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A Stochastic Programming Approach for Transportation Network Protection

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A Stochastic Programming Approach for Transportation Network Protection

by

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A Stochastic Programming Approach for Transportation Network Protection

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Dedication

To Zhiyu, Xuanyu and my parents

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Abstract

A Stochastic Programming Approach for Transportation Network Protection

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DOCTOR OF PHILOSOPHY in Civil and Environmental Engineering

University of California, Davis

Finding effective strategies of allocating limited mitigation resources to critical infrastructure system components for protection, response, and recovery is among the central tasks of disaster mitigation and management. This dissertation tackles the pre-disaster network protection problem, a specific instance of the above general resource allocation problem, of determining which network components should be protected (e.g. retrofitted or strengthened) before disasters given resource constraints. The most prominent feature of this problem is decision making under uncertainty since disasters are not realized yet and hence uncertain at the time of making protection decisions. A popular method for dealing with uncertainty in the practice of disaster mitigation is scenario analysis. System cost is evaluated under each disaster scenario and scenario dependent policies may be generated. One then can aggregate these scenario dependent policies into an implementable policy or simply take the policy from the most likely scenario. This scenario analysis approach has little possibility to ensure an optimal policy in the sense of optimizing mathematically well defined system measures (e.g. expected loss from disasters).

This dissertation develops a rigorous approach based on stochastic programming and network optimization with the capability of capturing system component interdependency and explicitly incorporating uncertainty. We study two variants of the network protection problem with different assumptions of network flows. Firstly assuming network flows are completely controllable to achieve system optimum (SO), we formulate the problem as a two-stage risk averse stochastic program with nonlinear recourse and binary variables in the first stage, which seeks a balance between minimizing expected system cost and reducing system cost variation. An efficient algorithm is designed via extending the well-known L-shaped method. Numerical experiment results demonstrate the superiority of the stochastic programming approach to the engineering method. Secondly assuming network flows are in the user equilibrium (UE) condition, we formulate the problem as a stochastic mathematical program with complementarity constraints (SMPCC), which is hard to solve due to its non-convexity and non-smoothness. The Progressive Hedging (PH) method is employed to solve the SMPCC, which iterates between the process of solving scenario (perturbed) subproblems and aggregating scenario solutions into an implementable policy. Each scenario subproblem, a mathematical program with complementarity constraints (MPCC), is solved via a relaxation approach.

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1 Introduction

Critical infrastructure systems are essential for the functioning of a society and economy. Examples include internet, electricity generation and distribution network, water supply network, and transportation system. However, these critical infrastructure systems are vulnerable to natural and man-made disasters (e.g., earthquakes, hurricanes and floods) and terrorist attacks. Such disruptive activities may cause facility damage, loss of service capability and significant economic and social losses (see e.g. White House 2003; Ham, Kim et al. 2005; Burby 2009), raising the importance of disaster management and mitigation. While technologies continue to play an important role in disaster mitigation, effective management of mitigation resources is equally important in order to make the best use of available mitigation technologies. This dissertation focuses on the problem of allocating mitigation resources for protecting transportation infrastructure systems against disasters, particularly seismic hazards.

1.1 Motivation

Certain components in a transportation infrastructure system such as highway bridges are often fragile under seismic hazards due to their special structural features. For example, 286 state highway bridges were damaged in the 1994 Northridge earthquake, of which seven major ones collapsed (Housner and Thiel 1994). A damaged transportation system directly affects the effectiveness of post-disaster rescue and repair activities, and also causes huge socio-economic losses (Okuyama and Chang 2004). Despite the unpredictable nature of disasters in terms of location, time, and magnitude, seismic retrofit appears to be one of the effective mitigation methods for highway bridges. Again using the 1994 Northridge earthquake as an example, the highway bridges that had been retrofitted survived the earthquake even though some were within 100 meters of collapsed structures (Yashinsky 1998). On the other hand, retrofitting highway bridges can be costly in terms of monetary and manpower resources. This naturally raises a **research question**: how should limited resources be allocated to candidate facilities for retrofit so that the total loss of the entire transportation system caused by future earthquakes is minimized?

Several challenges need to be addressed in order to answer the above question. First of all, individual bridges should be considered as a whole system instead of being treated separately. The Federal Highway Administration (FHWA) seismic retrofit manual (Werner, Taylor et al. 1999) states that retrofit decisions are made according to seismic hazard and the importance of individual components. The importance is mainly judged by the daily traffic volume that a highway segment carries, and some other subjective judgments such as its connectivity to critical facilities. However, individual components in a transportation system are actually not independent of each other. Any change in one component of the system may cause redistribution of the traffic and thus affect the traffic on other remote components as well. Thus a segment could be more important in a system sense than its traffic volume alone would indicate, if the loss of that segment would have a disproportionately adverse impact on system performance. A rigorous retrofit decision should be made at a system level, where a spatially distributed transportation infrastructure system may be modeled as a network and the interrelations between different components can be captured by network flow theory. Such system issues are not currently considered in seismic retrofit practice, due primarily to the lack of adequate system-based evaluation and decision tools (Werner, Taylor et al. 1999).

Another challenge in retrofit decision making is the high uncertainty induced by the nature of most disasters, which makes deterministic modeling techniques less relevant. Most existing research in disaster mitigation is scenario specific (e.g. Shinozuka, Juran et al. 2000; Beavers 2003; Ham, Kim et al. 2005). A few representative scenarios are first identified by domain experts. Then damage assessment and a mitigation plan are developed for each scenario. However, the scenario-specific mitigation plans are not useful to policy makers in the case of pre-disaster retrofit since one would not know exactly which scenario would happen at the time of decision making. A common engineering approach is to implement the mitigation plan specific to the most likely scenario. But this plan may not even be feasible for other possible scenarios, and the penalty for encountering an infeasible solution can be extremely high. Moving beyond current scenario-specific engineering approaches to arrive at a more rigorous stochastic approach is another focus of this dissertation.

In summary, optimizing the resource allocation for protecting transportation infrastructure is an important but challenging problem. In view of the limitations of the current disaster mitigation practice and research, this dissertation is devoted to developing stochastic optimization models which explicitly incorporate interdependency among infrastructure system components and cope well with uncertainty. The next section will formally define the problem and then present a brief introduction to our approach.

1.2 Research Design

In recognizing the interdependency of transportation infrastructure and uncertainty involved in retrofit decision making, we refer to the problem of optimizing resource allocation for retrofitting transportation infrastructure as **Stochastic Transportation Network Protection (STNP).** The primary *goal* of this dissertation is to establish a mathematical modeling framework for this problem with the capability of capturing infrastructure interdependency and incorporating uncertainty, and to evaluate the solution by comparing it with the one from the engineering approach. We first mathematically define the problem and then introduce the main methodologies developed in this dissertation.

Consider a transportation network which consists of links (e.g. road segments, bridges and tunnels) and nodes (e.g. intersections). Denote this network as $G(N, A)$, where N is the set of nodes with size n and A is the set of network links with size m. Suppose the network is under the risk of seismic hazards. Let the random vector $\boldsymbol{\xi}$ describe the uncertain link damage conditions under earthquakes. Each realization of $\boldsymbol{\xi}$, denoted by ξ, and the corresponding probability $p(\xi)$ define a damage *scenario*. Advanced seismic and structural analysis can lead to probabilistic estimations of the damage condition of links and thereby construct the data on damage scenarios. Damage scenarios are illustrated by the following Figure 1-1.

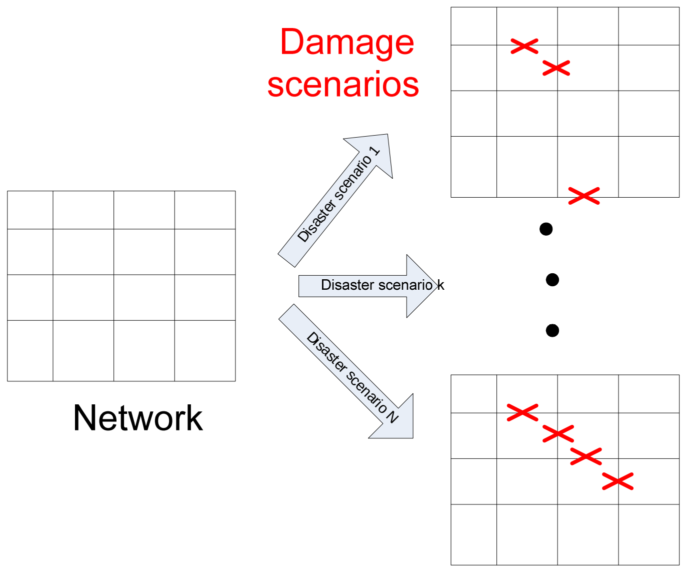

Figure 1-1: An illustration of network damage scenarios

(Crosses indicate that the links are damaged)

Now consider the option of retrofitting network links before disasters. The survivability of retrofitted links in future earthquakes is increased and consequently system resilience and robustness are enhanced. However, given a limited budget, only a subset of links can be retrofitted. Let u_a be the decision variable representing the retrofit action on link a, which could be a continuous variable if the decision is on the amount of retrofit resource to be allocated to link a, an integer variable if the decision is on the level of retrofit efforts, or a binary variable if the decision is simply whether or not to retrofit link a. The STNP is then to determine the optimal value of u_a under budget constraints so that the loss from future disasters is minimized.

To tackle the first challenge of incorporating interdependency among system components, we formulate the transportation network protection problem as a network optimization model. It is closely related to the class of network design models in the sense that they both optimize resource allocation over network components given limited resources for improving network performance. The network design problem (NDP) is extensively studied in the literature of operations research and transportation planning (Magnanti and Wong 1984; Yang and Bell 1998). However, the special feature of the STNP is that decision makers face an uncertain network configuration. More specifically, a given design decision in the NDP results in a deterministic new network configuration, while in the STNP, design decision and disaster co-determine network configuration. At the time of decision making, we can at best obtain some probabilistic estimations of disasters and resultant post-disaster network configurations.

In order to cope with the high uncertainty involved in decision making, we built the model in a two-stage stochastic programming framework. Stochastic programming is one of the most popular methods for modeling optimization problems involving uncertainty. Stochastic programming was first introduced by Dantzig (1955) to handle linear programming with uncertainty, and was further developed both in theory and computational aspects by subsequent work (e.g. Wets 1966; Vanslyke and Wets 1969; Wets 1974). We found that the STNP fits into the framework of two-stage stochastic programming very well. In the first stage, planning agencies make decisions on the choice of links to be retrofitted, then a disaster occurs and changes network configuration. In the second stage, network users or system controllers make routing decisions based on the post-disaster network configuration. The objective of the STNP problem is to make retrofit decisions in a way that the sum of the first-stage costs and the expected value of the random second-stage costs is minimized.

Combining network optimization and stochastic programming, the resultant stochastic network optimization model is able to capture infrastructure interdependency and explicitly incorporate uncertainty. A conceptual model for the STNP problem reads as follows:

$$\min_{u} \quad g_1(u) + \underset{\xi\in\Xi_d}{E}\{Q(u,\boldsymbol{\xi})\} \tag{1.1}$$

$$s.t. \qquad u \in U \tag{1.2}$$

with

$$Q(u,\xi) := \min_{f} \ \{ \quad g_2(u,\xi,f) \mid f \in G(u,\xi)\} \tag{1.3}$$

where $g_1(u)$ is the first stage cost function which may include retrofit cost, and $Q(u,\boldsymbol{\xi})$ is the optimal value function of the second stage problem. The second stage is a network optimization problem, where g_2 is the second stage cost function, f is network flow and $G(u,\xi)$ is the feasible region for flow. The first stage problem is to find an optimal retrofit decision such that the summation of the first stage cost $g_1(u)$ and expected second stage cost $\underset{\xi\in\Xi_d}{E}\{Q(u,\boldsymbol{\xi})\}$ is minimized.

Variants of the model (1.1)-(1.3) are developed in this work depending on modeling assumptions. Chapter 3 focuses on the STNP assuming flows are completely controllable and achieve system optimum (SO); while chapter 4 studies the problem assuming flows are in user equilibrium (UE) condition. Different flow assumptions in these two chapters lead to different formulations and numerical algorithms. In the context of infrastructure protection where extremely severe consequences should be avoided, we also include some risk measures in the model to improve the robustness of the solution.

System uncertainty has been largely ignored in the area of transportation network modeling until recently a few studies have started to account for uncertainty. For example, Patil and Ukkusuri (2007) propose a stochastic programming model for the network design problem under demand uncertainty assuming SO flows; Ukkusuri, Mathew et al.(2007) study the same problem as Patil and Ukkusuri (2007) but assuming UE flows, and formulate a stochastic bi-level programming model. However, the numerical experience for solving these models is quite limited. Most of these works either only solve very small networks by solving deterministic equivalent programs (DEP) (e.g. Patil and Ukkusuri 2007) or solve the problem using evolutionary algorithms (e.g. Chen and Yang 2004; Ukkusuri, Mathew et al. 2007). It is another *goal* of this dissertation to enrich the numerical experience for solving these stochastic transportation network optimization models by developing numerical algorithms which explore the special structure of the problems and decompose them into manageable subproblems.

1.3 Contributions

This dissertation advances the research in disaster mitigation and transportation network modeling in several ways.

