Research Report – UCD-ITS-RR-08-51

An Efficiency-Equity Solution to the Integrated Corridor Control Problem

August 2008

Jingtao Ma
Michael H. Zhang

Institute of Transportation Studies ◦ University of California, Davis
One Shields Avenue ◦ Davis, California 95616
PHONE (530) 752-6548 ◦ FAX (530) 752-6572
www.its.ucdavis.edu

UCDAVIS
SUSTAINABLE TRANSPORTATION CENTER
of the Institute of Transportation Studies

Final Research Report [**DO1-6**]

An Efficiency-Equity Solution to the Integrated Corridor Control Problem

August, 2008

Jingtao Ma
Michael H. Zhang

Department of [Civil and Environmental Engineering]
University of California, Davis
One Shields Ave., Davis, CA 95616

ABSTRACT

Efficiency has been considered as the single most important measure in designing traffic control systems for over five decades. As traffic congestion spreads wider and spans longer in most urban areas, the traveling public is more and more concerned with how the benefits from the new control design or existing control system updates are distributed among them; and the systems that favor certain groups over the others are certainly considered "inefficient" to the disadvantaged groups. Within this context, an efficiency-equity bi-criterion corridor control design program is developed in this study. Firstly a network flow dynamics study tool based on the finite difference solution to the kinematic wave model has been developed, with prevalent control measures including signal controllers, ramp meters and priority rule controls adapted in a coherent manner. The system efficiency measures, the user equity measures at both aggregate and disaggregate levels are then developed and calculated based on the flow dynamics model. The bi-criterion control program has been solved by a heuristic search method to compute the signal green split and ramp metering rates. The numerical experimentation indicates that 1) the system efficiency measure is independent from the user equity measures to a great extent; 2) using only the disaggregate equity measure cannot characterize the user equity holistically, and rather both aggregate and disaggregate equity measures must be present simultaneously; 3) the introduction of equity measures can be well justified from the analysis of the relative gains and losses of system efficiency and user equity.

INTRODUCTION

Efficiency has been considered as the single most important measure in designing traffic control systems for years. From the system administrators' viewpoint, it is so natural to use measures like the total generalized cost (e.g., travel time/delay, fuel consumption and so on) to compare and select the “best” control strategy, that nearly all control systems since the seminal work of Webster (1958) are designed with this philosophy. However, recent practices begin to question whether it is appropriate to design a control system relying solely on efficiency. Questioned by urban travelers who endured long delays on metered ramps, for instance, the Minnesota DOT was mandated by the Minnesota legislature to conduct an eight-week ramp metering shut-off experiment to compare the system performances with and without metering (Levinson et al 2002). This experiment confirmed that ramp metering would favor long-distance travelers at the expense of short-distance ones. Understandably, those controls are not considered “efficient” from the perspective of those short-distance travelers. To gain public acceptance and support, therefore, new or updated control systems must not only consider the overall efficiency improvements, but also how the efficiency gains are distributed among the system's user groups who differ from one another in their departure time, origin-destination, trip purposes, and value of time. Equity, or user fairness, begins to emerge as an important issue in traffic control design.

Compared with the overwhelming number of studies in addressing control efficiency, the analysis of equity issues in traffic control is far less, and their findings are mostly qualitative and sometimes even conflicting. For instance, efficiency and equity are usually considered as two competing requirements: the more efficient a control system is, the less equitable it becomes (Kotsialos & Papageorgiou 2004, Meng & Yang 2002). However, some microscopic simulation (e.g., Yin, Liu & Benouar, 2000) finds that the above claim may not be true because a control algorithm could be more equitable than another, yet maintains a similar level of overall efficiency. Apparently, there is a need to resolve these contradictory findings, and develop proper equity measures to be used in the design of control systems that balance efficiency and equity.

The entanglement around the two dimensions originates from several threads. Firstly, the control objectives themselves, especially the equity objectives, are not clearly identified. The equity measures applied in the literature are inherently deficient because they are borrowed mainly from social welfare studies in economics, where an appropriate measure is not generally agreed upon (Bowman 1945). Meanwhile, the equity measures only address one aspect of the travelers and may not characterize the users' fairness in a holistic manner. Secondly, no control design has taken equity explicitly as its objectives and consequently the equity requirements are only marginally taken care of by some practical constraints such as minimum queuing time and so on. Thirdly and most importantly, control systems implemented in isolation hinder a more efficient manipulation of control elements and the achievement of system efficiency and user equity goals from the perspective of the entire system, since current control systems only take care of a sub-network of urban signalized intersections or metered amps. To disentangle theses complex issues, we lay out a comprehensive study framework to identify the needs of efficient and equitable control, develop unambiguous measures of equity and efficiency, and design control methods that consider explicitly both efficiency and equity requirements. Taken into the transportation corridor context, the proposed framework and solution could also be applied to study this important dimension of user equity in other applications such as route guidance (Jahn et al 2005) and especially regional planning of the transportation system.

The remainder of the paper is organized as follows. In the next section, one network flow dynamics model is firstly introduced and then the traffic control measures prevalent within a general corridor are adapted into this flow dynamics model, which will serve as the basis to evaluate the system efficiency and user equity under various control strategies. Next the study formulates the measures on both dimensions of system efficiency and user equity and presents the integrated corridor control formulations that can balance both dimensions. Then a heuristic solution algorithm, genetic algorithm is developed to calculate the optimal control plan. A real network example is followed in the third section to investigate the study the system efficiency and user equity performances of various control strategies. The last section concludes the research and discusses potential expansion possibilities.

EFFICIENCY-EQUITY BICRITERION CORRIDOR CONTROL

FORMULATION

Network Representation and Assumptions

A transportation corridor is given in the node-link network representation $G\,(\mathcal{N},\mathcal{L})$, where $\mathcal{N}$ and $\mathcal{L}$ are the sets of nodes and links, respectively. The demand is assumed to be known a priori in the form of (time-dependent) origin-destination (O-D) matrix ***Q (R, S)***, where ***R*** and ***S*** are the sets of trip origins and destinations, respectively. $q^{r,s}$ is the demand between O-D pair (r,s). Let K_{rs} denote the set of paths connecting the O-D pair $r-s$ and the entire path set $K=\bigcup K_{rs},\forall r,s$. In this way, the network users are differentiated by their origin-destination and path choice characteristics.
Starting with an empty network at $t=0$, each network user group $q^{r,s}(t)$ will be assigned onto the network, and we shall have

$$q^{r,s}=\int_0^T q^{r,s}(t)d_t \tag{1}$$

Assume all travelers will finish their journey at T', and following certain network study conventions (Nie 2006), we first clarify the following time segment terms in the modeling:

[Assignment interval ϕ_a] A discrete period during which any departure flows will hold constant. The assignment horizon T consists of m_a assignment intervals $T=m_a\times\phi_a$.
[Loading interval ϕ_l] A discrete period during which the network flow conditions are stationary. The loading horizon T' consists of m_l loading intervals $T`=m_l\times\phi_l$.
[Control interval ϕ_c] A discrete period during which all control variables are fixed. The control horizon T_c consists of m_c assignment intervals $T_c=m_c\times\phi_c$, where T_c could coincide with T'. This also means our study focus is time-of-day control.

A subset of the nodes $\mathcal{N}_C$ contains all the nodes with certain kinds of control measures (urban signals, ramp meters or yield/STOP signs and so on). The system designer can have the following control variables at disposal to affect the network flow:

- Signal green splits $g_i^m(t)$ for the phase m signalized intersection i or metering rates $R_i(t)$;
- Phasing and phasing sequence $\eta_i(t)$ for the signal controller at signalized intersection i ;
- Cycle length $C_i(t)$ of the signal controller i ;
- Offset $\Delta_i(t)$ of the signal controller at intersection i;

We collectively denote the trajectory of the control variables
$(g_i^m(t),R_i(t),\eta_i(t),C_i(t),\Delta_i(t))$, , $i\in\mathcal{N}_C$, $t\in[0,T_c]$
as vector θ.

