
(demand1[i,j,k]*(probability1[i,j,1,k]*time1 + probability1[i,j,2,k]*time1+
probability1[i,j,3,k]*time1+probability1[i,j,4,k]*time2+probability1[i,j,5,k]*time2) +
demand2[i,j,k]*(probability2[i,j,1,k]*time1 + probability2[i,j,2,k]*time1+

```
probability2[i,j,3,k]*time1+probability2[i,j,4,k]*time2+probability2[i,j,5,k]*time2) +
demand3[i,j,k]*(probability3[i,j,1,k]*time1 + probability3[i,j,2,k]*time1+
probability3[i,j,3,k]*time1+probability3[i,j,4,k]*time2+probability3[i,j,5,k]*time2));

# total vehicles/vmt definition #
subject to totveh_definition:
totveh = sum {i in income ,j in ttype ,k in hovgrp} (demand1[i,j,k]*(probability1[i,j,1,k]
+ 0.5*probability1[i,j,2,k]+
0.333*probability1[i,j,3,k]+probability1[i,j,4,k]+0.5*probability1[i,j,5,k]) +
demand2[i,j,k]*(probability2[i,j,1,k] + 0.5*probability2[i,j,2,k]+
0.333*probability2[i,j,3,k]+probability2[i,j,4,k]+0.5*probability2[i,j,5,k]) +
demand3[i,j,k]*(probability3[i,j,1,k] + 0.5*probability3[i,j,2,k]+
0.333*probability3[i,j,3,k]+probability3[i,j,4,k]+0.5*probability3[i,j,5,k]));

# individual cost definitions #
subject to totcost_definition1:
totcost[1] = sum {i in income, j in ttype, k in hovgrp} (demand1[i,j,k]*(sum {l in
1..altves} (probability1[i,j,l,k]*altcost1[i,j,l,k])));

subject to totcost_definition2:
totcost[2] = sum {i in income, j in ttype, k in hovgrp} (demand2[i,j,k]*(sum {l in
1..altves} (probability2[i,j,l,k]*altcost2[i,j,l,k])));

subject to totcost_definition3:
totcost[3] = sum {i in income, j in ttype, k in hovgrp} (demand3[i,j,k]*(sum {l in
1..altves} (probability3[i,j,l,k]*altcost3[i,j,l,k])));

# Total cost definition #
subject to totalcost_definition:
totalcost = sum{m in 1..3} totcost[m];

subject to nonneg1:
totalcost >= 0;

subject to nonneg2:
profit >= 0;

subject to nonneg3:
totvehtime >=0;

subject to nonneg4:
totveh >= 0;
##############################################################################
##############################################################
```

pre-conversion
##################################
subject to tim01:
time01 = length*(1/65)*(1+alpha*(prop3*demand/capacity1)^beta);

subject to tim02:
time02 = length*(1/65)*(1+alpha*((prop1+prop2)*demand/capacity2)^beta);

subject to behavior_model011 {i in income, j in ttype, k in hovgrp}:
altcost01[i,j,1,k] = wage[i]*value[j]*time02 + ocost[1];
subject to behavior_model012 {i in income, j in ttype, k in hovgrp}:
altcost01[i,j,2,k] = wage[i]*value[j]*time02 + ocost[2] +
wage[i]*value[j]*(hovcost[2,k]+oppcost[1]+hovbias1[2]);
subject to behavior_model013 {i in income, j in ttype, k in hovgrp}:
altcost01[i,j,3,k] = wage[i]*value[j]*time01 + ocost[3] +
wage[i]*value[j]*(hovcost[3,k]+oppcost[2]+hovbias1[3]);

subject to probability_comp01 {i in income, j in ttype, l in 1..3, k in hovgrp}:
probability01[i,j,l,k] = (exp(-coeff*altcost01[i,j,l,k]))/(exp(-coeff*altcost01[i,j,1,k]) +
exp(-coeff*altcost01[i,j,2,k]) + exp(-coeff*altcost01[i,j,3,k]));

subject to behavior_model021 {i in income, j in ttype, k in hovgrp}:
altcost02[i,j,1,k] = wage[i]*value[j]*time02 + ocost[1];
subject to behavior_model022 {i in income, j in ttype, k in hovgrp}:
altcost02[i,j,2,k] = wage[i]*value[j]*time02 + ocost[2] +
wage[i]*value[j]*(hovcost[2,k]+oppcost[1]+hovbias2[2]);
subject to behavior_model023 {i in income, j in ttype, k in hovgrp}:
altcost02[i,j,3,k] = wage[i]*value[j]*time01 + ocost[3] +
wage[i]*value[j]*(hovcost[3,k]+oppcost[2]+hovbias2[3]);

subject to probability_comp02 {i in income, j in ttype, l in 1..3, k in hovgrp}:
probability02[i,j,l,k] = (exp(-coeff*altcost02[i,j,l,k]))/(exp(-coeff*altcost02[i,j,1,k]) +
exp(-coeff*altcost02[i,j,2,k]) + exp(-coeff*altcost02[i,j,3,k]));