1. First of all, it integrates multiple disciplines to establish a conceptual framework for optimizing the decision making among infrastructure protection activities. This framework overcomes the main limitations in the current disaster mitigation research and practice of ignoring interdependency among system components and uncertainty.
2. The dissertation also adds substantially to the methodologies of transportation network modeling by systematically investigating the issues involved with incorporating uncertainty. Stochastic network optimization models are developed including a novel mean-risk stochastic programming model and a stochastic

programming model with equilibrium constraints (SMPEC). Much effort has been devoted to designing and testing decomposition-based numerical algorithms with the goal of complementing rather limited numerical experience for solving these models.

3. Through case studies, we demonstrate that the proposed models generate more efficient and robust solutions than the engineering approach used in practice.
4. The proposed methodologies are general and can be applied to other network optimization problems under uncertainty.

1.4 Thesis Outline

The remainder of this dissertation is organized as follows. Chapter 2 reviews related literature including critical infrastructure protection, network design and stochastic programming.

Chapter 3 formulates the STNP problem in the framework of the two-stage stochastic programming assuming SO flows. A risk-averse measure is incorporated into the model to improve solution robustness. A numerical algorithm based on an extension to the well known L-shaped method is designed to solve large scale problems.

Chapter 4 tackles the STNP problem assuming network flow is in UE condition. The problem is formulated as a SMPEC. Equilibrium constraints introduce nonconvexity and make the problem much more difficult to solve. We will show the effectiveness of a numerical algorithm based on the Progressive Hedging (PH) method.

Finally in chapter 5 we conclude this dissertation with a summary of results and discussion of future research extensions.

2 Literature Review

Protecting the transportation network against disasters is one class of general critical infrastructure protection (CIP) problems. In this chapter, we shall first review the relevant research on CIP with the objective to find what has been studied and what has not been addressed. Then we will review the network design problem and stochastic programming, which provide foundations for our proposed methodologies.

2.1 Critical Infrastructure Protection

Engineers and physicists study CIP problems from different perspectives. Engineers often adopt a scenario analysis approach which first identify disaster scenarios and then evaluate system loss accordingly. Physicists, however, are more interested in understanding how network behaves under some assumed disruption strategies. Their research approach could be categorized as network vulnerability analysis. In the following sections, we will review these two approaches respectively.

2.1.1 Engineering Scenario Analysis Approach

Scenario analysis approach is a prevalent method in the practice of disaster mitigation. Firstly potential location and level of disasters are identified and disaster scenarios are constructed. Then detailed analysis on system loss is conducted for each scenario. A typical example is the work by Kiremidjian et al. (2007). It studies the impact on highway bridges and related highway network of a magnitude 7.0 scenario earthquake in the Hayward fault in California. Damage to bridges from ground shaking and ground displacements are first computed. Then direct loss from bridge damage and increased travel delays in transportation network are estimated. Similarly, Ham et al. (2005) assess transportation network damage and economic impacts of a potential earthquake with several hypothetical scenarios.

The major advantage of scenario analysis approach is its simplicity and capability of conducting relatively detailed analysis for each scenario. The analysis can include the impact of disruptions on facilities, network flows, economic activities, and even the measures unique to each scenario. Analysis results can provide insights on the potential value of certain facilities and facilitate effective response and recovery plans. However, as we point out in chapter 1, this approach provides little guidance on making pre-disaster protection plans since nobody knows which scenario will happen at the time of decision making.

2.1.2 Network Vulnerability Analysis

Assessment of network vulnerability to disruptions has attracted a lot of attention from physics research (e.g. Albert, Jeong et al. 2000; Crucitti, Latora et al. 2004; Latora and Marchiori 2005; Jenelius, Petersen et al. 2006; Grubesic, Matisziw et al. 2008; Murray, Matisziw et al. 2008;

Matisziw, Murray et al. 2009). Different from engineering evaluation to the physical loss of disasters, physicists pay more attention to understanding statistical properties of different types of networks under disruptions. A common approach is to simulate the process of network performance change (e.g. network connectivity loss) with gradual removal of network facilities (nodes or links) following certain removal strategies. For example, Crucitti et al. (2004) examine the change of network performance under two disruption strategies of attack and random failure. Here attack refers to removing nodes according to their degrees (number of links connected to the node), while failure refers to randomly removing nodes.

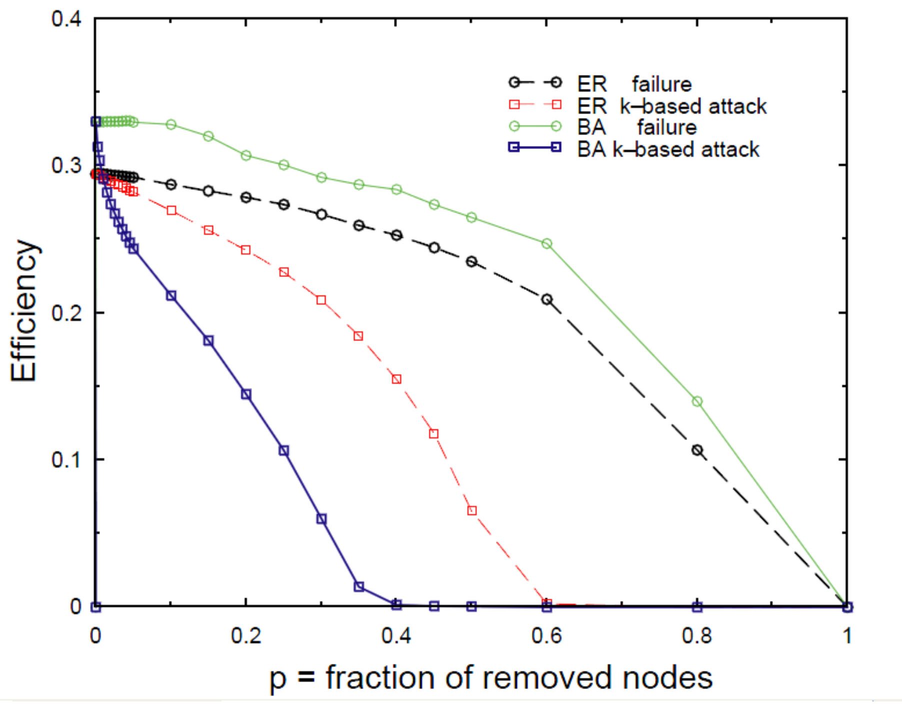

Figure 2-1: Change of Network performance (efficiency) with nodes removal

(Source: Crucitti, Latora et al. 2004)

As visualized by Figure 2-1, BA network (a few super nodes are responsible for network connectivity) and ER network (most nodes have approximately same degree) behave very differently under attack and failure: BA network is robust to failures but quite vulnerable to attacks, whereas ER network is relatively indifferent to attacks and failures.

Another class of network vulnerability studies aims to identify critical components in a network. The importance of a network component is indicated by the drop in network performance if the component is removed. If importance indicators for all the components in a network are calculated, one can easily obtain a rank of network components in the order of importance. This idea is first explored by Latora and Marchiori (2005), and then extended to transportation networks (see e.g. Jenelius, Petersen et al. 2006; Nagurney and Qiang 2008), where travel demand and spatial distribution of network flows are considered. We note that importance indicators described here contain information on network components interdependency and network flow characteristics. Hence the rank based on these indicators is more valuable than the one based on simple measurements (e.g. node capacity and link volume) as discussed in Chapter 1. However, each re-evaluation of network performance after the removal of a network component includes a non-trivial computation of flow re-distribution, and an enumeration of all possible component removal combinations is necessary for obtaining the rank of components. The high computational burden restricts the application of this method to any realistic-size networks.

Network vulnerability study is beneficial to disaster management and mitigation planning. Firstly, it can help to design networks which are robust to potential disruptions. The vulnerability of different network configurations can be evaluated under certain disruption strategy, which offers insight on what type of configuration is more resilient to this specific disruption strategy.

Secondly, the importance of facilities and their ranking are useful information for guiding disaster mitigation including pre-disaster protection and post-disaster response and recovery activities. However, as we pointed out, the requirement of enumeration prohibits their application to even moderately sized networks. In fact, the optimization method, which will be discussed in the next section, is designed to avoid enumeration and quickly obtain desired results.

2.2 Network Design

The network design problem (NDP) is to choose an optimal subset from a set of feasible link additions and/or expansions to an existing network. The objective is to optimize a system performance measure subject to limited resources, while accounting for the route choice behavior of network users. NDPs have many variants depending on the types of decision variables, routing behavior assumptions, and the forms of objective functions. For example, the NDP could be classified into two types of problems: the continuous network design problem (CNDP), where decision variables are continuous and represent continuous capacity increase to links; the discrete network design problem (DNDP), where decision variables are integers and may represent the number of lanes added to a link or the binary decision of adding a link or not. The NDP could be also classified into NDP with system optimal flows (NDP-SO) where flows are completely controllable to achieve system optimum and NDP with user equilibrium flows (NDP-UE) where flows are assumed to follow equilibrium conditions. We will review the formulations and solution methods for these two types of problems in subsequent sections.

2.2.1 NDP-SO

For a network where flows are completely controllable to achieve system optimum, the NDP is usually formulated as an integer or linear programming problem (Magnanti and Wong 1984). We first introduce some notation and then present a typical formulation of NDP-SO.

Consider a network $G(N, A)$ under an improvement project, where N is the set of nodes with size n and A is the set of links with size m. Denote $\overline{A}$ ($\overline{A} \subset A$) as the set of links that are candidates of the network improvement project. The size of $\overline{A}$ is $\overline{m}$. The flow between each origin-destination pair is considered as one commodity. For each commodity $k \in \kappa$, denote $O(k)$ as its origin, $D(k)$ as its destination, and q^k as the travel demand from origin $O(k)$ to destination $D(k)$. The model contains two types of variables representing design decisions and operational decisions respectively. Here we assume discrete design decisions, i.e. the binary decision variable u_{ij} is 1 if link (i, j) is selected by the network design project and 0 otherwise. The operational decisions specifically refer to flow variables. For each commodity $k \in \kappa$, x_{ij}^k is the flow of commodity k on link (i, j). Denote f_{ij} as the total flow on link (i, j), i.e., $f_{ij} = \sum_{k \in \kappa} x_{ij}^k, \forall (i, j) \in A$, u and f vectors of design and flow variables. Finally, the network design project has a budget of B, and we assume a linear construction cost function $\sum_{(i,j) \in \overline{A}} c_{ij} u_{ij}$ with c_{ij} as the investment cost of link (i, j). Then a general NDP-SO could be formulated as the following integer program:

$$\min \varphi(f,u) \tag{2.1}$$

$$s.t. \sum_{(i,j)\in \bar{A}} c_{ij}u_{ij} \le B \tag{2.2}$$

$$u_{ij} \in \{0,1\},\ \forall (i,j) \in \bar{A} \tag{2.3}$$

$$\sum_{j\in N} x_{ij}^k - \sum_{j\in N} x_{ji}^k = \begin{cases} q^k & if\ i \in O(k) \\ -q^k & if\ i = D(k), \\ 0 & otherwise \end{cases} \quad \forall i \in N, \forall k \in \kappa \tag{2.4}$$

$$x_{ij}^k \le Mu_{ij}, \forall k \in \kappa, \forall (i,j) \in \bar{A} \tag{2.5}$$

$$x_{ij}^k \ge 0, \forall k \in \kappa, \forall (i,j) \in A, \tag{2.6}$$

$$f_{ij} = \sum_{k\in\kappa} x_{ij}^k, \forall (i,j) \in A, \tag{2.7}$$

where M is a large positive number. Equation (2.2) represents the budget constraint. Equation (2.3) simply restricts decision variable to be binary. Equation (2.4) is flow conservation constraint. Equation (2.5) restricts the link flow to be zero if the link is not added to the network. Finally $\varphi(f,u)$ is a function of network design decision and traffic flow.

Using a matrix notation, model (2.1)-(2.7) can be rewritten as

$$\min \varphi(f,u) \tag{2.8}$$

$$s.t. < c,u > \le B \tag{2.9}$$

$$u \in \{0,1\}^{\bar{m}} \tag{2.10}$$

$$Wx^k = q^k, \forall k \in \kappa \tag{2.11}$$

$$x^k \le Mu, \forall k \in \kappa \tag{2.12}$$

$$x^k \geq 0 \tag{2.13}$$

$$f = \sum_{k \in \kappa} x^k, \tag{2.14}$$

where $c \equiv (c_{ij})$, $u \equiv (u_{ij})$, $f \equiv (f_{ij})$, $x^k \equiv x_{ij}^k$ and W is the node-link adjacency matrix.

2.2.2 NDP-UE

For networks where users may not behave consistently with system optimum, the NDP is usually formulated as a bi-level programming problem with the capability of incorporating user routing behavior. A general bi-level framework may be presented as follows (Yang and Bell 1998):

$$\min_u \ F(u, v(u)) \tag{2.15}$$

$$s.t \ \ G(u, v(u)) \leq 0 \tag{2.16}$$

where $v(u)$ is defined by

$$\min_v \ f(u, v) \tag{2.17}$$

$$s.t. \ \ g(u, v) \leq 0 \tag{2.18}$$

In model (2.15)-(2.18), F and u are the objective function and decision vector of upper-level problem respectively, and G is the constraint set of upper-level problem; f and v are the objective function and decision vector of lower-level problem, and g is the constraint set of lower-level problem.