Since Allsop's work on the interaction between network traffic flows and signal control (Allsop 1974), it has been well understood that by changing the control vector θ, the system engineer can dictate the travel time on the road sections leading to the controls. Consequently, the users $q^{r,s}(t)$ perceive the changes in their journey cost $c_k^{r,s}(t)$ and could then switch their routes. Tackling this interaction has been generally been classified into equilibrium network design, either in a static study (van Vuren and van Vliet 1992) or a dynamic one (Chen 1998), where the traffic assignment and traffic control problems are solved alternatively with the solution from one problem feeding as the input into solving the other until a *mutually consistent point* (van Vuren and van Vliet 1992) can be reached. In this study, our focus is to develop the efficiency and equity bi-criterion corridor control program, and thus the discussion of the traffic assignment is reduced to a minimum, assuming one simple user behavior to solve the traffic assignment problem to obtain a realistic traffic flow pattern.

As recognized in (Stephanedes & Cang 1993) and then discussed in (Lo 1999, Ma, Nie & Zhang 2007), evaluation of the effectiveness of control strategies is largely dependent on a reliable dynamic traffic flow model. This is especially important when the network becomes heavily loaded and the traffic queues evolve in a complex manner due to the shockwave formation and dissipation across the network. A variety of traffic flow dynamics models have been developed and used in prevalent control programs, for example, vertical queuing model in TRANSYT (Robertson 1969), OPAC(Gartner 1983), spatial queuing model in RHODES (Shelby, 1999), INC-TUC (Diakaki, Papageorgiou & McLean, 2000), cell transmission model (CTM) (Lo 1999, Gomez 2004, Ma, Nie and Zhang 2007), and high-order model (Kotsialos et al 2002). In all these models, the cell transmission model is deemed able to realistically model the network flow dynamics yet maintaining the computation efficiency, which leads to a number of recent studies on traffic control (Lo 1999, Gomez 2004a, 2004b, Ma, Nie and Zhang 2007). Particularly in (Ma, Nie and Zhang 2007), a CTM based generalized network dynamics study tool has been built to study the integrated corridor control strategies. This tool continues to serve as the basis for the bi-criterion corridor control program. In the next section, the tool is briefly introduced and the control measures including urban traffic signals and ramp meters are adapted into the framework in a coherent way.

Traffic Flow Dynamics under Control

Flow Dynamics on a General Corridor Roadway Section

The cell transmission model (CTM) uses a set of finite difference equations to approximate the well-accepted lighthill-whitham-Richards (LWR) model numerically. The well accepted LWR model states the following:

$$\frac{\partial q}{\partial x}+\frac{\partial \rho}{\partial t}=0\,,\; q=f(x,\rho,t) \qquad (2)$$

where q is the flow rate on a road section, ρ is the density, x and t are the space and time variables, respectively. Daganzo (1994) shows that, if the relationship between traffic flow q and density ρ is in the form

$$q=\min\{v\rho, q_{\max}, w(\rho_j-\rho)\} \qquad (3)$$

where v is the free flow speed, $q_{\max}$ is the maximum flow rate, w is the backward shockwave speed and ρ_j is the jam density, then LWR model can be approximated by a set of difference equations. The model discretizes the entire time horizon T (assignment period) into *loading interval* ϕ_l. Conforming to the loading interval, the model divides every road section of the network into homogeneous segments called cells, in a way that the cell length equals the distance traversed by one typical vehicle at free flow speed in one loading interval. The flows are updated by the following difference equations:

$$y_i(t)=\min\{n_{i-1}(t), q_{i,\max}, \delta(N_i-n_i(t))\} \qquad (4)$$

and

$$n_i(t+1)=n_i(t)+y_i(t)-y_{i+1}(t) \qquad (5)$$

where $y_i(t)$, $y_{i+1}(t)$ are the number of vehicles entering cell i and $i+1$ at time t, respectively, $n_{i-1}(t), n_i(t), n_{i+1}(t)$ are the numbers of vehicles in the cell $(i-1)$, i and $i+1$ at time t, respectively, $q_{i,\max}$ is the capacity flow into i at t, N_i-n_i is the space available in i, $\delta=w/v$.

Essentially equation (4) tells that the number of vehicles staying in cell i at loading interval $t+1$ is the number of vehicles from interval t plus the incoming vehicles and minus the outgoing vehicles. Daganzo (1995) extended the model to a general network by carefully dividing various roadway junctions into basic merges and diverges. Since certain types of control measures exist at any roadway junction within a general corridor network, including urban signals, ramp meters or priority rules (e.g., Yield/STOP signs), next the flow updating rules at various types of junctions will be discussed in detail.

Flow Updating at Signalized Urban Intersections

Lo (1999) showed that CTM can be deployed to model the flow updates at urban intersections with a few changes. If the flow capacity $q_{\max}$ in equation (2) is replaced by one that depends on the signal timing variable $g_i(t)$,

$$q_{\max}(t) = \begin{cases} q_{\max} & t \in \text{green} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where it switches between $q_{\max}$ (green) and zero (red), the end cell of an intersection approach will serve as a functioning signal, and the flow dynamics still approximates the LWR model. At a typical intersection, traffic is grouped into movements *or* streams. At a generalized intersection (Figure 1), the traffic movements can be decomposed into simple merges and diverges, where different flow updating rules must apply.

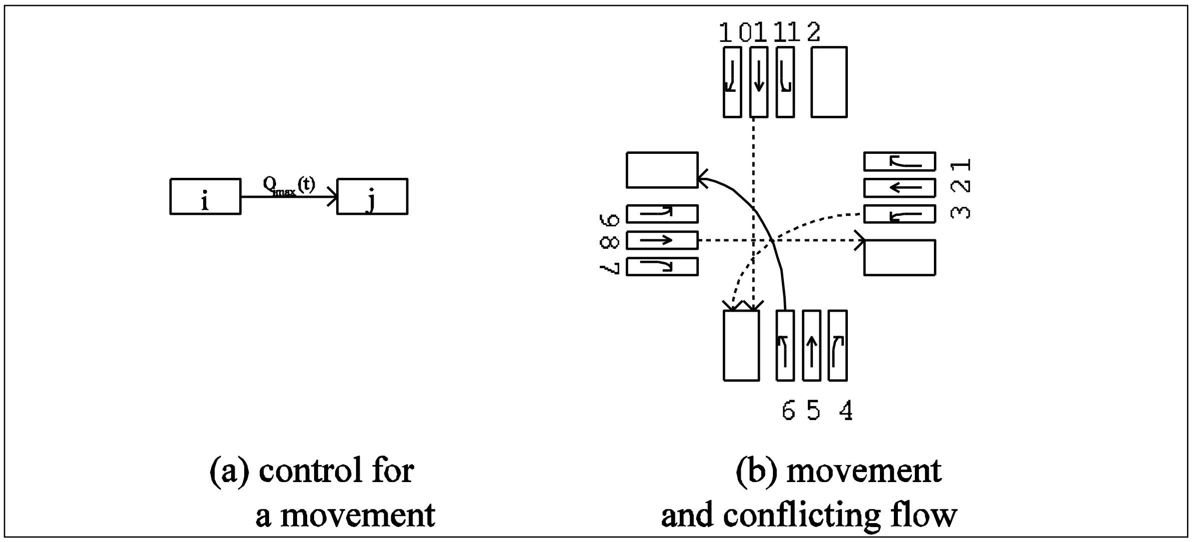

Figure 1 A general representation of cell-based intersection movements

Signalized Diverges

The diverging flows occur where the traffic stream on a single link splits into left turn, through and right turn movements. Left or right turn bays are common to store the incoming vehicles, and these short sections must also be accommodated in the generalized model. Denote the end cell C_s^j of a link j approaching a signalized intersection, the flow conservation equation then reads:

$$n_s(t+1) = \sum_{m=L,R,T} n_{s-1}^m(t) + y_{s-1,s}^m(t) - \sum_{m=L,R,T} y_s^m(t) \tag{7}$$

The superscripts of L, R, T denote the left turn, right turn and through movement, respectively. The cell C_{s-1}^j is the preceding cell of C_s^j . The number of vehicles into and out of cell s are stated as:

$$y_{s-1,s}^m(t+1) = \min\{n_{s-1}(t), q_{s,\max}, \delta_s(N_s^m - n_s^m)\} \tag{8}$$

$$y_{s,s+1}^m(t+1) = \min\{n_s^m(t), q_{s,\max}(t), \delta_{s+1}(N_{s+1}^m(t) - n_{s+1}^m(t))\},\ m = L, R, T \tag{9}$$

where the notation naming convention follows (4) and (5). Note that $N_{s+1}^m, m = L, R, T$ in equation (8) are the different storage capacities for various movements, ensuring that different sizes of turning bays can be modeled accurately.