subject to behavior_model031 {i in income, j in ttype, k in hovgrp}:
altcost03[i,j,1,k] = wage[i]*value[j]*time02 + ocost[1];
subject to behavior_model032 {i in income, j in ttype, k in hovgrp}:
altcost03[i,j,2,k] = wage[i]*value[j]*time02 + ocost[2] +
wage[i]*value[j]*(hovcost[2,k]+oppcost[1]+hovbias3[2]);
subject to behavior_model033 {i in income, j in ttype, k in hovgrp}:
altcost03[i,j,3,k] = wage[i]*value[j]*time01 + ocost[3] +
wage[i]*value[j]*(hovcost[3,k]+oppcost[2]+hovbias3[3]);

subject to probability_comp03 {i in income, j in ttype, l in 1..3, k in hovgrp}:

probability03[i,j,l,k] = (exp(-coeff*altcost03[i,j,l,k]))/(exp(-coeff*altcost03[i,j,1,k]) +
exp(-coeff*altcost03[i,j,2,k]) + exp(-coeff*altcost03[i,j,3,k]));

subject to dem_constraint1 {i in income, j in ttype, k in hovgrp}:
demand1[i,j,k]=(prop1*probability01[i,j,1,k]+2*prop2*probability02[i,j,1,k]+3*prop3*pr
obability03[i,j,1,k])*demand*0.25*0.25*tprop[j];

subject to dem_constraint2 {i in income, j in ttype, k in hovgrp}:
demand2[i,j,k]=(prop1*probability01[i,j,2,k]+2*prop2*probability02[i,j,2,k]+3*prop3*pr
obability03[i,j,2,k])*demand*0.25*0.25*tprop[j];

subject to dem_constraint3 {i in income, j in ttype, k in hovgrp}:
demand3[i,j,k]=(prop1*probability01[i,j,3,k]+2*prop2*probability02[i,j,3,k]+3*prop3*pr
obability03[i,j,3,k])*demand*0.25*0.25*tprop[j];

subject to arbit1:
dem1 = sum{i in income, j in ttype, k in hovgrp} demand1[i,j,k];

subject to arbit2:
dem2 = sum{i in income, j in ttype, k in hovgrp} demand2[i,j,k];

subject to arbit3:
dem3 = sum{i in income, j in ttype, k in hovgrp} demand3[i,j,k];

subject to totvehtime0_definition:
totvehtime0 = sum {i in income ,j in ttype ,k in hovgrp}
(demand1[i,j,k]*(probability01[i,j,1,k]*time02 + 0.5*probability01[i,j,2,k]*time02+
0.333*probability01[i,j,3,k]*time01) + demand2[i,j,k]*(probability02[i,j,1,k]*time02 +
0.5*probability02[i,j,2,k]*time02+ 0.333*probability02[i,j,3,k]*time01) +
demand3[i,j,k]*(probability03[i,j,1,k]*time02 + 0.5*probability03[i,j,2,k]*time02+
0.333*probability03[i,j,3,k]*time01));

subject to totpsngrtime0_definition:
totpsngrtime0 = sum {i in income ,j in ttype ,k in hovgrp}
(demand1[i,j,k]*(probability01[i,j,1,k]*time02 + probability01[i,j,2,k]*time02+
probability01[i,j,3,k]*time01) + demand2[i,j,k]*(probability02[i,j,1,k]*time02 +
probability02[i,j,2,k]*time02+ probability02[i,j,3,k]*time01) +
demand3[i,j,k]*(probability03[i,j,1,k]*time02 + probability03[i,j,2,k]*time02+
probability03[i,j,3,k]*time01));

subject to totveh0_definition:
totveh0 = dem1+0.5*dem2+0.3333*dem3;

subject to totcost0_definition1:

totalcost0 = (sum {i in income, j in ttype, k in hovgrp} (demand1[i,j,k]*(sum {l in 1..3} (probability01[i,j,l,k]*altcost01[i,j,l,k])))) + (sum {i in income, j in ttype, k in hovgrp} (demand2[i,j,k]*(sum {l in 1..3} (probability02[i,j,l,k]*altcost02[i,j,l,k])))) + (sum {i in income, j in ttype, k in hovgrp} (demand3[i,j,k]*(sum {l in 1..3} (probability03[i,j,l,k]*altcost03[i,j,l,k]))));

subject to grptime_definition0 {i in income}:
grptime0[i] = (1/(sum{j in ttype, k in hovgrp} (demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp} (demand1[i,j,k]*(probability01[i,j,1,k]*time02+probability01[i,j,2,k]*time02+probability 01[i,j,3,k]*time01) +
demand2[i,j,k]*(probability02[i,j,1,k]*time02+probability02[i,j,2,k]*time02+probability 02[i,j,3,k]*time01) +
demand3[i,j,k]*(probability03[i,j,1,k]*time02+probability03[i,j,2,k]*time02+probability 03[i,j,3,k]*time01)));

subject to grpcost_definition0 {i in income}:
grpcost0[i] = (1/(sum{j in ttype, k in hovgrp} (demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp} (demand1[i,j,k]*(1/(wage[i]*value[j]))*(altcost01[i,j,1,k]*probability01[i,j,1,k]+altcost01 [i,j,2,k]*probability01[i,j,2,k]+altcost01[i,j,3,k]*probability01[i,j,3,k]) +
demand2[i,j,k]*(1/(wage[i]*value[j]))*(altcost02[i,j,1,k]*probability02[i,j,1,k]+altcost02[i,j,2,k]*probability02[i,j,2,k]+altcost02[i,j,3,k]*probability02[i,j,3,k]) +
demand3[i,j,k]*(1/(wage[i]*value[j]))*(altcost03[i,j,1,k]*probability03[i,j,1,k]+altcost03[i,j,2,k]*probability03[i,j,2,k]+altcost03[i,j,3,k]*probability03[i,j,3,k])));