The upper level of model (2.15)-(2.18) is to determine the optimal investment so as to maximize social welfare, and the lower level is to determine traffic flows given route choice behavior assumptions. Given a decision vector u, there is a responsive flow pattern v(u) obtained from the lower level problem. The whole problem is to find an optimal decision *u*, such that the objective function F attains its optimum, while simultaneously taking account of the reactions of network users.

The most widely used assumption of routing behavior in transportation literature, known as Wardrop's first principle (Wardrop 1952) states that every user chooses the least cost path and as a result a stable traffic flow pattern called *user equilibrium* (UE) will be attained, where no one can reduce his/her cost by the unilateral action of changing routing decisions. An UE flow pattern is the solution to the following mathematic program (Beckmann, McGuire et al. 1956):

$$\min \sum_{(i,j)\in A} \int_0^{f_{ij}} t_{ij}(w)dw \tag{2.19}$$

$$s.t. \;\; \sum_{j\in N} x_{ij}^k - \sum_{j\in N} x_{ji}^k = \begin{cases} q^k \; if \; i = O(k) \\ -q^k \; if \; i = D(k), \\ 0 \;\; otherwise \end{cases} \;\; \forall i \in N, \forall k \in \kappa \tag{2.20}$$

$$x_{ij}^k \geq 0, \forall k \in \kappa, \forall (i,j) \in A, \tag{2.21}$$

$$f_{ij} = \sum_{k\in\kappa} x_{ij}^k, \tag{2.22}$$

where t_{ij} represents link travel time, a convex and non-decreasing function of link flow f_{ij}. One of the most well-know function is the Bureau of Public Road (BPR) formula, $t_{ij}^0[1+\alpha(\frac{f_{ij}}{c'_{ij}})^\beta]$, where t_{ij}^0 and f_{ij} are free flow link travel time and flow for link (i, j) respectively, α and β are parameters, and $c_{ij}^{'}$ is the "practical capacity" of link (i, j).

Alternatively, UE flow pattern can be depicted by the following complementary conditions:

$$0 \le x_{ij}^k \perp (t_{ij} + \tau_j^k - \tau_i^k) \ge 0, \forall k \in \kappa, \forall (i,j) \in A, \quad (2.23)$$

where the operator $\perp$ indicates that its operands are perpendicular, i.e., $x_{ij}^k \perp (t_{ij} + \tau_j^k - \tau_i^k)$ is equivalent to $x_{ij}^k (t_{ij} + \tau_j^k - \tau_i^k) = 0$, τ_j^k is the minimum travel time from node *j* to destination *D(k)*, and τ_i^k the minimum travel time from node *i* to destination *D(k)*. The complementary conditions (2.23) indicates that if a positive amount of flow travels on link *ij* toward destination *D(k)* (i.e. $x_{ij}^k > 0$), then link *ij* must be on the shortest path from *i* to *D(k)*.

Now we can write the bi-level network design model with user equilibrium assumption according to the framework (2.15)-(2.18):

$$\min \varphi(f, u) \quad (2.24)$$

$$s.t. \sum_{(i,j)\in \bar{A}} c_{ij} u_{ij} \le B \quad (2.25)$$

$$u_{ij} \in \{0,1\} \quad (2.26)$$

and network flow *f* is defined by the lower level problem

$$\min \sum_{(i,j)\in A} \int_0^{f_{ij}} t_{ij}(w,u)dw \quad (2.27)$$

$$s.t. \sum_{j\in N} x_{ij}^k - \sum_{j\in N} x_{ji}^k = \begin{cases} q^k \ if\ i = O(k) \\ -q^k \ if\ i = D(k) \\ 0 \ \ otherwise \end{cases}, \forall i \in N, \forall k \in \kappa \quad (2.28)$$

$$x_{ij}^k \ge 0, \forall k \in \kappa, \forall (i,j) \in A, \quad (2.29)$$

$$f_{ij} = \sum_{k \in \kappa} x_{ij}^k, \tag{2.30}$$

where t_{ij} is a function of both link flow f_{ij} and design decision variable u_{ij}. If we use BPR type time function, one possible form of t_{ij} is $t_{ij}^0[1+\alpha(\frac{f_{ij}}{u_{ij}c'_{ij}+\varepsilon})^\beta]$ with ε as a small number. When $u_{ij}=1$, it does not affect the link performance formula; when $u_{ij}=0$, the link capacity becomes small and make the cost of travelling through this link prohibitively high.

Replacing the lower level problem using its complementary conditions, model (2.24)-(2.30) is converted into a mathematical program with complementarity constraints (MPCC) or mathematical program with equilibrium constraints (MPEC):

$$\min \varphi(f,u) \tag{2.31}$$

$$s.t. \sum_{(i,j)\in \bar{A}} c_{ij}u_{ij} \le B \tag{2.32}$$

$$u_{ij} \in \{0,1\} \tag{2.33}$$

$$\sum_{j\in N} x_{ij}^k - \sum_{j\in N} x_{ji}^k = \begin{cases} q^k \ if\ i = O(k) \\ -q^k \ if\ i = D(k) \\ 0 \ \ otherwise \end{cases}, \forall i \in N, \forall k \in \kappa \tag{2.34}$$

$$0 \le x_{ij}^k \perp (t_{ij} + \tau_j^k - \tau_i^k) \ge 0, \forall k \in \kappa, \forall (i,j) \in A. \tag{2.35}$$

$$f_{ij} = \sum_{k \in \kappa} x_{ij}^k, \tag{2.36}$$

2.2.3 Solution Methods for NDPs

2.2.3.1 Solving NDP-SO

The mixed integer programming model for discrete NDP-SO can be readily solved by commercial solvers, such as CPLEX. Numerical algorithms are also designed to explore the special structure of NDPs. Of particular interest to us is Benders decomposition (BD) (Benders 1962), which has been successfully applied to a variety of network design applications (see e.g. Magnanti and Wong 1984). BD was originally proposed to solve mixed-integer linear problem through decomposition and cutting plane method, and later extended by Geoffrion (1972) to mixed-integer nonlinear programs, known as generalized Benders decomposition. Our numerical algorithm for solving stochastic transportation protection problem in chapter 3 is based on generalized Benders decomposition, so we provide an introduction here.

BD is appropriate for problems with complicating variables, which, when temporarily held constant, render the remaining problem more tractable. It decomposes the problem into two parts through the projection of original problem onto the space of complicating variables. For example, consider the following problem:

$$\min f_1(u) + f_2(u,x) \;\; s.t. \; G(x,u) \le 0, x \in \mathrm{X}, u \in U \tag{2.37}$$

Assume that $f_1(u)$, $f_2(u,x)$, and $G(x,u)$ are convex functions, and that X is convex set. Let the vector u represent the complicating variables. The projection of problem (2.37) onto the u-space is

$$\min f_1(u) + v(u) \;\; s.t. \; u \in U \bigcap V \tag{2.38}$$

where

$$v(u) := \inf_{x} \{ f_2(u,x) \} \quad s.t. \;\; G(x,u) \le 0, x \in \mathrm{X} \tag{2.39}$$

and

$$V := \{ u \mid G(x,u) \le 0, \; for \;\; some \;\; x \in \mathrm{X} \} \tag{2.40}$$

Note that V is the set of induced constrains, which restricts u to guarantee that $v(u)$ is feasible. Function $v(u)$ is the objective value of the optimization problem parameterized by u, which is called the value function. Both $v(u)$ and V are convex since they are projections of convex function and convex set respectively. By the designation of u as complicating variables, evaluating $v(u)$ is much easier than solving problem (2.37). Problem (2.38)-(2.40) can simply be reformulated as

$$\min_{\theta, u} \; f_1(u) + \theta \quad s.t. \; \theta \ge v(u), u \in U \bigcap V \, . \tag{2.41}$$

The original problem (2.37) is equivalent to problem (2.41) (see theorem 2.1,Geoffrion 1972). The problem (2.41) is then solved by a cutting-plane method which explores the approximate representation of the convex set V and convex function $v(u)$.

2.2.3.2 Solving NDP-UE

The NDP-UE is difficult to solve due to the complexity of the bi-level programming structure. Additionally, each computation of low-level flow variables requires solving a traffic assignment problem. Due to these difficulties, researchers often seek sub-optimal solutions using heuristic methods and approximation techniques. Heuristic search methods include iterative optimization- assignment (IOA) algorithms (e.g. Allsop 1974; Gartner, Gershwin et al. 1980) and global search algorithms (e.g. simulated annealing algorithm of Friesz et al. (1993), tabu search algorithm of Mouskos (1991), and genetic algorithm of Xiong and Schneider (1995)). IOA algorithms do not necessarily converge to optimum (see e.g. Marcotte 1981) and global heuristic search methods, although promising in achieving near global optimal solutions, have to solve a large number of traffic assignment problems and hence are very computationally demanding.

More recent efforts on solving NPD-UE try to convert the bi-level program into single level problem in the form of MPCC or MPEC. For example, Marcotte and Zhu (1996) formulate NDP-UE as a MPEC using a variation inequality (VI) to represent lower level problem and are able to solve the MPEC by the penalty approach. Ban et al. (2006; 2009) formulate NDP-UE as a MPCC and solve it by a relaxation scheme using nonlinear programming (NLP) solver. The authors report that the method is promising in terms of computation efficiency and solution accuracy compared with other popular methods in the literature.

2.3 Stochastic Programming

A general form of stochastic programming model takes the form of

$$\min\{E[f(x,\xi)]: x \in X\}, \tag{2.42}$$

where ξ represent system uncertainty. The reader is encouraged to refer to the monographs (Prékopa 1995; Birge and Louveaux 1997; Ruszczynski and A.Shapiro 2003) for a systematical introduction to stochastic programming. In the following sections, we shall focus on two-stage stochastic programming, which is the most widely applied stochastic programming model.

2.3.1 Two-stage stochastic Programming with Recourse

In two-stage stochastic programming with recourse, we classify decision variables according to whether they are implemented before or after the realization of system uncertainties. Decisions that are made before are known as *first-stage decisions* while those after are *second-stage* or *recourse* decisions. In applications, first-stage decisions are often associated with system planning and second-stage decisions are often related to system operations.

Under the standard two-stage stochastic programming paradigm, the first-stage decision has to be made before the actual realization of system uncertainties, after which a random event occurs and affects the outcome of the first-stage decision. A recourse decision can then be made in the second stage, which is typically interpreted as corrective measures against any infeasibility caused by a particular uncertainties realization. Since the recourse decision is scenario-dependent, the second-stage cost is also a random variable. The objective of a typical two-stage programming model is to make the first stage decision in a way that the sum of the first-stage costs and the expected value of the random second-stage costs is minimized. The concept of recourse has been applied to linear, integer, and non-linear programming.

A standard formulation of a two-stage stochastic linear program with recourse can be presented as follows (Wets 2009):

$$\min_{x} < c, x > + EQ(x) \tag{2.43}$$

$$s.t. \quad Ax = b \tag{2.44}$$

$$x \geq 0 \ , \tag{2.45}$$

with

$$EQ(x) = E_{\xi}\{Q(x, \boldsymbol{\xi})\} \tag{2.46}$$

$$Q(x,\xi) = \inf_{y} \{< q(\xi), y >| \, w(\xi)y = d(\xi) - T(\xi)x, y \geq 0\}, \tag{2.47}$$

where $\boldsymbol{\xi}$ represents system uncertainties, ξ is a particular realization, q, w, d and T are components of vector ξ. Equations (2.43) - (2.45) constitute the first stage problem which needs to be decided prior to the realization of the uncertain parameter $\boldsymbol{\xi}$. Equation (2.47) defines the second stage (recourse stage) problem. $Q(x,\xi)$ is called recourse cost function and $EQ(x)$ expected recourse cost function. If the parameter w, known as recourse matrix, is fixed (deterministic), then the model (2.43)-(2.47) is noted as stochastic linear program with fixed recourse (SLPFR). We shall now discuss some properties of SLPER and expected recourse cost function EQ, laying a foundation for the introduction of solution techniques in the subsequent section. The first question is about the feasibility of SLPER. The *first-stage feasible set* is the feasible set of x defined in the first stage problem. Namely,

$$K_1 = \{x \in R^n \mid Ax = b, x \geq 0\} \tag{2.48}$$

To make the second stage problem feasible, some constraints may have to be added to the first stage decisions. This is the *induced constraints*, defined by the set

$$K_2 = \{x \in R^n \mid EQ(x) < \infty\} = \operatorname{dom} EQ \tag{2.49}$$

The feasible set for the recourse cost functions $Q(.,\xi)$ is given by

$$K_2(\xi) = \{x \in R^n \mid \exists y \text{ so that } wy = d(\xi) - T(\xi)x, y \geq 0\} = \operatorname{dom} Q \tag{2.50}$$

A problem is said to have relative complete recourse if $K_1 \subset K_2$, i.e., the induced constraints don't impose any additional constraints on the problem.

Now let us analyze the properties of feasible sets and recourse cost function $Q(.,\xi)$. It is easy to know that K_1 and $K_2(\xi)$ are convex polyhedral sets. Under the assumption of finite distribution of random parameter ξ,

$$K_2 = \bigcap_{\xi} K_2(\xi). \tag{2.51}$$

So K_2 is also a convex polyhedral set as an finite intersection of polyhedral sets. Recourse cost function $Q(.,\xi)$ is a piecewise linear, convex function as the value of the linear program

$$\min < q(\xi), y > \text{so that } wy = d(\xi) - T(\xi)x, y \geq 0 \tag{2.52}$$

Readers are referred to Theorem 10.9 (Wets 2009) for a proof from duality view point. Finally expected recourse cost function EQ is also a piecewise linear, convex function as the expectation of $Q(.,\xi)$ under the assumption of the finite distribution of ξ. The convexity of EQ and feasible sets K_1 and K_2 implies the existence of global optima. Moreover, it facilitates the application of solution techniques like cutting plane, as we will see in section 2.3.4.1 The L-shaped Method.