Signalized Merges

In this study, the right turns are explicitly considered in the signal timing optimization. In this way, the flow updating at intersections is simplified to be the same as a set of coupled consecutive links, which then reads:

$$n_{i+1}(t+1) = n_{i+1}(t) + y_{i+1}(t) - y_{i+2}(t) \tag{10}$$

where $(i+1)$ is the start cell index for the downstream link, i.e., the first cell of the downstream link that receives the stream with cell index of i serviced by the signal. The incoming flux $y_{i+1}(t)$ is then determined by the signal timing plan but shares the same updating rules as in (4) with $q_{i,\max}$ replaced by $q_{i,\max}(t)$ in (6).

The above defined flow dynamics model can conveniently accommodate all four types of signal control actions, namely cycle length $\boldsymbol{C}$, phase sequencing, phase duration $\boldsymbol{g}$ and offset $\boldsymbol{\Delta}$ between two adjacent signalized intersections. In this study, the offset is in reference with respect to the start of the analysis horizon; the numerical values of each variable are also calculated in the multiples of the loading interval ϕ_l .

Metered Freeway Onramp

Modeling ramp meters has only one control variable to deal with, the metering rate at on-ramp link j at time t . For notational simplification, the ramp subscript j is omitted in the following development. Modifying the demand-supply method for merges, we apply one generic flow updating rule to represent the flow dynamics at a freeway merge section:

$$y_i(t) = \min\{n_i(t), q_{i,\max}(t), \delta(N_i - n_i)\} \tag{11}$$

$$D_R^t = \min(D_R^t, R^t) \tag{12}$$

$$D^t = D_M^t + D_R^t \tag{13}$$

$$S^t = \min(S_M^t, D^t) \tag{14}$$

$$f_M^t = \frac{D_M^t}{D^t} S^t \tag{15}$$

$$f_R^t = \frac{D_R^t}{D^t} S^t \tag{16}$$

where the ramp metering R^t is embedded, and other notations are:

D_R^t : Ramp demand at time t;

D^t : Demand upon the beginning cell of the link downstream of the ramp;

D_M^t : Competing demand on mainline;

S_M^t : Supply of the beginning cell of the downstream link;

S^t : Total service flow rate;

f_R^t : Outflow from ramp;

f_M^t : Outflow from upstream mainline.

The modification mainly lies in two aspects: (i) the ramp demand to the merge point is bounded not only by actual demand and the flow capacity, but also by the metering rate executed at that time step (Equation 11); (ii) in the overflow or congestion situation, the freeway mainline and ramp flows will be distributed proportionally to their relative demand (Equations 14-16) (Zhang & Jin 2003). The ramp metering takes effect in the form of R^t .

Priority rule controlled merges and diverges: all-way STOP and yield merge

In addition to the junctions controlled by traffic lights, a large number of junctions are controlled by the rules that drivers must follow in order to go through the junction. Adapted into this study framework, these priority rule-based flow controls can be classified into two categories: all-way stops and yielding merges. Different flow updating mechanisms follow at these two types.

Stop sign control operates on a ``first-come-first-serve'' basis, where the flow is thus discharged in an ordered manner. At each loading interval, the right-of-way (ROW) is allocated according to the order the flow at each approach arrives. Once the approach gets the ROW, the flow will be discharged according to (4) and (5) again.

However, some exceptions, such as the zero-demand approach at certain interval must be handled separately. One flow updating algorithm for this all-way stop sign has been developed in (Ma 2008) and will not be detailed here.

The updating rule for competing flows at yield sign control is essentially merges under priority rules (Figure 2).

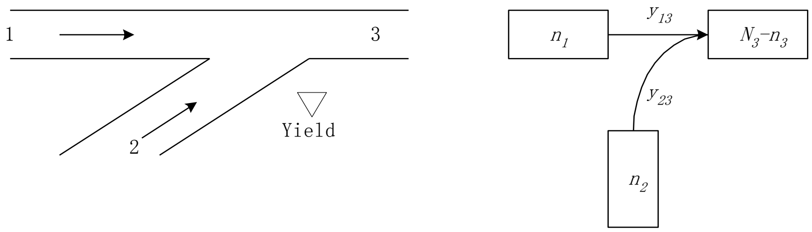

Figure 2 Flow Updating by Priority Control (e.g., Yield Sign)

Under a yield sign, the yielding flow will only be able to take the remainder of the available space at each loading interval. At any loading interval t, the flow updates at a yield sign will be specified by:

$$y_{13}^t = \min\{n_1^t, N_3 - n_3^t, q_{1,\max}\} \tag{17}$$

$$y_{23}^t = \min\{n_2^t, \max\{N_3 - n_3^t - y_{13}^t, 0\}, q_{2,\max}\} \tag{18}$$

where the flow on approach 1 has the priority and the flow on 2 has to yield to flow 1.

Adaptation of Control Methods

The control vector θ, namely the trajectory of the control actions $(g_i^m(t), R_i(t), \eta_i(t), C_i(t), \Delta_i(t))$, $,i \in \mathcal{N}_C$, $t \in [0, T_c]$ is all taken into the above traffic flow dynamics from equations (6, 11-12). Thanks to the discrete nature of this study tool, the phasing order $\eta_i(t)$ and the offset $\Delta_i(t)$ for the signal controller i at a certain intersection is also reflected in the green duration for each phase m, since the control variables of any urban signal controller ($g_i^m(t)$, $\eta_i(t), C_i(t), \Delta_i(t)$) are all rounded to be multiples of the loading interval duration ϕ_l. Basic control methods such as pre-timed, vehicle actuated controllers have been developed and tested in detail in (Ma 2008), indicating that this flow dynamics tool is capable of capturing the complex queuing dynamics under the control actions.

In practice, traffic controls usually enforce some physical constraints including the maximum and minimum duration of the cycle length and green duration for any phase, and the max/min metering rates as follows:

$$C_{i,\min} \le C_i \le C_{i,\max} \tag{19}$$

$$g_{i,\min} \le g_i \le g_{i,\max} \tag{20}$$

$$R_{i,\min} \le R_i \le R_{i,\max} \tag{21}$$

Furthermore, it is assumed that cycle length and phasing sequences are fixed. The cycle length constraint for any intersection then reads:

$$\sum_{h=1}^{N} g_h^j = C^j - NL \tag{22}$$

It tells that the sum of the effective green duration of the phases $h = 1...N$ at intersection j has to be equal to the available green time $C^j - NL$, i.e., the cycle length deducted by the loss time of all phases.

Traffic assignment and Vehicle routing

The traffic assignment process maps the time-dependent demand onto the network to obtain the network flow pattern. Its structure usually contains the following components:

- user behavior model and route guidance generation, which takes the time-dependent demand and route guidance information as input and assigns the trips to a set of paths available to each O-D pair;
- dynamic network loading (DNL), as specified in the flow dynamics equation (1-18).

In this study the users are assumed to select only the (time-dependent) shortest path based on their prior experience to the network. Correspondingly, one particular time-dependent shortest path calculation method (Chabini 1998) is used to update the shortest paths by relying on an space-time expanded network (STEN) (Nie, Nie & Zhang 2004). The STEN is the expansion of the static node-link network by the time dimension, where the time-dependent link travel costs obtained from dynamic network loading is added. Because of its reduced computation complexity, this algorithm is chosen to compute the time-dependent shortest path for loading the path flows.

After the successful traffic assignment and network loading process, the flow discharge profile can be recorded at the cell level for each link. This profile will be used to compute the travel costs of each traveler group and then the system efficiency and equity performances under control.