subject to grpcostall_definition0 {i in income, j in ttype, k in hovgrp}:
grpcostall0[i,j,k] =
(1/(wage[i]*value[j]))*(1/(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k]))*(demand1[i,j ,k]*(altcost01[i,j,1,k]*probability01[i,j,1,k]+altcost01[i,j,2,k]*probability01[i,j,2,k]+altco st01[i,j,3,k]*probability01[i,j,3,k]) +
demand2[i,j,k]*(altcost02[i,j,1,k]*probability02[i,j,1,k]+altcost02[i,j,2,k]*probability02[i ,j,2,k]+altcost02[i,j,3,k]*probability02[i,j,3,k]) +
demand3[i,j,k]*(altcost03[i,j,1,k]*probability03[i,j,1,k]+altcost03[i,j,2,k]*probability03[i ,j,2,k]+altcost03[i,j,3,k]*probability03[i,j,3,k]));

subject to grptimeall_definition0 {i in income, j in ttype, k in hovgrp}:
grptimeall0[i,j,k] =
(1/(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k]))*(demand1[i,j,k]*(time02*probabilit y01[i,j,1,k]+time02*probability01[i,j,2,k]+time01*probability01[i,j,3,k]) +
demand2[i,j,k]*(time02*probability02[i,j,1,k]+time02*probability02[i,j,2,k]+time01*pro bability02[i,j,3,k]) +
demand3[i,j,k]*(time02*probability03[i,j,1,k]+time02*probability03[i,j,2,k]+time01*pro bability03[i,j,3,k]));

lane travel time constraints

subject to BPR_constraints1:
time1 = length*(1/65)*(1 + alpha * ((sum {i in income,j in ttype,k in hovgrp} (demand1[i,j,k]*probability1[i,j,1,k] + 0.5*demand1[i,j,k]*probability1[i,j,2,k] + 0.33*demand1[i,j,k]*probability1[i,j,3,k] + demand2[i,j,k]*probability2[i,j,1,k] + 0.5*demand2[i,j,k]*probability2[i,j,2,k] + 0.33*demand2[i,j,k]*probability2[i,j,3,k] + demand3[i,j,k]*probability3[i,j,1,k] + 0.5*demand3[i,j,k]*probability3[i,j,2,k] + 0.33*demand3[i,j,k]*probability3[i,j,3,k]))/capacity1)^beta);

subject to BPR_constraints2:
time2 = length*(1/65)*(1 + alpha * ((sum {i in income,j in ttype,k in hovgrp} (demand1[i,j,k]*probability1[i,j,4,k] + 0.5*demand1[i,j,k]*probability1[i,j,5,k] + demand2[i,j,k]*probability2[i,j,4,k] + 0.5*demand2[i,j,k]*probability2[i,j,5,k] + demand3[i,j,k]*probability3[i,j,4,k] + 0.5*demand3[i,j,k]*probability3[i,j,5,k]))/capacity2)^beta);

subject to time_constraint:
time2 >= time1;

subject to quality_constraint:
time1 <= length*(1/65)*(65/50);

subject to volume_constraint1:
volume1=(sum {i in income,j in ttype,k in hovgrp} (demand1[i,j,k]*probability1[i,j,1,k] + 0.5*demand1[i,j,k]*probability1[i,j,2,k] + 0.3333*demand1[i,j,k]*probability1[i,j,3,k] + demand2[i,j,k]*probability2[i,j,1,k] + 0.5*demand2[i,j,k]*probability2[i,j,2,k] + 0.3333*demand2[i,j,k]*probability2[i,j,3,k] + demand3[i,j,k]*probability3[i,j,1,k] + 0.5*demand3[i,j,k]*probability3[i,j,2,k] + 0.3333*demand3[i,j,k]*probability3[i,j,3,k]));

subject to volume_constraint2:
volume2=(sum {i in income,j in ttype,k in hovgrp} (demand1[i,j,k]*probability1[i,j,4,k] + 0.5*demand1[i,j,k]*probability1[i,j,5,k] + demand2[i,j,k]*probability2[i,j,4,k] + 0.5*demand2[i,j,k]*probability2[i,j,5,k] + demand3[i,j,k]*probability3[i,j,4,k] + 0.5*demand3[i,j,k]*probability3[i,j,5,k]));;

subject to vol_constraint1:
vol[1]=(sum {i in income,j in ttype,k in hovgrp} (demand1[i,j,k]*probability1[i,j,1,k] + demand2[i,j,k]*probability2[i,j,1,k] +demand3[i,j,k]*probability3[i,j,1,k]));

subject to vol_constraint2:
vol[2]=(sum {i in income,j in ttype,k in hovgrp} (0.5*demand1[i,j,k]*probability1[i,j,2,k] + 0.5*demand2[i,j,k]*probability2[i,j,2,k] + 0.5*demand3[i,j,k]*probability3[i,j,2,k]));

subject to vol_constraint3:

vol[3] = sum {i in income,j in ttype,k in hovgrp}
(0.333*demand1[i,j,k]*probability1[i,j,3,k] + 0.333*demand2[i,j,k]*probability2[i,j,3,k]
+ 0.333*demand3[i,j,k]*probability3[i,j,3,k]);

subject to vol_constraint4:
vol[4] = sum {i in income,j in ttype,k in hovgrp} (demand1[i,j,k]*probability1[i,j,4,k] +
demand2[i,j,k]*probability2[i,j,4,k] + demand3[i,j,k]*probability3[i,j,4,k]);

subject to vol_constraint5:
vol[5] = sum {i in income,j in ttype,k in hovgrp}
(0.5*demand1[i,j,k]*probability1[i,j,5,k] + 0.5*demand2[i,j,k]*probability2[i,j,5,k] +
0.5*demand3[i,j,k]*probability3[i,j,5,k]);

subject to prob_inc {i in income}:
probinc[i]=sum{j in ttype, k in hovgrp} (
tprop[j]*0.25*((dem1/(dem1+dem2+dem3))*(probability1[i,j,1,k]+probability1[i,j,2,k]+p
robability1[i,j,3,k])) +
tprop[j]*0.25*((dem2/(dem1+dem2+dem3))*(probability2[i,j,1,k]+probability2[i,j,2,k]+p
robability2[i,j,3,k])) +
tprop[j]*0.25*((dem3/(dem1+dem2+dem3))*(probability3[i,j,1,k]+probability3[i,j,2,k]+p
robability3[i,j,3,k])));

subject to prob_inc1 {i in income}:
probinc0[i]=sum{j in ttype, k in hovgrp} (
tprop[j]*0.25*((dem1/(dem1+dem2+dem3))*(probability01[i,j,3,k])) +
tprop[j]*0.25*((dem2/(dem1+dem2+dem3))*(probability02[i,j,3,k])) +
tprop[j]*0.25*((dem3/(dem1+dem2+dem3))*(probability03[i,j,3,k])));

subject to prob_inc01 {i in income}:
probinc01[i]=sum{j in ttype, k in hovgrp} (
tprop[j]*0.25*((dem1/(dem1+dem2+dem3))*(probability1[i,j,3,k])) +
tprop[j]*0.25*((dem2/(dem1+dem2+dem3))*(probability2[i,j,3,k])) +
tprop[j]*0.25*((dem3/(dem1+dem2+dem3))*(probability3[i,j,3,k])));

subject to prob_ttpye {j in ttype}:
probtyp[j]=sum{i in income, k in hovgrp} (
0.25*0.25*((dem1/(dem1+dem2+dem3))*(probability1[i,j,1,k]+probability1[i,j,2,k]+prob
ability1[i,j,3,k])) +
0.25*0.25*((dem2/(dem1+dem2+dem3))*(probability2[i,j,1,k]+probability2[i,j,2,k]+prob
ability2[i,j,3,k])) +
0.25*0.25*((dem3/(dem1+dem2+dem3))*(probability3[i,j,1,k]+probability3[i,j,2,k]+prob
ability3[i,j,3,k])));

subject to prob_hov {k in hovgrp}:
probhov[k]=sum{i in income, j in ttype } (
tprop[j]*0.25*((dem1/(dem1+dem2+dem3))*(probability1[i,j,1,k]+probability1[i,j,2,k]+p

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robability1[i,j,3,k])) +
tprop[j]*0.25*((dem2/(dem1+dem2+dem3))*(probability2[i,j,1,k]+probability2[i,j,2,k]+p
robability2[i,j,3,k])) +
tprop[j]*0.25*((dem3/(dem1+dem2+dem3))*(probability3[i,j,1,k]+probability3[i,j,2,k]+p
robability3[i,j,3,k])));
###
####################################

toll constraints
###
subject to sovtoll_constraint:
sov1toll >= 0;
subject to hov2toll_constraint:
hov2toll >= 0;
subject to sovhov2toll_constraint:
hov2toll <= sov1toll;
###
####################################

behavior model constraints for
SOVs ###############################
subject to behavior_model11 {i in income, j in ttype, k in hovgrp}:
altcost1[i,j,1,k] = wage[i]*value[j]*time1 + ocost[1] + (sov1toll/incoeff[i]);
subject to behavior_model12 {i in income, j in ttype, k in hovgrp}:
altcost1[i,j,2,k] = wage[i]*value[j]*time1 + ocost[2] + (0.5*hov2toll/incoeff[i]) +
wage[i]*value[j]*(hovcost[2,k]+hovbias1[2]+oppcost[1]);
subject to behavior_model13 {i in income, j in ttype, k in hovgrp}:
altcost1[i,j,3,k] = wage[i]*value[j]*time1 + ocost[3] +
wage[i]*value[j]*(hovcost[3,k]+hovbias1[3]+oppcost[2]);
subject to behavior_model14 {i in income, j in ttype, k in hovgrp}:
altcost1[i,j,4,k] = wage[i]*value[j]*time2 + ocost[4];
subject to behavior_model15 {i in income, j in ttype, k in hovgrp}:
altcost1[i,j,5,k] = wage[i]*value[j]*time2 + ocost[5] +
wage[i]*value[j]*(hovcost[2,k]+hovbias1[2]+oppcost[1]);