2.3.2 Stochastic Programming with Risk-adverse Measures

Classical stochastic programming models are risk neutral in the sense of optimizing expected system measures. Recent applications of stochastic programming especially in finance engineering have motivated the studies on incorporating risk-averse measures into the objective functions of SP (see e.g. Mulvey, Vanderbei et al. 1995; Schultz and Tiedemann 2003; Ahmed 2006; Schultz and Tiedemann 2006; Gotoh and Takano 2007; Shapiro 2008). The resultant models are called risk-averse or mean-risk stochastic programs which seek a balance between optimizing expectation and reducing variability:

$$\min E\{f(x,\xi)\}+\eta D\{f(x,\boldsymbol{\xi})\} \quad s.t.\, x\in X, \tag{2.53}$$

where $f(x,\xi)$ is a measure of system cost depending on uncertainties realization, D is a measure of system dispersion, and η a weighting coefficient between expected cost and dispersion statistic.

Effective disaster mitigation strategies should not only be efficient in terms of minimizing expected system cost but also robust to the full range of possible disaster scenarios. Developing such efficient and robust strategies are one of our research goals of this dissertation and we will propose a mean-risk stochastic programming model in chapter 3.

2.3.3 Stochastic Mathematical Programming with Equilibrium Constraints (SMPEC)

As shown in section 2.2, the bi-level network design problem can be formulated as a mathematical program with equilibrium constraints (MPEC) or mathematical program with complementarity constraints (MPCC) if the lower level user equilibrium problem is described by a set of complementary conditions. If certain parameters are random, then the problem can be formulated as a stochastic mathematical program with equilibrium constraints (SMPEC) or stochastic mathematical program with complementarity constraints (SMPCC). The general form of a SMPCC is as follows:

$$\min E\{f(x,y,\boldsymbol{\xi})\} \tag{2.54}$$

$$s.t.\;\; x\in X, \tag{2.55}$$

$$0\le y\perp F(x,y,\xi)\ge 0, \tag{2.56}$$

where f is the scenario cost function, F a continuous function defined in the second stage problem, x is the first stage decision variable, y is the second stage variable which is a solution to the equilibrium problem defined by complementary conditions (2.56).

2.3.4 Solution techniques for stochastic programming

Under the assumption of finite discrete distributions of the uncertain parameters, stochastic programming models can be converted to their deterministic equivalent form. For example, the deterministic equivalent form to model (2.43)-(2.47) reads as follows

$$\min_x EQ(x, y^{\xi}) \tag{2.57}$$

$$s.t. \quad Ax = b \tag{2.58}$$

$$x \geq 0 \tag{2.59}$$

with

$$Q(x, y^{\xi}) = < q(\xi), y^{\xi} > | \, \omega(\xi) y^{\xi} = d(\xi) - T(\xi)x, y^{\xi} \geq 0 \tag{2.60}$$

It is possible to solve stochastic programs through solving there deterministic equivalent form. However, as the size of the network and the number of scenarios increase, these deterministic equivalent programs (DEP) become prohibitively large. Numerical algorithms based on decomposition methods are often designed for solving large scale problems.

2.3.4.1 The L-shaped Method

With the assumption that random elements have a discrete distribution with finite support, Van Slyke and Wets (1969) proposed L-shaped method for solving the following stochastic linear program with fixed recourse:

$$\min_x < c, x > +EQ(x) \tag{2.61}$$

$$s.t. \quad Ax = b \tag{2.62}$$

$$x \geq 0 \tag{2.63}$$

with

$$EQ(x) = E_{\xi}\{Q(x, \boldsymbol{\xi})\} = \sum_{l=1}^{L} p_l Q(x, \xi^l) \tag{2.64}$$

for $l = 1,..., L$ with L finite, $p_l = prob[\boldsymbol{\xi} = \xi^l]$ and $\xi^l = (q^l, d^l, T^l)$

$$Q(x, \xi^l) = \min_y \{< q^l, y >| wy = d^l - T^l x, y \geq 0\}, \tag{2.65}$$

The L-shaped method takes advantages of the properties of the stochastic programs with recourse and greatly reduces computational efforts. We know that *EQ* and domain of *EQ* (dom *EQ*) are convex polyhedral and could be represented by finite but possibly extremely large number of cuts. The idea is to successively approximate *EQ* by so called optimality cuts and dom *EQ* by feasibility cuts. One basic version of the L-shaped method proceeds as follows.

L-Shaped Algorithm

Step 0. Set $\upsilon = r = s = 0$.

Step1. Set $\upsilon = \upsilon + 1$ and solve the master linear program:

$$\min \; < c, x > +\theta \tag{2.66}$$

$$s.t.\ Ax = b \tag{2.67}$$

$$< E_k, x > \geq e_k,\ k = 1,...,r, \quad (feasibility\ \ cuts) \tag{2.68}$$

$$< F_k, x > + \theta \ \geq f_k,\ k = 1,...,s, \quad (optimality\ \ cuts) \tag{2.69}$$

$$x \geq 0, \quad \theta \in R \tag{2.70}$$

Let ($x^{\upsilon}, \theta^{\upsilon}$) be an optimal solution. If $s = 0$, θ^{υ} is set to $-\infty$ and not included in the objective.

Step 2. With $e = (1,...,1)$, for $l = 1,..., L$, solve the linear program

$$\min \omega' = e^T v^+ + e^T v^- \tag{2.71}$$

$$s.t.\ Wy + Iv^+ - Iv^- = d^l - T^l x^{\upsilon} \tag{2.72}$$

$$y \geq 0, v^+ \geq 0, v^- \geq 0, \tag{2.73}$$

If for all *l*, $\omega' = 0,$ go to step 3. Otherwise, there exist a first l where $\omega' > 0$. In this case, let z^l be associated multipliers to the above linear program and one then generates a feasibility cut:

$$< E_{r+1}, x > \geq e_{r+1}, \tag{2.74}$$

where

$$E_{r+1} = (T^l)^T z^l,\ e_{r+1} =< d^l, z^l > \tag{2.75}$$

Add the feasibility cut to the constraint set (2.68), set $r = r + 1$ and return to step 1.

Step 3. For $l = 1,..., L$, solve the linear program

$$\min \omega = (q^l)^T y \tag{2.76}$$

$$s.t.\ Wy = d^l - T^l x^{\upsilon} \tag{2.77}$$

$$y \geq 0. \tag{2.78}$$

Let π^l be the multipliers associated with the linear program(2.76). Let

$$F_{s+1} = \sum_{l=1}^{L} p_l (T^l)^T \pi^l, \; f_{s+1} = \sum_{l=1}^{L} p_l < d^l, \pi^l > \tag{2.79}$$

If $\theta^{\upsilon} \geq f_{s+1} - < F_{s+1}, x^{\upsilon} >$, the algorithm stops and x^{υ} is an optimal solution. Otherwise, set $s = s+1$, add the optimality cut

$$< F_{s+1}, x > + \theta \geq f_{s+1} \tag{2.80}$$

to the constraint set (2.69) and return to step 1.

2.3.4.2 The Progressive Hedging Method

The L-shaped type methods require that expected recourse cost function EQ be convex. While this requirement is not satisfied, one shall resort to other methods. We explore the Progressive Hedging method (PH) of Rockafellar and Wets (1991) in this section, which is based on augmented Lagrangian and not limited to convex problems. It works with the following scenario based formulation:

$$\min \sum_{s \in S} p_s f_s (x^s, y^s) \tag{2.81}$$

$$s.t. \;\; (x^s, y^s) \in G_s, \forall s \tag{2.82}$$

$$x^s - z = 0, \forall s \tag{2.83}$$

where equation (2.83) is *non-anticipativity* constraint, which requires that first stage decisions cannot depend on any particular realization of system uncertainty.

In light of the popularity and limitations of practical scenario analysis, Rockafellar and Wets (1991) try to first decompose the problem into scenario subproblems, then scientifically aggregate scenario dependent solutions and eventually come up with a “well hedged” solution which works well under all scenarios. The non-anticipativity constraints are the major obstacle to decomposition. An augmented Lagrangian representation partially overcomes this difficulty by integrating non-anticipativity constraints into objective function. Define augmented Lagrangian as

$$L_r(X,Y,z,W) = \sum_{s\in S} p_s\,(f_s(x^s,y^s) + (W^s)' \cdot (x^s - z) + \frac{1}{2} r \left\| x^s - z \right\|^2) \qquad (2.84)$$

Now the difficulty of decomposing L_r into scenarios lies in the appearance of coupling variable z in penalty term. PH then adopts a scheme of alternatingly fixing (X,Y) and z. The formal procedure proceeds as follows.

<u>The Progressive Hedging Method</u>

Step 1.

Set the iteration index ν to 0. Initialize multiplier W to 0. Solve for each scenario sub-problem and obtain $(x^s, y^s)\ \ \forall s \in S$. Initialize $z^{\nu} = \sum_{s\in S} p_s x^s$. If $(x^s)^{\nu} = z^{\nu}, \forall s \in S$, then the optimal solution is found, otherwise continue with step 2.

Step 2.

If the termination criterion

$$\varepsilon = [\| z^{\upsilon} - z^{\upsilon-1} \|^2 + \sum_{s\in S} p_s \| (u^s)^{\upsilon} - z^{\upsilon}]^{1/2} \approx 0 \qquad (2.85)$$

is not satisfied, repeat step 2.

Solve for each scenario

$$((x^s)^{\nu+1}, (y^s)^{\nu+1}) \in \arg\min_{(x^s,y^s)\in G_s} \left\{ f_s(x^s,y^s) + ((w^s)^{\nu})' \cdot x^s + \frac{r^{\nu}}{2} \left\| x^s - z^{\nu} \right\|^2 \right\}, \forall s \in S \qquad (2.86)$$

Obtain a new implementable solution

$$z^{\nu+1} = \sum_{s \in S} p_s (x^s)^{\nu+1} \tag{2.87}$$

Update the dual variable estimates

$$(w^s)^{\nu+1} = (w^s)^{\nu} + r^{\nu}((x^s)^{\nu+1} - z^{\nu+1}), \forall s \in S \tag{2.88}$$

Increase the iteration index ν by 1.

2.4 Summary

A review of related literature reveals that scenario analysis approach is still the most prevalent methodology in the current research and practice of disaster management and critical infrastructure protection. However, this methodology is less relevant to pre-disaster transportation network protection problem, where decisions have to be made before the realization of uncertainty event and a solution contingent on a specific scenario is not appropriate. We also noticed the large amount of literature in physics which applies the tools of statistical analysis, network theory and computer simulation trying to understand network behavior under disruptions, extract physical and statistical properties and identify critical facilities in networks. It increases our understanding to the vulnerability of networks and hence shed light on disaster management, but does not directly lead to an answer to the network protection problem studied in this dissertation. Most of the research in this area does not concern about obtaining probabilistic estimations of disaster uncertainty but rather assume some structured disruption strategies (e.g. successive removal of nodes according to their degrees). If regarding one strategy as a scenario, the analysis is still scenario specific. More importantly, the focus of these studies is on understanding network properties but not on optimal decision making involved in infrastructure protection activities.

In summary, there is still missing a rigorous approach to model uncertainty in the research and practice of infrastructure protection. This dissertation made an important first step toward this direction by integrating stochastic programming techniques with probabilistic assessment of facility damage under disasters to produce effective protection decisions from a system viewpoint.

Network design and stochastic programming are two important foundations of developed methodologies in this work. Network protection is similar in the nature to network design in the sense that both concern about resource allocation over network components for a better system performance. We shall see in chapter 3 that stochastic network protection problem fits into the framework of two-stage stochastic programming very well. In the first stage, network planners make decisions on the choice of links to be retrofitted, then a disaster occurs and system uncertainty is realized. In the second stage, network users or system controllers make routing decisions based on the network configuration resultant from retrofit decision and uncertainty realization.

Solution techniques reviewed in this chapter are also essential to the development of numerical algorithms in chapter 3 and chapter 4. Benders decomposition and the L-shaped method share the common techniques of decomposition, outer linear approximation and successive approximation. Our algorithm developed in chapter 3 also integrates these three techniques. The proposed SMPEC model in chapter 4 is difficult to solve. We demonstrate the applicability of PH method with scenario MPCC subproblems solved by a relaxation approach. . Both methods have been reviewed in this chapter.

3 Transportation Network Protection Problem with SO Flows

This chapter studies the stochastic transportation network protection (SNTP) problem with the assumption that network flows are completely controllable to achieve system optimum (SO). This assumption ensures that the objective of network users is consistent with that of the system planner and makes a one-level formulation possible. The developed models are then best suited for networks with a central system controller such as power grid, oil/gas pipeline, airline and water distribution networks. For transportation road networks, network users may have different objectives other than system optimum. For example, a well known assumption is user equilibrium (UE), discussed in chapter 2, where every user chooses the least cost path and in equilibrium nobody can reduce his/her cost by the unilateral action of changing routing decisions. However, an UE assumption will inevitably lead to a stochastic bi-level program or stochastic mathematical program with equilibrium constraints (SMPEC), which is notoriously hard to solve. A SO assumption brings huge savings in the efforts of formulating and solving the problem. The system cost estimated under the SO assumption can be considered as a lower bound to the cost in reality.