Measurements of System Efficiency and User Equity

Calculation of Travel Costs

Link Travel Cost

The fundamental diagram indicates two regions that traffic flow status can fall into, the free flow region and the forced flow region. Once the flow falls in the forced flow region, the vehicles will not operate at the free flow speed any more, and delays are incurred to these vehicles. Within the DNL framework built on cell transmission model, at a given time t, a vehicle, *or* flow quantum, can either move to the next cell, or it has to stay in the cell because of the occupied successor cell downstream. Then at the cell level, the delay is calculated as the following:

$$d_i(t) = d(n_i(t) - y_i(t)) = (n_i(t) - y_i(t)) \times \phi_l \, , \; t \in [0, T'] \qquad (23)$$

where $d_i(t)$ is the delay occurring at cell i at loading interval t, and n_i, y_i has the same meaning as before. At the link level, the travel time is the free flow journey time, expressed as the integer multiples of loading interval, plus the delay incurred on the link. Given link l and its cells that are ordered from link entry to exit as $i = 1,...K$, the link traversal time of a certain vehicle entering the link at time t will then be calculated from the following function:

$$\tau_l^1(t) = d_1(t) + \phi_l$$
$$\tau_l^2(t) = d_2(\tau_l^1(t)) + \phi_l, ...$$
$$\tau_l(t) = d_K(\tau_l^{K-1}(t)) + \phi_l$$

In a simple but recursive form, it would be written as:

$$\tau_l(t) = \sum_{i=2}^{K} d_i(c_l^{i-1}(t)) + K\phi_l \qquad (23)$$

where $\tau_l^i(t)$ denotes the time the vehicles spend within the cell i on link l when entering the cell at time t.

Path Travel Cost

Similar to the calculation of link traversal time, the path travel time for a certain travel groups, the travelers with the same origin-destination in this study, can also be calculated recursively from the dynamic network loading results. Consider a path p consisting of sorted nodes $N^{(}r,s)_p = (r,1,...,s-1,s)$, from r to s. When a user departs from origin r at time t,

$$
\begin{aligned}
&c^{r,1}(t) = \tau_{r,1}(t) \\
&c^{r,2}(t) = c^{r,1}(t) + \tau_{1,2}(t + c^{r,1}(t)),\ldots \\
&c^{r,i}(t) = c^{r,i-1}(t) + \tau_{i-1,i}(t + c_p^{r,i-1}(t)),\ldots \\
&c_p^{r,s}(t) = c_p^{r,s-1}(t) + \tau_{s-1,s}(t + c^{r,s-1}(t))
\end{aligned}
\tag{24}
$$

where $\tau_{i,j}$ is the actual link travel time on link (i, j) calculated from Equation (23).
For later analysis, we also give the following definition of relative path travel cost.

[Relative path travel cost (RPTC)] is the ratio of the travel cost of certain path with regard to its nominal travel cost. The nominal travel cost can be that under equilibrium flow pattern or free flow conditions.

RPTC is a better measure to compare the path travel costs between different O-D trips. For a path p with sorted nodes $N_p^{r,s} = (r,1,\ldots,s-1,s)$, the *nominal path travel cost* can be defined as the sum of the free flow travel time traversing all associated links:

$$\tau_{p,0}^{r,s} = \sum_{i=r}^{s-1} \tau_0^{i,i+1} \tag{25}$$

The relative path travel cost is then computed as:

$$d_p^{r,s}(t) = \frac{c_p^{r,s}(t)}{\tau_{p,0}^{r,s}} \tag{26}$$

In the later development, a variation of (26) will also be used:

$$D_p^{r,s}(t) = \frac{c_p^{r,s}(t) - \tau_{p,0}^{r,s}}{\tau_{p,0}^{r,s}} \tag{27}$$

where $D_p^{r,s}(t)$ is the relative path travel delay. To note that minimization of (27) is equivalent to that of (26) when the path is fixed as in this study.

<u>System Efficiency: Total Travel Time</u>

For the entire system, the total travel time is the summation over all O-D pairs through the entire analysis horizon.

$$TTT = \sum_{(r,s)} \sum_{p} \sum_{t} c_p^{r,s}(t) \times f_p^{r,s}(t) \tag{28}$$

TTT from formula (28) will be our system efficiency measure.

Development of User Equity Measurements

A generally accepted taxonomy for evaluation of equity issues in transportation systems identifies two categories (Litman 2007):

- Horizontal equity, or *egalitarianism* is concerned with treating everybody equally, regardless of factors like race and income. It implies that public policies should avoid favoring certain individual or groups over the others.
- Vertical equity, or *social justice* is concerned with the distribution of the benefits or losses between individuals or groups that differ in needs and abilities such as income, social class, and in particular, mobility needs and ability.

Translated in the corridor control context, improving the horizontal equity will imply that the traveler groups experience equal delays without regard to their *nominal travel costs*. On the other hand, improving vertical equity implies that the travel costs of different O-D pairs will be proportional to their nominal travel cost (e.g., travel cost under free-flow conditions). From the development of travel costs in the previous section, we can immediately conclude that path travel cost and relative path travel cost (RPTC) conveniently correspond to the horizontal equity and vertical equity, respectively.

It is recognized that equity can be evaluated at both the aggregate (system) level and disaggregate (user group) level. Aggregate equity characterizes the overall benefits distribution with respect to all user groups. On the contrary, disaggregate level equity measures are concerned with how much benefits or losses each individual traveler groups harvest or suffer. Commonly used aggregate measures are: Gini Coefficient (Gini, 1936) and the associated Lorenz Curve (Lorenz, 1905); Pareto Coefficient and the Pareto Chart. However, different measures could draw conflicting conclusions for the same population and income data set, as revealed in economics studies (Bowman, 1945). As a result, any single measure could distort the understanding of the equity of the travelers. In fact, one of the study objectives was to try to resolve these conflicting results.

For all traveler groups according to their departure time and origin-destination characteristics $\forall(r,s)\in(\mathbf{R},\mathbf{S})$, the path travel costs (23) or RPTC (26, 27) can be reordered in their ascending order:

$$c_{(1)} < c_{(2)} < \ldots < c_{(\mathbf{W})}, \tag{29}$$

and

$$d_{(1)} < d_{(2)} < \ldots < d_{(\mathbf{W})}, \tag{30}$$

where $\mathbf{W}$ denotes the total number of (time-dependent) O-D pairs in $(\mathbf{R},\mathbf{S})$. Their corresponding number of trips is denoted as $f_{(w)}, i = 1, 2 \ldots \mathbf{W}$ successively. Three aggregate equity measures are defined as follows.

<u>Gini Coefficient and Lorenz Curve</u>

Referring to Figure 3, the Lorenz Curve concerning the path travel cost is drawn from reorganizing the path travel cost. Then the Lorenz Curve is a cumulative distribution of a population, drawn to show how much percentage $y\%$ of the total travel delay is experienced by bottom $x\%$ percentage of the driving population. After reordering the path travel cost according to (29) or (30), the Lorenz Curve for the transportation corridor system will be easily drawn as in Figure 3.