subject to probability_comp1 {i in income, j in ttype, l in 1..altves, k in hovgrp}:
probability1[i,j,l,k] = (exp(-coeff*altcost1[i,j,l,k]))/(exp(-coeff*altcost1[i,j,1,k]) + exp(-
coeff*altcost1[i,j,2,k]) + exp(-coeff*altcost1[i,j,3,k]) + exp(-coeff*altcost1[i,j,4,k]) +
exp(-coeff*altcost1[i,j,5,k]));
###
####################################

behavior model constraints for HOV2

subject to behavior_model21 {i in income, j in ttype, k in hovgrp}:
altcost2[i,j,1,k] = wage[i]*value[j]*time1 + ocost[1] + (sov1toll/incoeff[i]);
subject to behavior_model22 {i in income, j in ttype, k in hovgrp}:
altcost2[i,j,2,k] = wage[i]*value[j]*time1 + ocost[2] + (0.5*hov2toll/incoeff[i]) +
wage[i]*value[j]*(hovcost[2,k]+hovbias2[2]+oppcost[1]);
subject to behavior_model23 {i in income, j in ttype, k in hovgrp}:
altcost2[i,j,3,k] = wage[i]*value[j]*time1 + ocost[3] +
wage[i]*value[j]*(hovcost[3,k]+hovbias2[3]+oppcost[2]);
subject to behavior_model24 {i in income, j in ttype, k in hovgrp}:
altcost2[i,j,4,k] = wage[i]*value[j]*time2 + ocost[4];
subject to behavior_model25 {i in income, j in ttype, k in hovgrp}:
altcost2[i,j,5,k] = wage[i]*value[j]*time2 + ocost[5] +
wage[i]*value[j]*(hovcost[2,k]+hovbias2[2]+oppcost[1]);

subject to probability_comp2 {i in income, j in ttype, l in 1..altves, k in hovgrp}:
probability2[i,j,l,k] = (exp(-coeff*altcost2[i,j,l,k]))/(exp(-coeff*altcost2[i,j,1,k]) + exp(-coeff*altcost2[i,j,2,k]) + exp(-coeff*altcost2[i,j,3,k]) + exp(-coeff*altcost2[i,j,4,k]) +
exp(-coeff*altcost2[i,j,5,k]));
###
####################################

behavior model constraints for HOV3

subject to behavior_model31 {i in income, j in ttype, k in hovgrp}:
altcost3[i,j,1,k] = wage[i]*value[j]*time1 + ocost[1] + (sov1toll/incoeff[i]);
subject to behavior_model32 {i in income, j in ttype, k in hovgrp}:
altcost3[i,j,2,k] = wage[i]*value[j]*time1 + ocost[2] + (0.5*hov2toll/incoeff[i]) +
wage[i]*value[j]*(hovcost[2,k]+hovbias3[2]+oppcost[1]);
subject to behavior_model33 {i in income, j in ttype, k in hovgrp}:
altcost3[i,j,3,k] = wage[i]*value[j]*time1 + ocost[3] +
wage[i]*value[j]*(hovcost[3,k]+hovbias3[3]+oppcost[2]);
subject to behavior_model34 {i in income, j in ttype, k in hovgrp}:
altcost3[i,j,4,k] = wage[i]*value[j]*time2 + ocost[4];
subject to behavior_model35 {i in income, j in ttype, k in hovgrp}:
altcost3[i,j,5,k] = wage[i]*value[j]*time2 + ocost[5] +
wage[i]*value[j]*(hovcost[2,k]+hovbias3[2]+oppcost[1]);

subject to probability_comp3 {i in income, j in ttype, l in 1..altves, k in hovgrp}:
probability3[i,j,l,k] = (exp(-coeff*altcost3[i,j,l,k]))/(exp(-coeff*altcost3[i,j,1,k]) + exp(-coeff*altcost3[i,j,2,k]) + exp(-coeff*altcost3[i,j,3,k]) + exp(-coeff*altcost3[i,j,4,k]) +
exp(-coeff*altcost1[i,j,5,k]));
###
#####################################

definitions of necessary outputs and other stuff

subject to avgtime_definition:
avgtime = totpsngrtime/(sum {i in income, j in ttype, k in hovgrp} (demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k]));

subject to avgeqvtimecost_definition:
avgeqvtimecost = (1/(sum{i in income, j in ttype, k in hovgrp} (demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{i in income, j in ttype, k in hovgrp}
(demand1[i,j,k]*(1/(wage[i]*value[j]))*(altcost1[i,j,1,k]*probability1[i,j,1,k]+altcost1[i,j,2,k]*probability1[i,j,2,k]+altcost1[i,j,3,k]*probability1[i,j,3,k]+altcost1[i,j,4,k]*probability1[i,j,4,k]+altcost1[i,j,5,k]*probability1[i,j,5,k]) +
demand2[i,j,k]*(1/(wage[i]*value[j]))*(altcost2[i,j,1,k]*probability2[i,j,1,k]+altcost2[i,j,2,k]*probability2[i,j,2,k]+altcost2[i,j,3,k]*probability2[i,j,3,k]+altcost2[i,j,4,k]*probability2[i,j,4,k]+altcost2[i,j,5,k]*probability2[i,j,5,k]) +
demand3[i,j,k]*(1/(wage[i]*value[j]))*(altcost3[i,j,1,k]*probability3[i,j,1,k]+altcost3[i,j,2,k]*probability3[i,j,2,k]+altcost3[i,j,3,k]*probability3[i,j,3,k]+altcost3[i,j,4,k]*probability3[i,j,4,k]+altcost3[i,j,5,k]*probability3[i,j,5,k])));