The STNP problem with SO network flows is formulated in the framework of two-stage stochastic programming, and then solved by a numerical algorithm based on an extension to the L-shaped method.

3.1 Problem Statement

Consider the case of a network under the risk of disasters. The probabilistic estimates of infrastructure damage can be obtained from domain experts (e.g. structural engineers). One common approach to disaster mitigation is to retrofit or strengthen network facilities before disasters so that the survivability of the individual facility is increased and hence system resilience is enhanced. Given budget constraints and hazard estimates, which network components should be retrofitted in order to minimize the potential loss from disasters? This is the **Stochastic Transportation Network Protection Problem (STNP).**

3.2 Mathematical Models

The proposed models are general and in principle can be used to address the question of how to protect any type of networks under limited resources. However, for the convenience of discussion, we only focus on the protection of transportation network links (e.g. bridges) under seismic hazards. We shall first analyze the underlying physical and decision process involved in this problem.

3.2.1 Underlying Physical and Decision Process

As depicted by Figure 3-1, the network is first retrofitted before disasters and its resilience is enhanced. However, due to resource constraints and/or technology limits, some facilities are still damaged as a combined effect of disasters and related ground motion &

liquefaction, structure vulnerability, and protection strategy & technology. A new network configuration forms. The next stage of the process is flow routing and loss evaluation. Network flows are routed by a system controller (assumed in this chapter) or driven by individual users (assumed in chapter 4). System cost $Q(u,\xi)$ is a random variable which is a function of protection decisions and system uncertainty. Policy makers are interested in finding good protection strategies that minimize disaster loss..

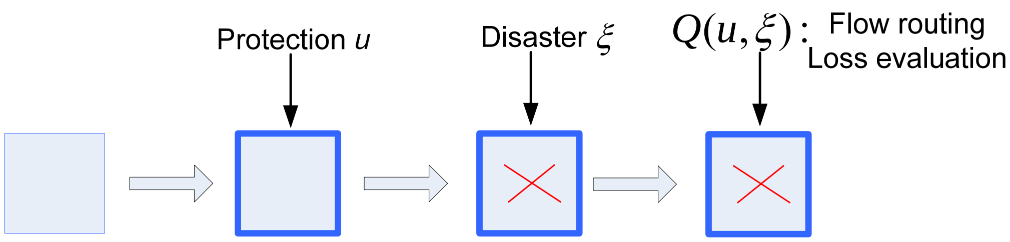

Figure 3-1: Underlying physical and decision process

The above process fits very well into the framework of two-stage stochastic programming. The first-stage protection decisions have to be made before the actual realization of system uncertainty (disaster occurrence), after which disasters occur and a new network configuration is formed. The recourse decisions in the second stage include routing flow and also calculating disaster loss. The second-stage cost $Q(u,\xi)$ is a random variable depending on protection decisions and uncertainty realization. The objective of the STNP problem, in the context of this framework, is to make protection decisions in a way that the sum of the first-stage costs and the expected value of the random second-stage costs is minimized.

3.2.2 Model Assumptions

3.2.2.1 Damage Scenarios

Risk assessment is the first step of disaster management, which includes the identification of threats and estimation of the consequence (loss) caused by them. For the case of seismic hazards, seismologists have predictions for the probabilities of various earthquake occurrences. Advanced structural analysis can lead to the probabilistic assessment of structural damage for a given earthquake, in the form of damage states and associated probabilities. The two sets of probabilistic estimations from earthquake-structural engineers and seismologists can be combined to prepare the damage prediction. Detailed description of assessing damage states of bridges in the study area, considering their spatial and structural correlation, can be found in Lee and Kiremidjian (2006).

Table 3-1: An illustration of damage scenarios

network link	scenario 1 (prob=0.2)	scenario 2 (prob=0.3)	scenario 3 (prob=0.5)
a	4	1	0
b	4	3	0
c	3	2	0

Let the random vector $\boldsymbol{\xi}$ describe the uncertain link damage states under earthquakes. Each realization of $\boldsymbol{\xi}$, denoted by ξ, and the corresponding probability $p(\xi)$ define a damage *scenario*. These damage scenarios are considered as input data. Table 3-1 provides an illustration. For example, for scenario 2, $\xi = (1,3,2)$ and $p(\xi) = 0.2$.

Seismic damage to a structure is usually classified into five categories, ranging from 0 (no damage) to 4 (complete collapse). For simplicity, we only consider binary damage states, with 1 indicating being damaged and 0 otherwise. This assumption is merely for the convenience of discussion. It can be easily relaxed without changing the structure of the proposed models, as long as the data supporting the more detailed analysis is available. We also assume the finite discrete distribution of $\boldsymbol{\xi}$, which is consistent with the data obtained from our collaborators.

Note that the damage states discussed in the last section are estimated without considering any retrofit actions. To distinguish it from the damages states discussed in the next section, we name it "pre-retrofit damage states".

3.2.2.2 Post-Retrofit Damage States

Denote u_a as the retrofit decision variable for link a. We assume that u_a is binary where $u_a = 1$ represents the decision of retrofitting link a and $u_a = 0$ represents the decision of not retrofitting link a. This assumption is reasonable considering that the fixed cost of retrofit is huge and thus it is cost-effective to adopt a strategy of retrofit or not.

We use $\boldsymbol{\Xi}$ to denote the uncertain link damage states under an earthquake given the implementation of any retrofit policy. Each realization of $\boldsymbol{\Xi}$, denoted by Ξ, and the corresponding probability $p(\Xi)$ define a post-retrofit damage *scenario*. The random vector $\boldsymbol{\Xi}$ should be a function of retrofit policy U and $\boldsymbol{\xi}$. We assume that if a link is retrofitted, its probability of being damaged is zero. The relationship between the pre-retrofit link damage state ξ, the retrofit decision *u*, and the post-retrofit damage state $\Xi(\xi, u)$ is described as

$$\Xi(\xi,u)_a = \begin{cases} \xi_a(\xi_a - u_a), & \forall a \in \bar{A} \\ 0, & \forall a \in A \setminus \bar{A} \end{cases} \quad (3.1)$$

where A is the set of all network links and $\bar{A}$ the set of retrofitted links. For a scenario, if the pre-retrofit damage state of link a is 1 ($\xi_a = 1$), but the link is retrofitted (u_a=1), the value of $\Xi(\xi,u)_a$ is 0, indicating that the link will be intact under this scenario. On the other hand, if the link is not retrofitted but its pre-retrofit damage state is 1, the value of $\Xi(\xi,u)_a$ would be 1, indicating that the link will be damaged. If the pre-retrofit damage state of link a is 0 ($\xi_a = 0$), then the link is always in good condition no matter whether it is retrofitted or not.

A more realistic way is to assume reduced but nonzero damage probabilities for retrofitted links. However, first there is no available study to quantify this relationship. Secondly, this more realistic assumption will make the problem Falls in the class of stochastic programming problems with decision-dependent uncertainty, which are significantly more difficult to solve and solution methods are only available for a special class of problems (e.g. Jonsbraten, Wets et al. 1998)

3.2.2.3 System Cost Evaluation

In the framework of stochastic programming, the effectiveness of the first stage retrofit decisions is measured by the expected system cost which arises from this strategy. The best retrofit decision is the one leading to the smallest expected system cost as a summation of the first stage cost and expected second stage cost. The first stage cost could be the retrofit cost, or zero if the retrofit cost is incorporated into budget constraints. The second stage cost may include loss from structure damage, increased system operation cost (e.g. increased travel delay), and other social & economic loss. However it is generally difficult to account for social & economic loss in an optimization model due to the difficulty of finding an analytical function relationship between such a loss and network damage condition. Herein we assume that the second stage cost only includes structure loss and increased travel delay. General social & economic costs could be incorporated into post-optimization analysis for further evaluating candidate retrofit strategies.

Structure loss is computed as the cost of repairing damaged links. We assume that the network is fully recovered. Increased travel delay is captured by system travel time, defined as $\sum_{a \in A} t_a f_a$, where t_a is link travel time and f_a is link flow. This quantity only reflects system travel time in a relatively short period (e.g. in peak hours of one day). We shall aggregate this quantity over the whole recovery period to get accumulated total travel time. Figure 3-2 illustrates the calculation of total travel time, where x-axis is time period and y-axis is system travel time. Point t1 is the time of disaster occurrence and t2 the time when the network is fully recovered. Corresponding to the period before time t1, the network is in intact condition (state0); corresponding to the period [t1,t2], the network is in damaged condition (state (u,ξ), depending on retrofit decision u and disaster realization ξ); after time t2, the network comes back to good condition (state0). The aggregated total travel time should be the area between time t1 and t2, as the one colored in the figure. To make it comparable with repair cost, aggregated total travel time should be converted to monetary value. A possible second stage cost expression is

$$Q(u,\xi) = < \rho, \Xi(\xi,u) > + \lambda < t, f > \tag{3.2}$$

where the operator < > represents inner product , ρ is the vector of repair cost for each link, $\Xi(\xi,u)$ is the damage state for each link, t is link travel time and f is link flow. The coefficient λ is the value of time, which converts total travel time to monetary value.

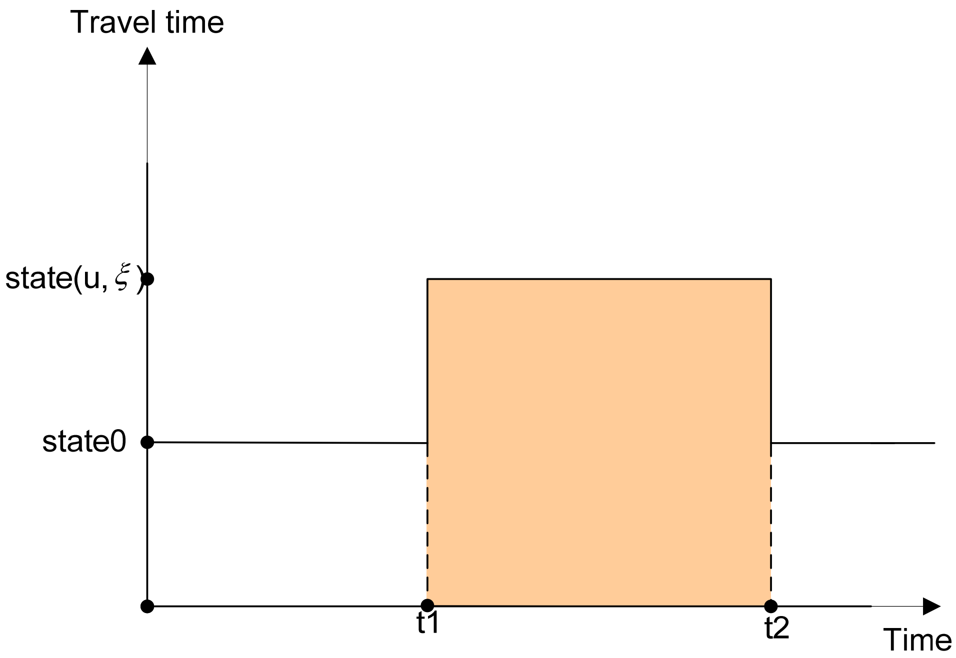

Figure 3-2: Calculation of total travel time

3.2.3 Stochastic Programming Formulation with System Optimal (SO) Flows

Let us introduce more notation before proceeding to the formulation. Consider a transportation network $G(N, A)$, where N is the set of nodes with size *n* and A is the set of all network links with size *m*. Denote $\overline{A}$ ($\overline{A} \subset A$) as the set of links that are subject to earthquake hazards and thus the candidates for retrofit. The size of $\overline{A}$ is $\overline{m}$. Consider the flow destined to different nodes as distinguished commodities. For each commodity $k \in \kappa$, $x^k \in R_+^m$ is the link flow vector, and $q^k \in R^n$ is the vector of travel demands destined to node *k*. Travel demands are assumed to be exogenous, i.e. not depending on changes in network configuration. Denote f_a as the total flow on link a, i.e., $f_a = \sum_{k \in \kappa} x_a^k, \forall a \in A$.

With the above assumptions and notations, the STNP is formulated as follows.

$$\min_{u} \quad 0 + \underset{\xi \in \Xi_d}{E} \{Q(u, \xi)\} \tag{3.3}$$

$$s.t. \quad < c_1, u > \leq B \tag{3.4}$$

$$u \in \{0,1\}^{\bar{m}}, \tag{3.5}$$

with

$$Q(u, \xi) := \min_{x^k} \; < c_2, \Xi(\xi, u) > + \gamma < f, t(f) > \tag{3.6}$$

$$s.t. \quad Wx^k = q^k, \;\; \forall k \in \kappa \tag{3.7}$$

$$x^k \leq (e - \Xi(\xi, u))M, \forall k \in \kappa \tag{3.8}$$

$$f = \sum_{k \in \kappa} x^k, x^k \in R_+^m \tag{3.9}$$

where c_1 is the retrofit cost vector, B is the total budget for retrofitting, and c_2 is the repair cost vector. Vector e has all entries 1, i.e. $e_a = 1, \forall a \in A$. The link travel time t_{ij} depends on the link flow f_{ij}. Their relationship is usually described by a non-decreasing function such as the Bureau of Public Roads (BPR) formula. The notation W represents the node-link adjacency matrix, and M is an arbitrarily large positive number.