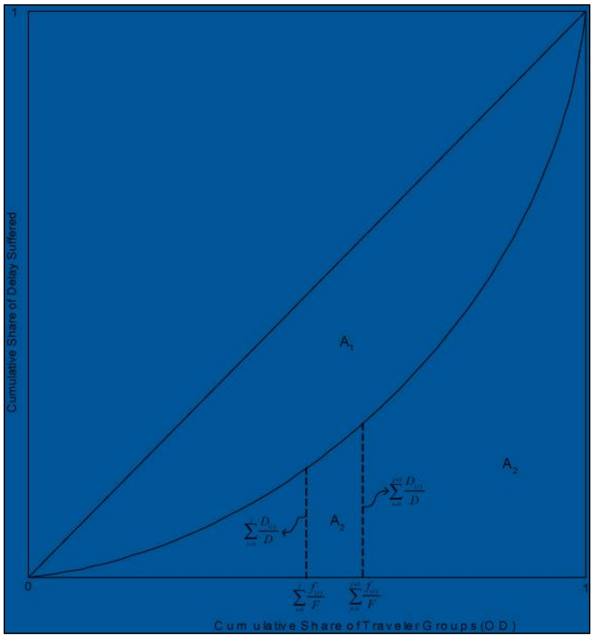

Figure 3 Lorenz Curve and Calculation of Gini Coefficient

Once the Lorenz Curve is drawn, the Gini Coefficient will then be calculated as:

$$Gini = \frac{A_1}{A_1 + A_2} = 1 - 2A_2$$

Differentiating the components, we have

$$A_{2,v}=\left[\sum_{w=0}^{v}\frac{D_{(w)}}{D}\frac{f_{(w)}}{F}+\sum_{w=0}^{v+1}\frac{D_{(w)}}{D}\frac{f_{(w)}}{F}\right]\bullet\frac{f_{(v+1)}/F}{2}$$

where

$$D=\sum_{i=1}^{W}D_{(w)}\qquad F=\sum_{w=1}^{\mathbf{W}}f_{(w)}$$

Then it results in:

$$Gini=1-2\frac{1}{DF^2}\left[\sum_{v=0}^{W-1}\sum_{w=0}^{v}D_{(w)}f_{(w)}f_{(v+1)}+\sum_{v=0}^{W-1}D_{(v+1)}\frac{f_{(v+1)}^2}{2}\right]\tag{31}$$

<u>Mean Difference and Relative Mean Difference</u>

Relative mean difference (RMD) is considered an estimate of the Gini Coefficient, and statistics text shows that it is approximately twice as large as Gini Coefficient. Its calculation is as follows:

$$RMD(D)=\frac{\sum_{v=1}^{\mathbf{W}}\sum_{w=1}^{\mathbf{W}}|D_w-D_v|}{(n-1)\sum_{w=1}^{W}D_w}\tag{32}$$

Similar to Gini Coefficient, it is a dimensionless measure.
For comparison purposes, the absolute mean difference is introduced as well:

$$MD=\sum_{w=1}^{\mathbf{W}}\sum_{w'=1}^{\mathbf{W}}|D_w-D_{w'}|\tag{33}$$

Apparently Mean Difference (MD) has the same unit as the path travel costs. In this sense, it may become a convenient measurement to be linearized with TTT so as to balance the efficiency and equity measures.

<u>Disaggregate Equity Measures</u>

Different from aggregate equity measures, disaggregate ones focuses on individual traveler groups. Mostly it is concerned with the most disadvantaged traveler groups as has been used in a few studies as the only equity measure (Meng 2002, Chen & Yang, 2004). It is called the *critical trip cost ratio* and simply the (relative) path travel cost of the most disadvantaged traveler group after the costs are reordered (29). That is,

$$CR=c_{(\mathbf{W})},or\,CR=d_{(\mathbf{W})}\tag{34}$$

depending on whether horizontal equity or vertical equity is concerned.

Another disaggregate measure quantifies the *range* of the distribution of the travel costs:

$$RG=c_{(\mathbf{W})}-c_{(1)},or\,RG=d_{(\mathbf{W})}-d_{(1)}\tag{35}$$

depending on which type of equities is concerned. The range RG is sometimes a weak complement to CR, since the best travel cost that can be achieved is the free-flow travel time. When vertical equity is concerned and the free-flow travel cost as nominal cost, the range is equivalent to the critical trip cost ratio.

Efficiency-Equity Control Formulation and Solution Algorithms

Balance Efficiency and Equity in Control Objectives

Achieving an efficient and equitable corridor control system implies balancing the measurements on both these dimensions. To formulate the efficiency-equity bi-criterion control problem into a solvable mathematical program, the measures of both dimensions must be present either as the control objective to be optimized or as the constraints to define certain threshold (Meng & Yang 2002). In this study, both efficiency and equity measures are defined in the control objective. While most control programs take the total travel time (TTT) as the sole control objective function, it is also worth mentioning that the equity measures themselves can also serve as the control objective. Therefore, a viable approach is to have the measures of both dimensions weighted into one single objective, e.g., taking MD and TTT into a weighted linear combination as:

$$TTT + W_M MD$$

where W_M is the weight for *MD*.

However, consider that efficiency measure *TTT* and equity measure *MD* cannot be known a priori when choosing the weight W_M , the above linearized combination of efficiency and equity is not always appropriate. Regarding this deficiency, a metric distance measure is developed to balance all three measures equally:

$$\alpha = \left(\left(\frac{TTT - TTT_{\min}}{TTT_{\max} - TTT_{\min}} \right)^{\beta} + \left(\frac{CR - CR_{\min}}{CR_{\max} - CR_{\min}} \right)^{\beta} + \left(\frac{MD - MD_{\min}}{MD_{\max} - MD_{\min}} \right)^{\beta} \right)^{\frac{1}{\beta}} \qquad (36)$$

where β is the power of the metric distance α that measures the effectiveness of the control system in balancing the three measures. Minimizing the metric distance α is to balance both the efficiency measure (TTT) and the equity measure (MD and CR). We apply $\beta = 2$ in this study.

As discussed above, various corridor control objectives, or *control design criterion*, can be defined as follows:

- Criterion I (C-I): total travel time (TTT) only, as defined by equation (28).
- Criterion II (C-II): Mean Difference only, as calculated in (33), and Gini Coefficient (31) will be the supplement measures to this aggregate equity measure;
- Criterion III (C-III): Critical Cost Ratio (CR) only, as calculated in (34).
- Criterion IV (C-IV): combined metric measure α as calculated in (36).

Note that C-I, CII, C-III are mainly defined for comparison purposes.

Efficiency-Equity Bi-criterion Control Program Formulation

Green splits ***g*** of the signal phases and metering rates ***R*** for ramp meters are the most important control variables within the control system, since the green splits and metering rates allocate the right-of-way to the conflicting traffic flows and thus determine the link and movement capacities. Computation of the green splits and metering rates is formulated as a mathematical program with the following structure:

(P 1)

Minimize *C-I, C-II, C-III or C-IV*

s. t.

Traffic demand input (1);

Dynamic link flow pattern from assignment and DNL (2-18);

Min and max green/metering rate constraint (19-21).

Cycle length (22);

Solution Algorithm: Genetic Algorithm based Heuristic Searching

The solution algorithm to compute the optimal control plan is highly tied with the underlying flow dynamics models (Ma, Nie & Zhang 2007, Stephanedes & Yang 1993), for example, the linear programming solver in IN-TUC (Diakaki 2000) global ramp metering control in (Gomez 2004a, 2004b), conjugate gradient algorithm for (Stephanedes & Yang, 1993, Papageorgiou & Kotsialos 2003), where all flow dynamics have been carefully built so as to take advantage of the efficient solution algorithms. Inevitably, many of these models and programs compromise the capability of the underlying dynamic traffic flow models to more realistically model the queuing evolution through the network. On the other hand, heuristic searching algorithms such as genetic algorithm (Foy 1992), simulated annealing (Meng 2002), simultaneous perturbation stochastic approximation (Ma, Nie & Zhang

2007) have been used to solve the control problems with complex traffic flow dynamics models like in this study, where the gradient information of the control objectives with respect to the control variables are not readily obtainable. For this reason, a widely used heuristic algorithm, genetic algorithm (GA) is used to solve the efficiency-equity bi-criterion corridor control problem as formulated above. As a well documented method, the readers can refer to (Goldberg 1989) for further algorithmic development details.

NUMERICAL EXPERIMENTATION

To investigate the effectiveness of the formulated bi-criterion control program, we take one real network to examine the network performances under all four previously specified control objectives. The network is a real one of SR-81 corridor at Fort Worth, Texas. A DynaSmart-P network, developed elsewhere and used in previous studies (Ma, Nie & Zhang 2007), is converted into the above network representation. The geometric layout of the network is illustrated in Figure 4.

Due to the differences in the network representation (e.g., the travel demand releasing mechanisms in DynaSmart-P and our DNL model are different) and lack of further data support, the network was slightly modified in the conversion. The most important modification is the controller type changes. In the original network, the signals are most vehicle-actuated controllers. Since herein only time-of-day corridor control is considered, all controllers are assumed pre-timed. Nevertheless, the same phasing sequence and phase diagrams are inherited from the original settings. The final model has 47 controllers and 2 ramp meters, resulting in a vector θ with 209 control variables.