subject to grptime_definition {i in income}:
grptime[i] = (1/(sum{j in ttype, k in hovgrp} (demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]*(probability1[i,j,1,k]*time1+probability1[i,j,2,k]*time1+probability1[i,j,3,k]*time1+probability1[i,j,4,k]*time2+probability1[i,j,5,k]*time2) +
demand2[i,j,k]*(probability2[i,j,1,k]*time1+probability2[i,j,2,k]*time1+probability2[i,j,3,k]*time1+probability2[i,j,4,k]*time2+probability2[i,j,5,k]*time2) +
demand3[i,j,k]*(probability3[i,j,1,k]*time1+probability3[i,j,2,k]*time1+probability3[i,j,3,k]*time1+probability3[i,j,4,k]*time2+probability3[i,j,5,k]*time2)));

subject to grpcost_definition {i in income}:
grpcost[i] = (1/(sum{j in ttype, k in hovgrp} (demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]*(1/(wage[i]*value[j]))*(altcost1[i,j,1,k]*probability1[i,j,1,k]+altcost1[i,j,2,k]*probability1[i,j,2,k]+altcost1[i,j,3,k]*probability1[i,j,3,k]+altcost1[i,j,4,k]*probability1[i,j,4,k]+altcost1[i,j,5,k]*probability1[i,j,5,k]) +
demand2[i,j,k]*(1/(wage[i]*value[j]))*(altcost2[i,j,1,k]*probability2[i,j,1,k]+altcost2[i,j,2,k]*probability2[i,j,2,k]+altcost2[i,j,3,k]*probability2[i,j,3,k]+altcost2[i,j,4,k]*probability2[i,j,4,k]+altcost2[i,j,5,k]*probability2[i,j,5,k]) +
demand3[i,j,k]*(1/(wage[i]*value[j]))*(altcost3[i,j,1,k]*probability3[i,j,1,k]+altcost3[i,j,2,k]*probability3[i,j,2,k]+altcost3[i,j,3,k]*probability3[i,j,3,k]+altcost3[i,j,4,k]*probability3[i,j,4,k]+altcost3[i,j,5,k]*probability3[i,j,5,k])));

subject to grpcostall_definition {i in income, j in ttype, k in hovgrp}:

grpcostall[i,j,k] =
(1/(wage[i]*value[j]))*(1/(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k]))*(demand1[i,j
,k]*(altcost1[i,j,1,k]*probability1[i,j,1,k]+altcost1[i,j,2,k]*probability1[i,j,2,k]+altcost1[i,
j,3,k]*probability1[i,j,3,k]+altcost1[i,j,4,k]*probability1[i,j,4,k]+altcost1[i,j,5,k]*probabil
ity1[i,j,5,k]) +
demand2[i,j,k]*(altcost2[i,j,1,k]*probability2[i,j,1,k]+altcost2[i,j,2,k]*probability2[i,j,2,k
]+altcost2[i,j,3,k]*probability2[i,j,3,k]+altcost2[i,j,4,k]*probability2[i,j,4,k]+altcost2[i,j,5
,k]*probability2[i,j,5,k]) +
demand3[i,j,k]*(altcost3[i,j,1,k]*probability3[i,j,1,k]+altcost3[i,j,2,k]*probability3[i,j,2,k
]+altcost3[i,j,3,k]*probability3[i,j,3,k]+altcost3[i,j,4,k]*probability3[i,j,4,k]+altcost3[i,j,5
,k]*probability3[i,j,5,k]));

subject to grptimeall_definition {i in income, j in ttype, k in hovgrp}:
grptimeall[i,j,k] =
(1/(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k]))*(demand1[i,j,k]*(time1*probability
1[i,j,1,k]+time1*probability1[i,j,2,k]+time1*probability1[i,j,3,k]+time2*probability1[i,j,4
,k]+time2*probability1[i,j,5,k]) +
demand2[i,j,k]*(time1*probability2[i,j,1,k]+time1*probability2[i,j,2,k]+time1*probabilit
y2[i,j,3,k]+time2*probability2[i,j,4,k]+time2*probability2[i,j,5,k]) +
demand3[i,j,k]*(time1*probability3[i,j,1,k]+time1*probability3[i,j,2,k]+time1*probabilit
y3[i,j,3,k]+time2*probability3[i,j,4,k]+time2*probability3[i,j,5,k]));

subject to loss_eval {i in income, j in ttype, k in hovgrp}:
loss[i,j,k]=(wage[i]*value[j])*(grpcostall[i,j,k]-
grpcostall0[i,j,k])*(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k]);

subject to lossv_eval {i in income}:
lossv[i]= sum{j in ttype, k in hovgrp} ((grpcostall[i,j,k]-
grpcostall['inc87',j,k])*(wage[i]*value[j])*(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k
]));
##
####################################

equity related
constraints ##################################
#subject to ttime_equity1 {i in income}:
#(1/(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]*(time1*probability1[i,j,1,k]+time1*probability1[i,j,2,k]+time1*probabili
ty1[i,j,3,k]+time2*probability1[i,j,4,k]+time2*probability1[i,j,5,k])+demand2[i,j,k]*(tim
e1*probability2[i,j,1,k]+time1*probability2[i,j,2,k]+time1*probability2[i,j,3,k]+time2*pr
obability2[i,j,4,k]+time2*probability2[i,j,5,k])+demand3[i,j,k]*(time1*probability3[i,j,1,
k]+time1*probability3[i,j,2,k]+time1*probability3[i,j,3,k]+time2*probability3[i,j,4,k]+ti
me2*probability3[i,j,5,k]))) <=(1+equity)*avgtime;