Condition (3.4) represents the budget constraint. Condition (3.5) simply restricts u to be binary. Expression (3.6) states the second-stage cost, including the repair cost term

$< c_2, \Xi(\xi, u) >$ and the weighted flow cost $\gamma < f, t(f) >$, where γ is a weight coefficient converting time to monetary value. This cost becomes known once the earthquake hazard has been realized, thus is the recourse cost quantifying the effectiveness of the first-stage decision. Condition (3.7) is the flow conservation constraint for the second stage problem. Condition (3.8) restricts the link flow to zero if the link is damaged by the earthquake. Finally expression (3.3) describes our objective as being to minimize the expected second stage cost.

3.2.4 The Mean-Risk Stochastic Programming Formulation

The above model (3.3) - (3.9) is risk neutral in the sense of minimizing expected system cost. In the context of disaster mitigation as targeted by this dissertation, policy makers are also concerned about the robustness of solutions, i.e., extremely severe consequences in worst –case scenarios should be avoided. This demand for robust solutions can be addressed by including risk-averse measures into the objective function, i.e., to optimize a weighted mean-risk objective:

$$\min E\{f(x,\xi)\} + \eta D\{f(x,\xi)\} \;\; s.t.\, x \in X, \tag{3.10}$$

where $f(x,\xi)$ is a measure of system cost depending on uncertainty realization, D is a measure of system cost dispersion, and η a weighting coefficient between expected cost and cost dispersion. In this section, we are adopting a mean-semideviation objective, defined as $E[f] + \eta(E[(f - Ef)_+^p])^{1/p}$. Ahmed (2006) shows that the mean-semideviation objective is convexity-preserving for all $p \geq 1$ and $\eta \in [0,1]$, which paves the way for an effective computation.

A two-stage mean-semideviation stochastic programming model for problem STNP is formulated as follows.

<u>Stochastic Transportation Network Protection Problem (*STNP*)</u>

$$\min_{u} \quad 0+\underset{\xi\in\Xi_d}{E}\{Q(u,\xi)\}+\eta\underset{\xi\in\Xi_d}{E}\{\,[Q(u,\xi)-\underset{\xi\in\Xi_d}{E}\{Q(u,\xi)\}]_+\,\} \tag{3.11}$$

$$s.t. \quad <c_1,u>\leq B \tag{3.12}$$

$$u\in\{0,1\}^{\bar{m}}, \tag{3.13}$$

with

$$Q(u,\xi):=\min_{x^k} \; <c_2,\Xi(\xi,u)>+\gamma<f,t(f)> \tag{3.14}$$

$$s.t. \quad Wx^k=q^k,\;\; \forall k\in\kappa \tag{3.15}$$

$$x^k\leq(e-\Xi(\xi,u))M,\forall k\in\kappa \tag{3.16}$$

$$f=\sum_{k\in\kappa}x^k,x^k\in R_+^m \tag{3.17}$$

The formulation is similar to model (3.3)- (3.9) except for the difference in objective functions. Note that model (3.3)- (3.9) is a special instance of model (3.11)-(3.17) as $\eta=0$.

For simplicity of the model presentation, we omit the superscripts in x^k and q^k in the remainder of this chapter. Hence constraints (3.15) and (3.16) are denoted as follows:

$$Wx=q \tag{3.18}$$

$$x\leq(e-\Xi(\xi,u))M \tag{3.19}$$

Under the assumption of finite discrete distributions of the uncertain parameters, used in this work, the deterministic equivalent program (DEP) of this formulation is a mixed-integer nonlinear program. As the size of the network and the number of damage scenarios increase, the DEP can become prohibitively large. Difficulties in solving large scale testing problems through direct usage of commercial software (e.g. Cplex 10.0 and GAMS/SBB with nonlinear sub-solvers) motivate us to use alternative solution methods based on decomposition and exploration of the problem structure, which can handle large size problems with reasonable computing time and memory requirements.

3.3 Solution Methods

Van Slyke and Wets (1969) introduced the L-shaped decomposition algorithm for stochastic linear programs, which greatly reduced the computational efforts required to generate a solution. The procedure takes advantage of the fact that the second-stage value function is convex and piecewise linear on a polyhedral domain, thus may be represented by a finite number of so-called feasibility and optimality cuts. It then proceeds to generating these cuts by solving successive linear programming problems. The L-shaped method was proposed for stochastic linear programming, and cannot be directly applied here for solving stochastic nonlinear programs with binary variables in the first stage. However, we are still able to design our numerical algorithm following the general idea of the L-shaped method, including decomposition, outer linearization and successive approximation. The developed algorithm can also be regarded as an extension of generalized Benders decomposition in the context of stochastic programming.

3.3.1 Problem Reformulation and Relaxation

The structure of the two-stage formulation of the STNP suggests a natural decomposition scheme: the network retrofit decisions are complicating variables, and once these are fixed, the sub-problem is a convex min-cost multicommodity network flow problem, for which efficient algorithms are available in the literature (Ouorou, Mahey et al. 2000). To see this, we simply rewrite the formulation (3.11)-(3.17) in a compact way:

Reformulated STNP (*R-STNP*)

$$\min_{\theta,u} \theta \quad s.t.\ \theta \geq EQ(u) + \eta DQ(u)_{+}, u \in U \bigcap V \tag{3.20}$$

with

$$p_l = prob[\boldsymbol{\xi} = \xi^l], l = 1,...,L \tag{3.21}$$

$$EQ(u) = E\{Q(u,\boldsymbol{\xi})\} = \sum_{l=1}^{L} p_l Q(u,\xi^l) \tag{3.22}$$

$$DQ(u)_{+} = DQ(u)_{+} = \sum_{l=1:Q(u,\xi^l)>EQ}^{L} p_l [Q(u,\xi^l) - EQ] \tag{3.23}$$

$$Q(u,\xi^l) := \min_{x} \{< c_2, \Xi(\xi^l,u) > +\gamma < f, t(f) > | x \leq (e - \Xi(\xi^l,u))M, x \in X\} \tag{3.24}$$

$$\mathrm{X} := \{x \mid Wx = q, x \in R_{+}^{m}\} \tag{3.25}$$

$$U := \{u \mid < c_1, u > \leq B, u \in \{0,1\}^{\bar{m}}\} \tag{3.26}$$

$$V := \bigcap_{l=1}^{L} V(\xi^l) = \bigcap_{l=1}^{L} \{u \mid x \leq (e - \Xi(\xi^l,u))M,\ for\ some\ x \in \mathrm{X}\} \tag{3.27}$$

Note that U is a binary set, X is a polyhedral set, and V is a convex set. We refer to expressions (3.24) and (3.25) as <u>sub-problem</u> ($SP(u,\xi^l)$), where $Q(u,\xi^l)$ is the value function of this sub problem. Set V defines the induced constraints to retrofit decisions such that the second stage min-cost network flow sub-problem $SP(u,\xi^l)$ is feasible. One way of representing the induced constraints V and $V(\xi^l)$ is to solve the following optimization problem given $u = \hat{u}$:

Feasibility Sub-problem ($FSP(\hat{u},\xi^l)$ **)**

$$Q_0(\hat{u},\xi^l) := \min_{x,s} \| s \|_1 \tag{3.28}$$

$$s.t.\;\; x \le (e - \Xi(\xi^l,\hat{u}))M + s \tag{3.29}$$

$$x \in X, s \ge 0 \tag{3.30}$$

where s is the slack variable, and $Q_0(\hat{u},\xi^l)$ is the value function of this minimization problem. Problem $FSP(\hat{u},\xi^l)$ is always feasible through constraints relaxation. If $Q_0(\hat{u},\xi^l) > 0$, it means that problem ($SP(\hat{u},\xi^l)$) is infeasible for this particular choice of $\hat{u}$ and ξ^l. Therefore an alternative way of expressing constraint (3.27) is

$$0 \ge Q_0(u,\xi^l),\;\; \forall l = 1,...L \tag{3.31}$$

Then problem (*R-STNP*) is equivalent to the following master problem with associated sub-problems $SP(u,\xi^l)$ and $FSP(u,\xi^l)$:

Master Problem (*M*)

$$\min_{\theta,u} \theta \;\;\; s.t.\; \theta \ge EQ(u) + \eta DQ(u)_+,\; 0 \ge Q_0(u,\xi^l),\;\; \forall l = 1,...L,\; u \in U \tag{3.32}$$

The optimal solution of problem (*M*) includes u^* and θ^*, which gives the first-stage optimal solution and the objective value to the original problem (NRP) respectively.

For each given scenario *l*, functions $Q_0(u,\xi^l)$ and $Q(u,\xi^l)$ are convex as inf-projections on convex sets of convex functions defined by $FSP(u,\xi^l)$ and $SP(u,\xi^l)$ respectively (c.f. Wets 2009). Function $EQ(u) + \eta DQ(u)_+$ is convex because function $Q(u,\xi^l)$ is convex and the mean-semideviation objective is convexity-preserving. The master problem (M) is solved through relaxation and outer linearization to $EQ(u)+\eta DQ(u)_+$ and $Q_0(u,\xi^l)$. At iteration step *k* we solve the following relaxed master problem M^k:

$$\min_{\theta,u} \theta \tag{3.33}$$

$$s.t.\quad 0 \ge Q_0(u^\nu,\xi^l) + < w_0^{\nu,l}, u-u^\nu >,\ \forall \nu \le k : SP(u^\nu,\xi^l)\, infeasible \tag{3.34}$$

$$\theta \ge [EQ(u^\nu)+\eta DQ(u^\nu)_+] + < w^\nu, u-u^\nu >,\ \forall \nu \le k : SP(u^\nu,\xi^l)\ feasible\ \ for\ l=1...L \tag{3.35}$$

$$u \in U \tag{3.36}$$

where $w_0^{\nu,l} \in \partial Q_0(u^\nu,\xi^l)$ and $w^\nu \in \partial\{EQ(u^\nu)+\eta DQ(u^\nu)_+\}$, and symbol ∂ represents the subgradient.

Let (u^ν,θ^ν) be the solution to the master problem M^k. We then check for every ξ^l if the sub-problem $SP(u^\nu,\xi^l)$ is infeasible, namely $Q_0(u^\nu,\xi^l)>0$. If the *l*-th sub-problem is infeasible, we add a constraint $0 \ge Q_0(u^\nu,\xi^l) + < w_0^{\nu,l}, u-u^\nu >$ to the relaxed master problem M^k. This constraint is also called a *feasibility cut*. If the sub-problem $SP(u^\nu,\xi^l)$ is feasible for every ξ^l, we then proceed to check the optimality of the current solution to M^k. If $[EQ(u^\nu)+\eta DQ(u^\nu)_+]-\theta^\nu$ is larger than a certain tolerance $\varepsilon>0$, we add the constraint $\theta \ge [EQ(u^\nu)+\eta DQ(u^\nu)_+] + < w^\nu, u-u^\nu >$ to M^k, which is also called an *optimality cut*. The algorithm proceeds by solving the relaxed master problem M^k and sub-problems (including $FSP(u^\nu,\xi^l)$ and $SP(u^\nu,\xi^l)$) iteratively. The optimal objective value of problem M^k, i.e. θ^ν, defines a non-decreasing sequence of lower bounds of the optimal objective value of the original problem (*STNP*), and the values of $EQ(u^\nu)+\eta DQ(u^\nu)_+$ define a sequence of upper bounds.

Thus the algorithm terminates when the gap between the upper and lower bounds is within the predefined tolerance ε. The detailed solution algorithm is provided in section 3.3.3.

The computation of the solution involves evaluation of the subgradients $\partial\{EQ(u^{v})+\eta DQ(u^{v})_{+}\}$ and $\partial Q^{0}(u^{v},\xi^{l})$. A derivation for obtaining these subgradients is given in the next section.

3.3.2 Calculation of Subgradients

We first derive the subgradient in the general case and then apply it to the present problem. Define the value function of a convex program as

$$v(u)=\min_{x}\{f_{0}(x,u)\} \tag{3.37}$$

$$\begin{aligned} s.t.\quad & f_{i}(x,u)\le 0,\quad i=1,...,s,\\ & f_{i}(x,u)=0,\quad i=s+1,...,m,\\ & (x,u)\in X\times U\subset R^{n_1}\times R^{n_2}, \end{aligned} \tag{3.38}$$

where X is a closed convex set, for $i=1,...,s,$ the functions $f_{i}:R^{n_1}\times R^{n_2}\rightarrow R$ are convex and for $i=s+1,...,m,$ the functions $f_{i}:R^{n_1}\times R^{n_2}\rightarrow R$ are affine. We note that the associated Lagrangian function to this program is

$$L(x,u,\lambda,\pi)=f_{0}(x,u)+\sum_{i=1}^{s}\lambda_{i}f_{i}(x,u)+\sum_{i=s+1}^{m}\pi_{i}f_{i}(x,u) \tag{3.39}$$

To evaluate $v(\hat{u})$, we may just fix u to be $\hat{u}$ and solve the resultant optimization problem. We have the following lemma on the subgradient of $v(u)$ at $\hat{u}$.