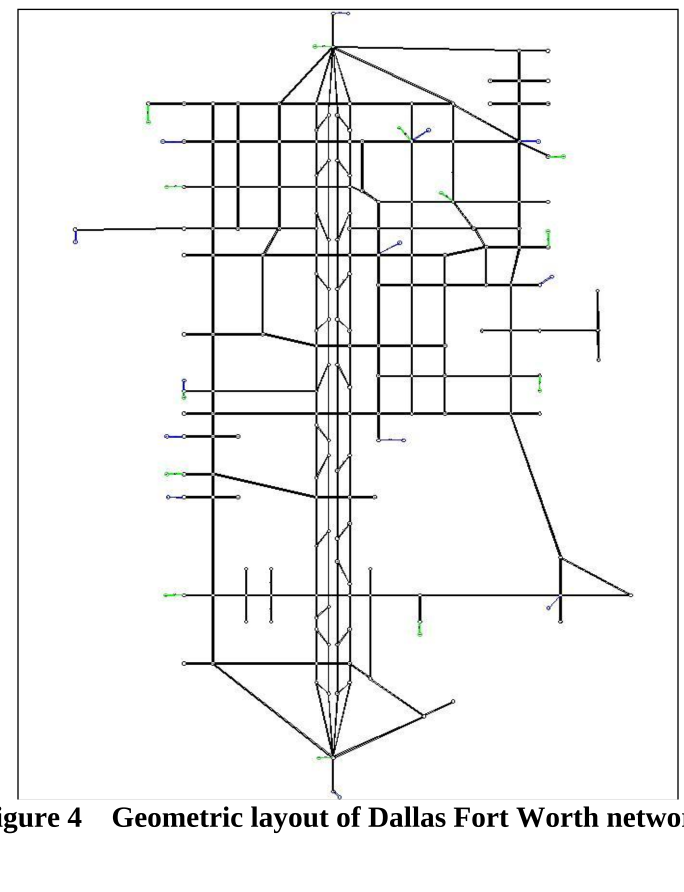

Figure 4 Geometric layout of Dallas Fort Worth network

A previous study (Ma, Nie & Zhang 2007) focused on the system efficiency of this corridor network, where a two-level optimization process to optimize the green splits at the intersection level and successively the intersection offset across the network, using several heuristic searching algorithms including the developed genetic algorithm (GA) tool. The process proved to improve the system efficiency significantly. The following table indicated the TTT reduction at both levels of optimization and the algorithmic parameters. The study also indicated that GA could reach a better and more stable solution than other heuristic searching algorithms.

Table 1 System Efficiency Improvement and Genetic Algorithmic (GA) Parameters

System Efficiency Improvement			GA parameters		
			Population size	# of generation	Mutation Rate
Level I: green split optimization	TTT at start	364.4	40	80	0.02
	TTT after optimization	323.1 (-11%)			
Level II: green split optimization	TTT at start	323.1	40	70	0.02
	TTT after optimization	306.9 (-5.0%)			
Overall Improvement	-15.8%				

Table 1 shows that integrating the signal control and ramp metering is able to improve the operational performance along the corridor. This efficiency-only corridor control design criterion (C-I) serves as the benchmark for comparing the performances under different criteria. However, under this criterion, the control plan after optimization led to the highest CR ratio, implying that some travelers are most severely penalized to make the system efficient.

Various Control Objective Specifications

Under a similar procedure using the developed genetic algorithm solver, the control criteria in (**P 1**) other than system efficiency (TTT) have also been selected and solved. The optimization processes of various programs are shown in Figure 5 – 7. The objective function values, or *fitness values* in GA, all see satisfactory convergences, e.g., the total travel time in the C-I process (Figure 5), the CR in the C-III process (Figure 6) and all measures in the C-IV process (Figure 7). However, Figure 5 and 6 also reveal that the corridor total travel time (TTT), the mean difference of relative path travel cost (RPTC) and the critical trip cost ratio (CR) are independent of each other in the optimization process: the measure selected for optimization sees steady decrease, while the values of the others could oscillate vibrantly and does not show convergence at all. This is particularly true for TTT vs. CR in Figure 5 and 6. In contrast, all processes show acceptable improvements in C-IV under the balanced objective function α.

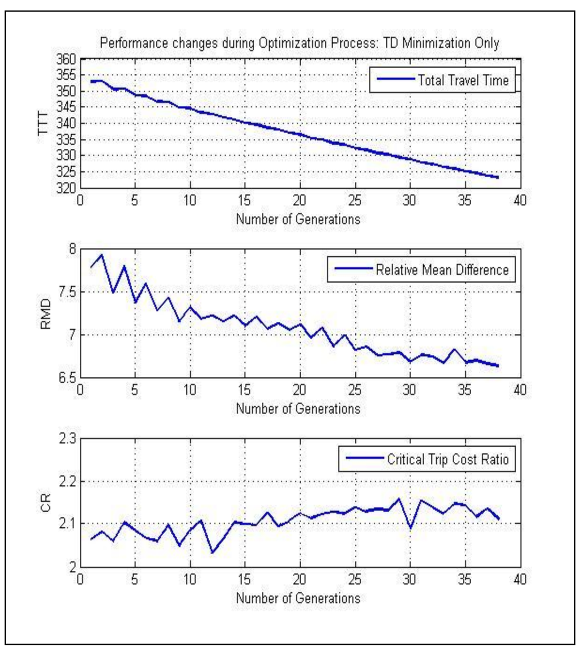

Figure 5 Convergence Process of Efficiency and Equity Measures: under Total Delay Only

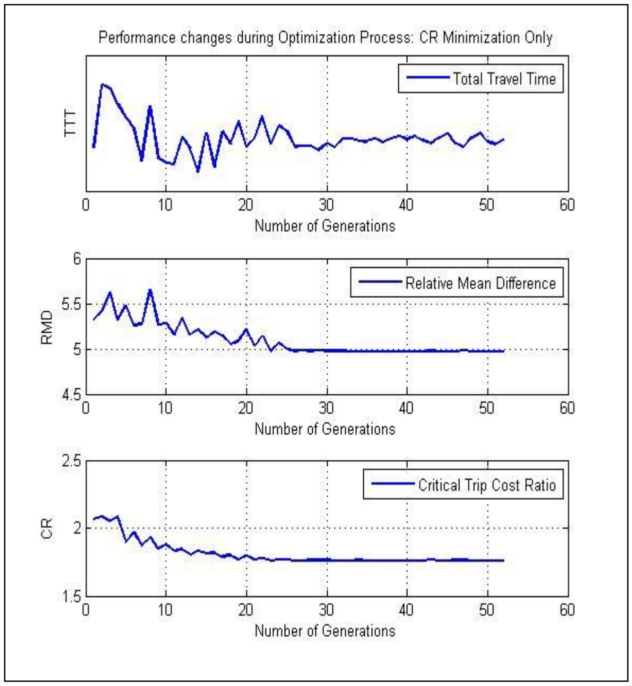

Figure 6 Convergence Process of Efficiency and Equity Measures: under Critical Trip Cost Ratio (CR) Only

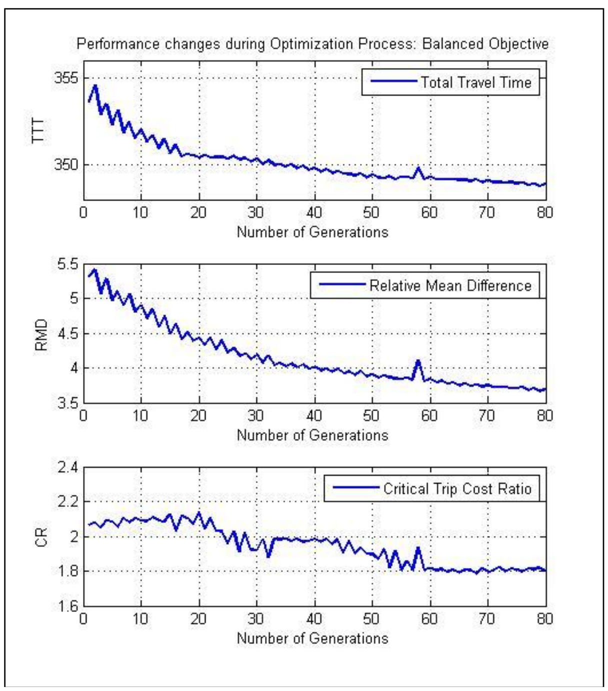

Figure 7 Convergence Process of Efficiency and Equity Measures: under Balanced Objective (Alpha)