#subject to ttime_equity2 {i in income}:
#(1/(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]*(time1*probability1[i,j,1,k]+time1*probability1[i,j,2,k]+time1*probability1[i,j,3,k]+time2*probability1[i,j,4,k]+time2*probability1[i,j,5,k])+demand2[i,j,k]*(time1*probability2[i,j,1,k]+time1*probability2[i,j,2,k]+time1*probability2[i,j,3,k]+time2*probability2[i,j,4,k]+time2*probability2[i,j,5,k])+demand3[i,j,k]*(time1*probability3[i,j,1,k]+time1*probability3[i,j,2,k]+time1*probability3[i,j,3,k]+time2*probability3[i,j,4,k]+time2*probability3[i,j,5,k]))) >=(1-equity)*avgtime;

subject to tcost_equity1 {i in income}:
(1/(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]*(1/(wage[i]*value[j]))*(altcost1[i,j,1,k]*probability1[i,j,1,k]+altcost1[i,j,2,k]*probability1[i,j,2,k]+altcost1[i,j,3,k]*probability1[i,j,3,k]+altcost1[i,j,4,k]*probability1[i,j,4,k]+altcost1[i,j,5,k]*probability1[i,j,5,k]) +
demand2[i,j,k]*(1/(wage[i]*value[j]))*(altcost2[i,j,1,k]*probability2[i,j,1,k]+altcost2[i,j,2,k]*probability2[i,j,2,k]+altcost2[i,j,3,k]*probability2[i,j,3,k]+altcost2[i,j,4,k]*probability2[i,j,4,k]+altcost2[i,j,5,k]*probability2[i,j,5,k]) +
demand3[i,j,k]*(1/(wage[i]*value[j]))*(altcost3[i,j,1,k]*probability3[i,j,1,k]+altcost3[i,j,2,k]*probability3[i,j,2,k]+altcost3[i,j,3,k]*probability3[i,j,3,k]+altcost3[i,j,4,k]*probability3[i,j,4,k]+altcost3[i,j,5,k]*probability3[i,j,5,k]))) <=(1+equity)*avgeqvtimecost;

subject to tcost_equity2 {i in income}:
(1/(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]+demand2[i,j,k]+demand3[i,j,k])))*(sum{j in ttype, k in hovgrp}
(demand1[i,j,k]*(1/(wage[i]*value[j]))*(altcost1[i,j,1,k]*probability1[i,j,1,k]+altcost1[i,j,2,k]*probability1[i,j,2,k]+altcost1[i,j,3,k]*probability1[i,j,3,k]+altcost1[i,j,4,k]*probability1[i,j,4,k]+altcost1[i,j,5,k]*probability1[i,j,5,k]) +
demand2[i,j,k]*(1/(wage[i]*value[j]))*(altcost2[i,j,1,k]*probability2[i,j,1,k]+altcost2[i,j,2,k]*probability2[i,j,2,k]+altcost2[i,j,3,k]*probability2[i,j,3,k]+altcost2[i,j,4,k]*probability2[i,j,4,k]+altcost2[i,j,5,k]*probability2[i,j,5,k]) +
demand3[i,j,k]*(1/(wage[i]*value[j]))*(altcost3[i,j,1,k]*probability3[i,j,1,k]+altcost3[i,j,2,k]*probability3[i,j,2,k]+altcost3[i,j,3,k]*probability3[i,j,3,k]+altcost3[i,j,4,k]*probability3[i,j,4,k]+altcost3[i,j,5,k]*probability3[i,j,5,k]))) >=(1-equity)*avgeqvtimecost;
##
##

improvement
constraints ###############################
#subject to grptimeimp_definition {i in income}:
#grptime[i] <= equityt*grptime0[i,j,k];

subject to grpcostimp_definition {i in income}:
grpcost[i] <= equityt*grpcost0[i];

```
#############################################################
#############################################
```

solve;

```
display equity;
display equityt;
display sov1toll;
display hov2toll;
display profit;
display totvehtime;
display totpsngrtime;
display totalcost;
display totveh;
display totvehtime0;
display totpsngrtime0;
display totalcost0;
display totveh0;
display avgtime;
display avgeqvtimecost;
display time1;
display time2;
display time01;
display time02;
display grptime;
display grpcost;
display grptime0;
display grpcost0;
display dem1;
display dem2;
display dem3;
display demand;
display volume1;
display volume2;
display vol;
display probinc;
display probinc0;
display probinc01;
display probtyp;
display probhov;
display grpcostall;
display grpcostall0;
display lossv;
```

APPENDIX B:

<u>I. MATLAB function for minimizing total conversion time:</u>
%given the current amount of money in hand, the current time period and the
%network status, this function gives the least amount of time necessary to
%upgrade all the links from this point onwards

```
function [T,actions] = phitest(nw,B,t)

theta=0.4;
revrate=theta*[6.356,6.514,5.364,3.837];
capcost=[18.5,11.22,23.32,19.46];
inrate=theta*6.71;
N=14;   %planning horizon



if(t>N)
  T=10;    %or T=11?
  actions=0;
else
  x=ones(length(capcost),1);
  x(nw)=0;
```

```
if(length(nw)==1) %if nw=1

    if(B>=capcost(nw))  %evaluating the 2 cases
       T=1;
       actions=[nw,zeros(1,(N+1-t))];
    else
       [T1,a]=phitest(nw,B+revrate*x+inrate,t+1);
       T=T1+1;
       actions=[0,a];
    end

else  %if nw>1

    for(i=1:length(nw))
       nw1=nw;
       B1=B;
       x1=x;
       if(B>=capcost(nw(i)))
          nw1(i)=[];
          B1=B-capcost(nw(i));
          x1(nw(i))=1;
          [T2,a]=phitest(nw1,B1+revrate*x1+inrate,t+1);
          axn(i,:)=[nw(i),a];
          T1(i)=T2+1;
       else
          [T2,a]=phitest(nw,B+revrate*x+inrate,t+1);
          axn(i,:)=[0,a];
          T1(i)=T2+1;
       end
    end    % end of for loop evaluating all links

    [T2,a]=phitest(nw,B+revrate*x+inrate,t+1);
    T1=[T1,T2+1];
    axn=[axn;[0,a]];
    [T,j]=min(T1);
    actions=axn(j,:);

  end

end
```

II. MATLAB function for minimizing total external funding:
%given the amount of money in hand (B), the current time period (t) and the network status (nw), this function gives the least amount of money (Bcum) necessary to

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%upgrade all the links from this point onwards by the end of a certain time period T

```
function [Bcum,flow,actions] = phiB(nw,B,t)

revrate=[0.3,0.3,0.2];
capcost=[1.2,0.9,0.6];
%revrate=[6.356,6.71,6.514,5.364,3.837];
%capcost=[18.5,59.2,11.22,23.32,19.46];
N=10;
cash=[0:0.1:max(capcost)];
T=4;

if(t>T+1)
    Bcum=1000;
    flow=1000;
    actions=0;
else        %if we are still within the time frame
    if(length(nw)==0)
        Bcum=0;
        flow=zeros(1,(N+1-t));
        actions=zeros(1,(T+1-t));
    else        %if we still have some links to upgrade

        x=ones(length(capcost),1);
        x(nw)=0;
        for(i=1:length(cash))   %I. for all possible cash amounts
            B1=B+cash(i);
            for(j=1:length(nw)) %II. for all possible links
                nw1=nw;
                x1=x;
                if(B1>=capcost(nw(j)))
                    nw1(j)=[];
                    B1=B1-capcost(nw(j));
                    x1(nw(j))=1;
                    [a,b,c]=phiB(nw1,B1+revrate*x1,t+1);
                    A(i,j)=cash(i)+a;

                    %f(i,j,:)=[cash(i),b];
                    b1=[cash(i),b];
                    siz=length(b1);
                    for(p=1:siz)
                        f(i,j,p)=b1(p);
                    end

                    %axn(i,j,:)=[nw(j),c];
                    c1=[nw(j),c];
```

```
          siz=length(c1);
          for(p=1:siz)
              axn(i,j,p)=c1(p);
          end
       else
          [a,b,c]=phiB(nw1,B1+revrate*x,t+1);
          A(i,j)=cash(i)+a;
          %f(i,j,:)=[cash(i),b];
          b1=[cash(i),b];
          siz=length(b1);
          for(p=1:siz)
              f(i,j,p)=b1(p);
          end

          %axn(i,j,:)=[nw(j),c];
          c1=[nw(j),c];
          siz=length(c1);
          for(p=1:siz)
              axn(i,j,p)=c1(p);
          end

     end %end of budget checking if
   end %end of for loop II
end %end of for loop I

A1=[];
for(k=1:length(cash))   %no action case
   [a,b,c]=phiB(nw,B+cash(k)+revrate*x,t+1);
   A1=[A1;cash(k)+a];
   f1(k,:)=[cash(k),b];
   axn1(k,:)=[0,c];
end

A=[A,A1];
siz=size(f1);
for(p=1:siz(1))
   for(q=1:siz(2))
      f(i+1,p,q)=f1(p,q);
   end
end
%f(i+1,:,:)=f1;
%f(:,:,length(cash)+1)=f1;
%axn(length(cash)+1,:,:)=axn1;
siz=size(axn1);
for(p=1:siz(1))
   for(q=1:siz(2))
```

```matlab
            axn(i+1,p,q)=axn1(p,q);
        end
    end
    [a,b]=min(A);
    %flow=f(b(1),b(2),:);
    siz=size(f);
    for(p=1:siz(3))
        flow(p)=f(b(1),b(2),p);
    end
    %actions=axn(b(1),b(2),:);
    siz=size(axn);
    for(p=1:siz(3))
        actions(p)=axn(b(1),b(2),p);
    end
    Bcum=sum(flow);
  end
end
```