Lemma 1 (subgradient of optimal value function):

Assume strong duality holds for $v(\hat{u})$ (for example in the case of satisfying Slater constraint qualification, i.e. $\exists$ feasible $(\hat{x})$, which in addition satisfies $f_i(\hat{x},\hat{u})<0, i=1,...,s.$). Letting x^*, λ^*, and π^* denote the optimal primal and dual solutions for $v(\hat{u})$, then

$$\nabla_u L(x^*,\hat{u},\lambda^*,\pi^*) = \nabla_u f_0(x^*,\hat{u}) + \sum_{i=1}^{s} \lambda_i^* \nabla_u f_i(x^*,\hat{u}) + \sum_{i=s+1}^{m} \pi_i^* \nabla_u f_i(x^*,\hat{u})$$

is a subgradient of value function $v(u)$ at $\hat{u}$, i.e.,

$$\nabla_u L(x^*,\hat{u},\lambda^*,\pi^*) \in \partial v(\hat{u})$$

Proof:

First note that $\nabla_x L(x^*,\hat{u},\lambda^*,\pi^*) = 0$ from the KKT conditions for problem (3.37)-(3.38) . The Lagrangian function $L(x,u,\lambda,\pi)$ is convex with respect to x and u

$$\begin{aligned}
&\Rightarrow L(x,u,\lambda,\pi) \geq L(x^*,\hat{u},\lambda,\pi) + \nabla_u L(x^*,\hat{u},\lambda,\pi)^T (u-\hat{u}) + \nabla_x L(x^*,\hat{u},\lambda,\pi)^T (x-x^*) \\
&\qquad = L(x^*,\hat{u},\lambda,\pi) + \nabla_u L(x^*,\hat{u},\lambda,\pi)^T (u-\hat{u}) \\
&\Rightarrow \inf_x L(x,u,\lambda,\pi) \geq L(x^*,\hat{u},\lambda,\pi) + \nabla_u L(x^*,\hat{u},\lambda,\pi)^T (u-\hat{u})
\end{aligned}$$

From strong duality we have

$$v(u) = \sup_{\lambda\geq 0,\pi} \inf_x L(x,u,\lambda,\pi) \geq \inf_x L(x,u,\lambda,\pi)$$

The combination of the last two expressions implies

$$v(u) \geq \inf_x L(x,u,\lambda,\pi) \geq L(x^*,\hat{u},\lambda,\pi) + \nabla_u L(x^*,\hat{u},\lambda,\pi)^T (u-\hat{u})$$

Evaluating the both sides of the last equation at λ^*,π^* yields

$$v(u) \geq L(x^*,\hat{u},\lambda^*,\pi^*) + \nabla_u L(x^*,\hat{u},\lambda^*,\pi^*)^T (u-\hat{u})$$

Invoking strong duality again we have that $L(x^*,\hat{u},\lambda^*,\pi^*)=v(\hat{u})$. Substituting this in the previous inequality gives $v(u)\geq v(\hat{u})+\nabla_u L(x^*,\hat{u},\lambda^*,\pi^*)^T(u-\hat{u})$, which is the definition of subgradient, thus $\nabla_u L(x^*,\hat{u},\lambda^*,\pi^*)\in\partial v(\hat{u})$. □

Applying the above lemma directly to the evaluation of $\partial Q_0(u^v,\xi^l)$ and $\partial Q(u^v,\xi^l)$ yields

$$\nabla_u[\|s\|_1+\sum_{i\in\bar{A}}\lambda_i^*(x_i^*-\xi_i^l Mu_i^v-M+M(\xi_i^l)^2)]=\begin{bmatrix}-\xi_{\bar{A}_1}^l\lambda_{\bar{A}_1}^* M\\ \vdots\\ -\xi_{\bar{A}_{\bar{m}}}^l\lambda_{\bar{A}_{\bar{m}}}^* M\end{bmatrix}\in\partial Q_0(u^v,\xi^l)$$

$$\nabla_u[<c_2,\Xi>+\gamma<x^*,t(x^*)>+\sum_{i\in\bar{A}}\lambda_i^*(x_i^*-\xi_i^l Mu_i^v-M+M(\xi_i^l)^2)]$$
$$=\begin{bmatrix}-\xi_1^l(c_{2_{\bar{A}_1}}+\lambda_{\bar{A}_1}^* M)\\ \vdots\\ -\xi_{\bar{A}}^l(c_{2_{\bar{A}_{\bar{m}}}}+\lambda_{\bar{A}_{\bar{m}}}^* M)\end{bmatrix}\in\partial Q(u^v,\xi^l).$$

According to the study by Ahmed (2006), the subgradient of the mean-semideviation function is calculated as

$$s\in\partial\{EQ(u)+\eta DQ(u)_+\}.$$

Denoting $$\pi(\xi^l)=\begin{bmatrix}-\xi_1^l(c_{2_{\bar{A}_1}}+\lambda_{\bar{A}_1}^* M)\\ \vdots\\ -\xi_{\bar{A}}^l(c_{2_{\bar{A}_{\bar{m}}}}+\lambda_{\bar{A}_{\bar{m}}}^* M)\end{bmatrix}\in\partial Q(u^v,\xi^l),$$

then

$$s=E[\pi]+\eta\underset{Q(u,\xi)\geq EQ(u)}{E}[(\pi(\xi)-E(\pi))]. \tag{3.40}$$

3.3.3 Solution Procedure

The detailed procedure for obtaining a numerical solution to the network retrofit problem is as follows.

Generalized Benders Decomposition (BD)-based Algorithm:

Step 0:

Initialization. Set $\nu = 0, k = 0$.

Step 1:

If $\nu = 0$, let u^{ν} be any feasible point in the domain U, and θ^{ν} be $-\infty$. Otherwise, solve the relaxed master problem M^k. Denote the current optimal solution as (u^{ν}, θ^{ν}).

Step 2:

For $l = 1...L$, solve feasibility sub-problem ($FSP(u^{\nu}, \xi^l)$).

1) If $Q_0(u^{\nu}, \xi^l) > 0$, it means that sub-problem $SP(u^{\nu}, \xi^l)$ is infeasible. The feasibility cut $0 \geq Q_0(u^{\nu}, \xi^l) + < w_0^{\nu,l}, u - u^{\nu} >$ is generated and added to the problem M^k.

Let $k = k+1, \nu = \nu + 1$. Return to step 1.

2) Otherwise, if $Q_0(u^{\nu}, \xi^l) = 0$ for all $l = 1...L$, go to step 3.

Step 3:

For $l = 1...L$, solve the sub-problem ($SP(u^{\nu}, \xi^l)$).

This problem must be feasible, since we have already passed the feasibility test.

1) If sub-problem is unbounded, then the original problem (NRP) is unbounded. Thus stop the process.

2) If sub-problem is bounded, we have the following cases:

a) If $[EQ(u^{v})+\eta DQ(u^{v})_{+}]-\theta^{v}\leq\varepsilon$, then stop. The solution (θ^{v},u^{v}) is the optimal solution of problem (NRP);

b) Otherwise, the optimality cut $\theta\geq[EQ(u^{v})+\eta DQ(u^{v})_{+}]+<w^{v},u-u^{v}>$ is generated and added to problem M^{k} . Let $k=k+1,v=v+1$. Return to step 1.

The *finite convergence* of our algorithm is a direct consequence of the finiteness of the discrete feasible set U and the fact that no u^{v} can ever repeat itself in a solution to the master problem (M) (see Theorem 2.4,Geoffrion 1972).

3.4 Numerical Examples

Numerical experiments were implemented using two case studies. The first case study has a relatively small size, and is used to validate the proposed solution procedure and to test the numerical efficiency of the algorithm. The second case study uses realistic seismic risk and cost data, and is included to demonstrate potential real world applications of the proposed methodologies.

3.4.1 Case Study I: Sioux Falls City Network

The first case study uses the well known Sioux Falls City road network (24 nodes and 76 links) (Leblanc 1975) as shown in Figure 3-3. It is assumed that six bi-directional highway bridges (labeled A to F) are under potential threat from future earthquakes and thus need to be retrofitted. The possible damage scenarios of these six bridges are considered as input data to the model. Here we use the independent probabilities given in Table 3-2 to generate a total of $2^6 = 64$ damage scenarios for the random vector ξ in problem ($STNP$). Note that the assumption of independent probabilities is only for the convenience of generating test data. Probabilities of damage scenarios generated with consideration of correlations between individual bridge damage states can be used in the same manner as an input to the model.

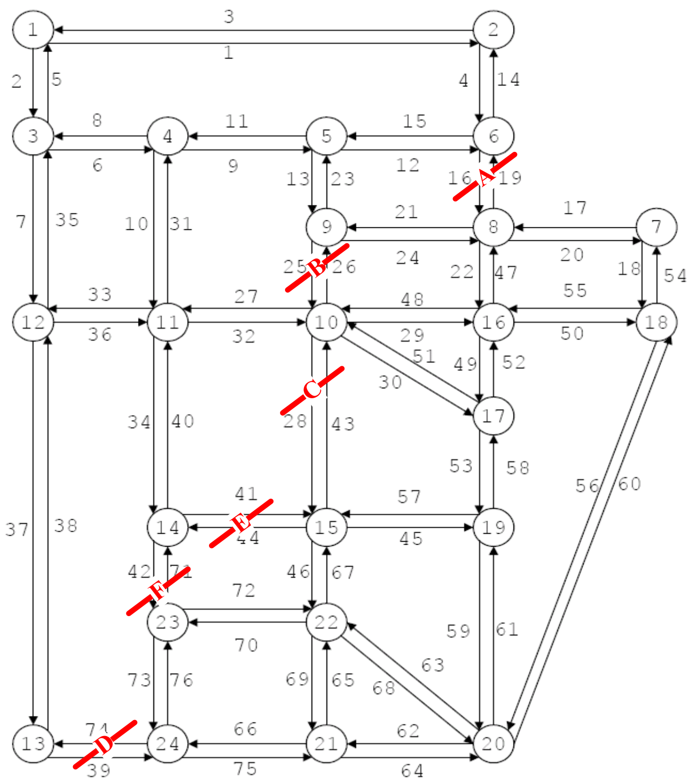

Figure 3-3 Aggregated Sioux Falls Road Network

Table 3-2 Independent probability of bridge damage for generating the set of damage scenarios

Bridge	A	B	C	D	E	F
Probability of damage	0.1	0.1	0.4	0.5	0.8	0.7

The *BPR* function is in the form of $t_a^0[1+\alpha(\frac{f_a}{c'_a})^\beta]$, where t_a^0 and f_a are free flow travel time and flow for link a respectively, c'_a is the "practical capacity" of link a and is set to be 90% of the design capacity. The values of other model parameters are: $c_{1ij}=1$, $c_{2ij}=1.5$, $\gamma=1$,

$\alpha = 0.15$, and $\beta = 4$. At this point we temporarily set the weighting coefficient η in the objective to zero and focus on the numerical performance of the decomposition algorithm. Later, η will be increased to show how consideration of risk may affect the retrofit strategies.

We consider two approaches to solve problem (*STNP*): using commercial solvers to solve the deterministic equivalent problem (DEP) directly or running the Benders Decomposition (BD) based algorithm presented in section 3.3.3. We investigate the efficiency of using these two approaches for different forms of the BPR function ($\beta = 4$ and $\beta = 1$). The DEP of the problem (*STNP*) considered is a mixed-integer nonlinear program with more than 110,000 variables (76 links x 24 destinations x 64 scenarios = 116,736). The commercial package GAMS SBB[1] (Simple Branch and Bound) solver is used to solve the DEP directly.

3.4.1.1 Results on the efficiency of the solution method:

When the budget for retrofit is set to be sufficient for only two bridges, there are fifteen (C_6^2) possible retrofit solutions, in which case all possible retrofit solutions can be easily enumerated. We use the results from this enumeration as a benchmark for validating the accuracy of the proposed solution algorithm. Directly solving the DEP using commercial optimization solvers and solving the problem using the BD-based algorithm both return the correct solution (to retrofit bridges D and E). However, the computational efficiency resulting from the decomposition method is much better than solving (DEP) directly. For the BPR link performance function with $\beta = 1$, solving (*DEP*) directly using GAMS SBB solver took 3,012 sec; the BD-

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[1] SBB is a GAMS solver for mixed integer nonlinear programming problems. It needs a nonlinear programming solver such as CONOPT to run.

based algorithm, with CONOPT[2] solving subproblems, solved the problem in 290 sec. For the BPR link performance function with $\beta = 4$, solving (*DEP*) directly using SBB solver took 22,817 sec; the BD-based algorithm, with CONOPT solving subproblems, solved the problem in 859 sec[3].

Additional numerical experiments are conducted to test the performance of the BD-based algorithm in problems of different size. The performance of the algorithm is measured by the number of optimality cuts since it determines the number of NLP subproblems to be solved. The problem difficulty is reflected by the size of the solution space of the first-stage integer variables, since these integer variables are the major complicating factors in the NRP problem. We now allow retrofit decisions to be associated with each directional link, thus increasing the number of first-stage decision variables from six to twelve. To speed up the experiment, only the ten most likely scenarios are included in this test. It is observed that the increase rate of the number of optimality cuts is smaller than the increase rate of the number of possible first-stage solutions. For example, as the number of possible retrofit solution increased about 40 times from C_{12}^{1} to C_{12}^{4}, the number of optimality cuts only increased about seven times from 9 to 61. This observation suggests that Benders decomposition based algorithms may be a favorable choice for problems where the first-stage integer variables are the major complicating factors. Numerical results from this case study also demonstrate that solving NRP problems through decomposition

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[2] CONOPT is a GAMS solver for nonlinear programs.
[3] All the numerical results reported in this chapter were computed using a Windows XP Dell Workstation with dual Intel(R) Xeon(R) CPU (2.40GHz) and 3.5 GB RAM.

is much more efficient than direct use of commercial solvers as long as the problem size is nontrivial.