The above comparison can answer some of the questions raised in the introduction and in other literature. The system efficiency measure (e.g., TTT) changes independent from the equity measures to a great extent, particularly the disaggregate measure such as critical trip cost ratio (CR). This implies that it is possible to improve the equity measures in the corridor control design while not drastically degrading the system efficiency. Table 2 summarizes the efficiency and equity performances in different control programs. The total network travel time varies within 7% under various control objectives; however, the mean differences (MD) of RPTC and CR vary significantly. It clearly indicates that introduction of the equity measures improves the network performance. This is particularly true when applying the balanced control objective of α, where the TTT, MD and CR are well balanced compared to other scenarios. One may still wonder if the loss of system efficiency when including equity measures into the design; we will introduce the equity elasticity with respect to efficiency changes to investigate the issue in the next section.

Table 2 System Efficiency and User Equity Measures under Various Control Objectives

Criterion	TTT	RMD	Gini Coefficient	CR
TTT Only (C-I)	323.1	6.63	0.85	2.1
RMD Only (C-II)	343.4	4.75	0.65	1.85
CR Only (C-III)	346.5	6.27	0.73	1.75
Alpha (C-IV)	342.8	4.83	0.71	1.76

Equity Elasticity Analysis

To examine the causality between the changing variables, the tool in economics studies is to look at the *elasticity*, i.e., the ratio of the proportional change of one variable with respect to that of another variable, for example, the price elasticity of demand. Similarly, we develop the following *efficiency elasticity of equity* (EEE) to answer the question whether the losses of system efficiency can be justified by the gains in user equity. EEE is defined as follows:

$$E_{TTT,\Psi} = \left| \frac{\partial \ln \Psi}{\partial \ln TTT} \right| = \left| \frac{\partial \Psi}{\partial TTT} \cdot \frac{TTT}{\Psi} \right|$$

where Ψ refers to any of the equity measures including *MD*, *RMD*, *CR* or *Gini Coefficient*. When no formulae are available to characterize both variables, $E_{TTT,\Psi}$ can also be defined in percentage changes as:

$$E_{TTT,\Psi} = \frac{\%\Delta\Psi}{\%\Delta TTT} = \frac{(\Psi - \Psi_0)/\Psi_0}{(TTT - TTT_0)/TTT_0} \qquad (37)$$

Following similar arguments in elasticity, here are the interpretations of the equity elasticity:

- $E_{TTT,\Psi} = 0$: perfectly inelastic;
- $0 < E_{TTT,\Psi} < 1$: relatively inelastic;
- $E_{TTT,\Psi} = 1$: unitary elastic;
- $1 < E_{TTT,\Psi} < \infty$: relatively elastic;
- $E_{TTT,\Psi} = \infty$: perfectly elastic.

When the equity is inelastic with respect to efficiency, the losses in efficiency after introducing equity cannot be compensated for by the gains in equity, and it would be arguable to do so. But when the equity is elastic, the gains in equity will be justified, implying a slight worse-off of system efficiency resulting in a greater improvement in user equity. The larger the equity elasticity, the better the balance will be.

For this analysis, the best possible efficiency performance, TTT in Criterion I (C-I), will be selected as TTT_0 in calculation of all elasticity (formula 37). The elasticity of various equity measures under different programs is computed in the following table.

Table 3 MD, CR & Gini Coefficient Equity Elasticity

Scenario	MD	CR	Gini Coefficient
C-II	4.5	3.7	1.9
C-III	0.7	1.9	2.3

C-IV	4.5	2.7	2.7

It is easily seen from Table 3 that optimizing mean difference (C-II) and the balanced objective α are relatively elastic with all EEE for aggregate and disaggregate equity measures greater than 1. It implies that introducing the equity measure as in both programs can be justified in comparison to the system efficiency optimization only. In particular, C-IV sees a well-balanced efficiency and equity performances. It is interesting to note that C-III, minimization of the critical trip cost ratio only, lead to a EEE of mean difference (MD) less than one; this means that restricting the most disadvantaged group (CR group) from being penalized too much may result in a worse-off of all the travelers. This finding implies that relying on disaggregate measure only (e.g., CR) may not be able to characterize the equity for the entire corridor system, as has been used in a few aforementioned literature (Meng & Yang 2002, Chen & Yang 2004). Instead, both aggregate and disaggregate measures must be present at the same time in order to depict the system performance.

Further statistical analysis is also performed to examine how the travel costs of have been changed under different programs. Figure 8 shows the RPTC scatter plots under various control programs and Table 4 summarizes the mean values and standard deviation of the RPTCs for all travelers under each control program. Even the critical trip cost ratio (CR) is confined under C-III, the average RPTC among all travelers is still the largest, implying a larger dispersion of RPTC among all travelers. This again confirms that compensating for only the most disadvantaged traveler group may not lead to the overall improvement of the system efficiency and user equity. In contrast, both balanced objective α (C-IV) and MD optimization (C-II) can reduce the RPTC dispersion across all travelers.

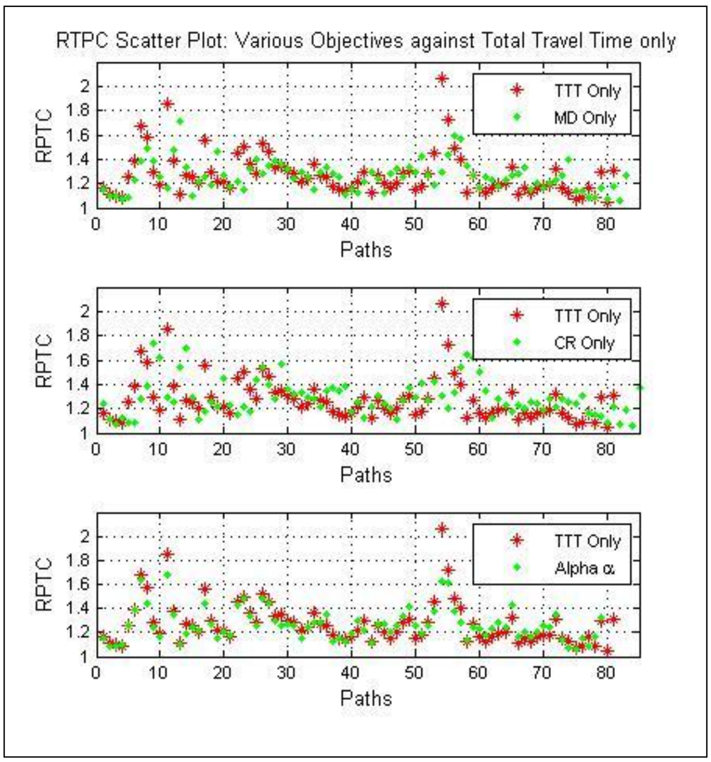

Figure 8 Relative Path Travel Cost (RPTC) Scatter Plots under Various Control Objectives after Optimization

Table 4 The Dispersion of Relative Path Travel Cost (RPTC) under Various Control Objectives

Scenario	C-I	C-II	C-III	C-IV
Avg. RPTC	1.27	1.24	1.28	1.26
Std. Dev.	0.18	0.12	0.15	0.13

CONCLUSION AND FUTURE RESEARCH

An integrated corridor control study tool is firstly built in this study. Based on the finite difference solution scheme to the well-accepted LWR model, this tool adapts the control measures including generalized signal controller, ramp meters and priority rules such as STOP/Yield signs in a coherent manner such that the traffic queuing evolution through a general corridor network can be studied holistically.

The system efficiency and user equity measures were then developed and calculated from the above flow dynamics model. The traveler groups are differentiated according to their origin-destination attributes and conveniently, the horizontal and vertical equity for corridor travelers can be distinguished from the traveler group' path travel cost under free flow condition. The literature review let us realize that the user equity must be characterized at both aggregate and disaggregate level; various measures were also developed to represent the equity measures at both levels.

An efficiency-equity bi-criterion corridor control program is set up to balance the system performance at both dimensions. Due to the complex nature of the underlying flow dynamics model, the program was solved using a heuristic searching algorithm. The green splits for signal controllers and ramp metering rates were computed under various control objectives: efficiency measure only, equity measures only and the balanced measure.

The numerical results from the experimentation on a real network are many folds. Firstly the optimization of only the system efficiency measures can generally lead to undesirable user equity performances. It implies that some traveler groups will sacrifice considerably to compensate for other traveler groups when the most efficient control plan is implemented. Secondly the system efficiency and user equity objectives are generally independent from each other. When optimizing one single control objective, the others could display random oscillation. This phenomenon confirms that an integrated control system is possible to balance the efficiency and equity. Particularly, minimization of total travel time is irrelevant to the critical trip cost ratios; this also implies that an efficient-equitable control program will have to combine both efficiency measures and equity measures. Thirdly, the usual treatment of having only the disaggregate equity measure is incomplete to model the user equity in general. With minimized critical trip cost ratio, the dispersion of the relative path travel cost can still be large, implying that only restricting the most disadvantaged group from being sacrificed too much is not enough for other traveler groups. Elasticity analysis in the numerical experiments also indicated that the gains in equity may not even be well justified when optimizing the disaggregate equity measure only. Therefore, equity measures must include both aggregate and disaggregate ones. The bi-criterion control program using the metric measure to combine both efficiency and equity measures in its objective can generate a balanced system. While the system efficiency may be slightly degraded, the equity among travelers can be significantly improved and well justified.

This study is by no means the end of the research on this topic; instead, it is rather the start point towards various efficient and equitable traffic management policies, and many more questions remain to be answered in the future. For example, an immediate question would be what the results and conclusion could change if different user behavior assumptions are applied and the resulting dynamic network flow patterns (e.g., dynamic network equilibrium) are changed? Furthermore, can we use traveler information and route guidance system to effect a more efficient and equitable transportation system; if the answer is yes, does more information fed into the system bring higher efficiency and greater equity among travelers? It is also hoped that introducing the bi-criterion design concept into the planning process can curb the research and practice towards a both efficient and equitable transportation system.

AKNOWLEDGEMENTS

The research was supported by a grant from the Sustainable Transportation Center at the University of California Davis, which receives funding from the U.S. Department of Transportation and Caltrans, the California Department of Transportation, through the University Transportation Centers program.

REFERENCES

Allsop, R. E. (1974), Some possibilities for using traffic control to influence trip distribution and route choice, *Proceedings of the 6th International Symposium on Transportation and Traffic Theory* pp. 345-372. University of New South Wales, Sydney, Australia.

Bowman, M. J. (1945). A graphical analysis of personal income distribution in the united states, *American Economic Review* XXXXV(4), 607-628.

Chen, O. J. (1998), Integration of Dynamic Traffic Control and Assignment, PhD Thesis, Massachusetts Institute of Technology, Department of Civil and environmental Engineering.

Chabini, I. (1998), Discrete dynamic shortest path problems in transportation applications, *Transportation Research Record* 1645, 170-175.

Daganzo, C.F. (1994). The cell transmission model: a dynamic representation of highway traffic consistent with the hydrodynamic theory. *Transportation. Research -B,* Vol. 28, pp.269:287.

Daganzo, C.F. (1995). The cell transmission model, part II: network traffic. *Transportation. Research -B*, Vol. 29, pp.79:93.

Diakaki, C., Papageorgiou, M. & McLean, T. (2000). Integrated traffic-responsive urban corridor control strategy in Glasgow, Scotland: application and evaluation, *Transportation Research Record* 1727, 101-111.

Gartner, N. H. (1983), OPAC: A demand-responsive strategy for traffic signal control, *Transportation Research Record* 906, 75-81.

Foy, M., Benekohal, R. & Goldberg, D. (1992), Signal timing determination using genetic algorithms, *Transportation Research Record* 1365, 108-115.

Gini, C. (1936), *On the measure of concentration with especial reference to income and wealth*, Cowles Commission.

Goldberg, D. (1989). *Genetic algorithms in search optimization and machine learning*, Addison-Wesley Publishing Co., Reading, Massachusetts.

Gomez, G., R. Horowitz (2004a). Globally optimal solutions to the onramp metering problem - part I. *IEEE on ITS'04*, Washington D.C. USA.

Gomez, G., R. Horowitz (2004a). Globally optimal solutions to the onramp metering problem - part II. *IEEE on ITS'04*, Washington D.C. USA.

Jahn, O., ☐Öhring, R. H. M., Schulz, A. S. & Stier-Moses, N. E. (2005). System optimal routing of traffic flows with user constraints in networks with congestion, *Operations Research* 53(4), 600-616.

Kotsialos, A. & Papageorgiou, M. (2004). Efficiency and equity properties of network-wide ramp metering with AMOC, *Transportation Research, -C* Vol. 12, 401-420.

Levinson, D., Zhang, L., Das, S. & Sheikh, A. (2002). Ramp meters on trial: evidence from the Twin Cities ramp meters shut-off, 81st TRB Annual Meeting CDROM. Washington, D.C.

Litman, T. (2007). *Evaluating transportation equity: Guidance for incorporating distributional impacts in transportation planning*, Research report, Victoria Transport Policy Institute, Victoria, BC, Canada.

Lo, H. K. (1999). A novel traffic signal control formulation. *Transportation Research, -A*, 33A:433–448.

Lorenz, M. (1905). Methods of measuring the concentration of wealth, *Publications of the American Statistical Association* 9(70), 209-219.

Ma, Jingtao (2008) *An Efficiency-Equity Solution to the Integrated Transportation Corridor Control Problem*, PhD Thesis, Department of Civil and Environmental Engineering, University of California at Davis, Davis, CA, USA.

Ma, Jingtao, Y. Nie, H.M. Zhang (2007) Solving the Integrated Corridor Control Problem Using Simultaneous Perturbation Stochastic Approximation, *TRB 86th Annual Meeting* CDROM paper #07-1282.

Meng, Q. & Yang, H. (2002). Benefit distribution and equity in road network design, *Transportation. Research -B* 36, 19-35.

Nie, Y., Nie, X. & Zhang, H. M. (2004). The relative performance of time-dependent shortest path algorithms: a network expansion perspective, *Proceedings of the 8th International Conference on Applications of Advanced Technologies in Transportation*. Beijing, China.

Nie, Y. (2006). *A Variational Inequality Approach For Inferring Dynamic Origin-Destination Travel Demands*, PhD Thesis, University of California at Davis, Department of Civil and environmental Engineering.

Kotsialos, A., M. Papageorgiou, M. Mangeals, and H. Haj-Salem (2002). Coordinated and integrated control of motorway networks via nonlinear optimal control. *Transportation. Research C*, Vol.10:65–84.

Robertson, D. I. (1969). *TRANSYT: a traffic network study tool*. Technical report 253, Transport Road Research Laboratory, UK.

Shelby, S. (2001). *Design and Evaluation of Real-time Adaptive Traffic Signal Control Algorithms*, PhD Thesis, University of Arizona, System and Industrial Engineering Department.

Stephanedes, Y. J. & Chang, K. (1993), Optimal control of freeway corridors, *ASCE Journal of Transportation Engineering* 119(4), 504-514.

van Vuren, T. & van Vliet, D. (1992). *Route Choice and Signal Control*, Athenaeum Press Ltd.

Webster, F. (1958), Traffic Signal Settings, Technical report 39, Transport Road Research Laboratory(TRRL), UK.

Yin, Y., Liu, H. & Benouar, H. (2000). A note on equity of ramp metering, *The 7th IEEE International Conference on Intelligent Transportation Systems* .

Shelby, S. (1999) PhD Thesis

Zhang, H. M., W. Jin (2003). On the distribution schemes for determining flows through a merge. *Transportation. Research -B.*, 37(6):521–540.