3.4.1.2 Sensitivity of the solution with respect to η:

When η increases within the interval [0,1], the retrofit solution remains the same (to retrofit bridges D and E). When the mean term is removed from the objective, or equivalently when the risk term is weighed very highly, the retrofit solution changes to bridges C and D. This risk-averse solution trades off 4% increase in expected cost (from 46.4 to 48.3) with 18% reduction in the semi-deviation (from 1.5 to 1.2) by comparison with the risk-neutral solution ($\eta = 0$). In this particular case study, the effect of risk consideration on retrofit strategy is observable, but not significant. However, the example demonstrates that (1) an optimal solution based on the mean criterion may not be the most reliable; (2) different risk preferences may affect retrofitting strategies.

3.4.2 Case Study II: Alameda County Network

The second case study uses a sub-network of the Alameda County, California road network (including highways and major local streets) as shown in Figure 3-4, which includes 510 nodes, 1424 links, and 2401 origin-destination pairs.

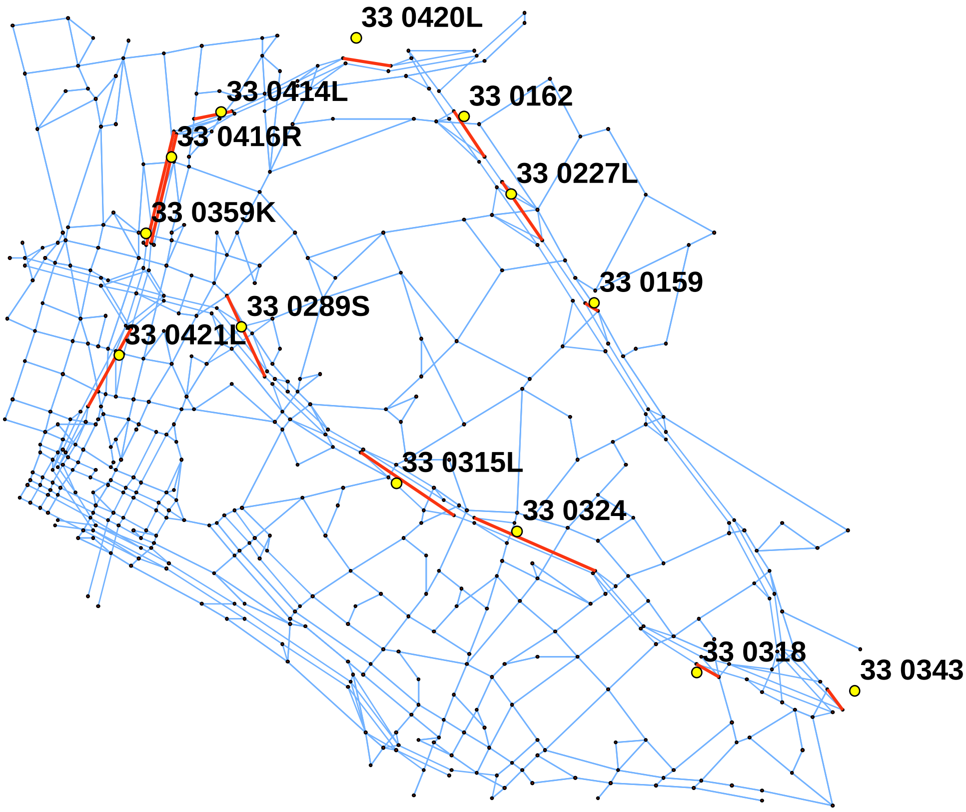

Figure 3-4 Alameda County road network

Thirteen highway bridges in the study area are found vulnerable while being evaluated under 31 potential earthquake events that are likely to affect Alameda County (see Lee and Kiremidjian 2006). Most of these earthquake events are not severe enough to cause functional damage to the bridges. After aggregating all no-damage scenarios, we have a total of six damage scenarios to consider. The probabilities of these damage scenarios are computed based on the Poisson arrival rates of the 31 earthquake events and a 10-year planning horizon.

Table 3-3 Model input data: damage scenarios and cost data

National Bridge Index (NBI)	Replacement Cost	Retrofit Cost	Scenarios						Engineering Ranking
			1	2	3	4	5	6	
33C0343	$833,833	$208,458	0	0	0	1	1	0	8
33C0318	$1,144,154	$286,038	0	1	0	1	1	0	4
33C0159	$1,024,100	$256,025	0	1	0	1	1	0	7
33C0324	$1,806,588	$451,647	0	1	1	1	1	0	2
33C0227L	$706,420	$176,605	0	1	0	1	1	0	5
33C0162	$1,980,990	$495,247	0	1	1	1	1	0	7
33C0315L	$3,878,490	$969,622	0	1	0	1	1	0	1
33C0420L	$1,737,450	$434,362	0	0	0	0	1	0	9
33C0289S	$489,940	$122,485	0	1	0	1	1	0	2
33C0414L	$1,361,008	$340,252	1	1	1	1	1	0	6
33C0416R	$9,007,614	$2,251,903	1	1	1	1	1	0	2
33C0359K	$1,746,030	$436,507	0	1	0	1	1	0	6
33C0421L	$5,871,690	$1,467,922	1	1	1	1	1	0	3
Probability of Each Damage Scenario (%)			7.6	11.3	6.2	7.7	1.6	66	

Table 3-3 provides information on the damage scenarios and the retrofit and replacement costs of each candidate bridge. The structure damage estimation was provided by Prof. Anne Kiremidjian's research group at Stanford University. The replacement costs were provided to us by the California Department of Transportation. The retrofit cost of a bridge is estimated to be one-fourth of the corresponding replacement cost.

Parameter γ converts two-hour peak time delay to yearly (assuming reconstruction of bridges takes one year) dollar value. It is set as (1/60)*8*365*20=973.3, where (1/60) is to convert minutes to hours, 365 is to convert daily to yearly value, 20 is the average value of time for travelers in the study area, and 8 is the two-peak-hour conversion factor to daily impact estimated for the San Francisco Bay Area[4]. Link performance function is in the BPR form with parameter $\beta = 4$ and practical capacity $c' = c$. Coefficient η takes a value within [0,1]. However, numerical experiments show that the solutions are the same when varying η. Its effects will be discussed later.

In this case study, there are 13 integer variables and 418,656 (1424 links x 49 origins x 6 scenarios = 418,656) continuous variables. Retrofit budget considered ranges from 0.5 to 8 million dollars, resulting in 15 to 2048 possible retrofit solutions. The optimal retrofit strategies under different budget constraints are reported in Table 3-4.

We observe that an optimal solution resulting from low budget may not necessarily be a subset of an optimal solution from high budget, which indicates that retrofit decisions based on simple engineering ranking approaches may be questionable. Let us consider 4 M$ budget as an example. A commonly used engineering approach is to rank the candidate bridges for retrofit based on the traffic volume they carry and their seismic risk estimates. Assuming equal importance of the two factors, the ranks of the thirteen bridges are computed and reported in Table 3-3. Bridge 33C0315L is ranked highly by the engineering method because of its large

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[4] This conversion factor is estimated based on peak duration and daily vehicle hours in year 2006 provided by Metropolitan Transportation Commission (unpublished).

traffic volume and the relatively high seismic risk it is subject to. However, this bridge is not chosen for retrofit by the stochastic programming model, demonstrating that a high-volume link may not be as critical as it seems depending on the redundancy of the network and consequently the flexibility it has to redistribute flow.

Table 3-4 Optimal retrofit strategies and expected system costs EQ

Budget (M$)	Retrofit Strategies	EQ (M$)	Budget (M$)	Retrofit Strategies	EQ (M$)
0.5	33C0414L	12.44	6	33C0414L	5.59
	33C0289S			33C0359K	
1	33C0414L	11.91		33C0416R	
	33C0289S			33C0421L	
	33C0162			33C0289S	
2	33C0414L	10.38		33C0227L	
	33C0421L			33C0162	
	33C0227L			33C0159	
3	33C0414L	9.13		33C0324	
	33C0416R		7	33C0414L	4.76
	33C0289S			33C0416R	
	33C0159			33C0421L	
4	33C0416R	7.69		33C0289S	
	33C0421L			33C0315L	
	33C0159			33C0227L	
5	33C0414L	6.54		33C0162	
	33C0359K			33C0318	
	33C0416R			33C0324	
	33C0421L		8	ALL	4.45
	33C0162				

We also observe the positive impact that a retrofit program may bring to society. Apparently, there is a tradeoff between the planning investment and the recourse cost. As plotted in Figure 3-5, the expected second stage recourse cost EQ (including repairing cost and travel delay cost) decreases as more retrofit funding is invested. As more retrofit funds become available, the total system cost (first-stage retrofit cost plus the expected second-stage cost) also decreases until a certain point (5M$ in this case) is reached. For example, as the retrofit budget increases from 0.5M$ to 4M$, the total system cost decreases from 12.9M$ to 11.7M$. The gained benefit is about 10%.

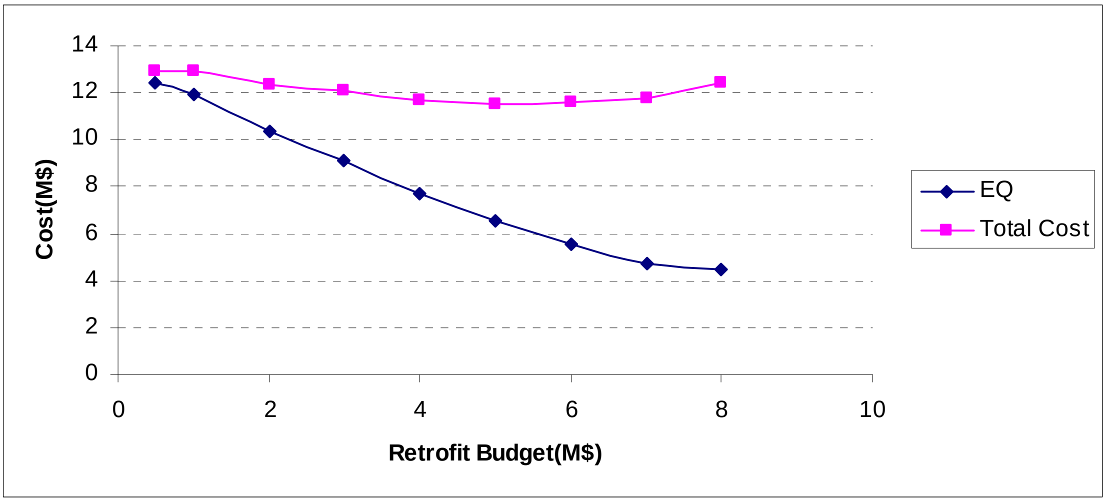

Figure 3-5 Retrofit budgets vs. system costs

3.4.2.1 Stochastic Programming Approach vs. Wait-and-see Approach

The wait-and-see approach (Birge and Louveaux 1997) is a commonly used deterministic approach which seeks an optimal solution for each scenario, as if we could wait and see the realization of random events and then make decisions accordingly. This is also the scenario analysis approach discussed in chapter 1 and 2. For this specific example, the wait-and-see approach generates 6 scenario dependent solutions or wait-and-see policies. We are interested in comparing their performance with the stochastic programming solution.

Table 3-5 Performance of wait-and-see and stochastic programming solutions

	Wait-and-See Policy	Scenario Cost with Perfect Information (million $)	Expected Cost over All Scenarios (million $)	Scenario Cost of SP Solution (million $)	Relative Regret of SP Solution
Scenario 1	33C0416R 33C0421L	5.81	7.90	5.81	0
Scenario 2	33C0359K 33C0421L 33C0289S 33C0315L 33C0159 33C0318 33C0324	17.51	8.79	17.65	0.8%
Scenario 3	33C0416R 33C0421L	9.61	7.90	9.61	0
Scenario 4	33C0414L 33C0359K 33C0421L 33C0289S 33C0315L 33C0343 33C0324	18.32	8.70	18.48	0.9%
Scenario 5	33C0414L 33C0359K 33C0421L 33C0289S 33C0315L 33C0343 33C0324	20.06	18.25	20.22	0.8%
Scenario 6	None	4.45	13.02	4.45	0
Stochastic Program (SP) Solution	33C0416R 33C0421L 33C0159		7.69		

Results are reported in Table 3-5, given 4M dollars of retrofit budget. Each of these solutions is evaluated under the six damage scenarios. Associated with each solution are six scenario-dependent system costs and an expected system cost. The column "Expected cost over all scenarios" in Table 3-5 reports expected system cost, and the column "scenario cost with perfect information" $Q(u(\xi)$ reports the system cost of each scenario when the corresponding wait-and-see policy is followed. This is the least possible cost for each scenario. As expected, the stochastic programming solution provides the least expected cost compared with wait-and-see policies. The difference ranges from 210K to 10.56M dollars.

The wait-and-see solution (Birge and Louveaux 1997), defined as $WS = E_{\xi}[Q(u(\xi),\xi)]$, is 5.03M$. Expected recourse cost from the stochastic programming solution is 7.69M$. Therefore, the expected value of perfect information (EVPI) is 7.69 - 5.03 = 2.66M$. The EVPI of the stochastic programming solution suggests that effort in improving estimates of uncertain parameters is worthwhile, even though stochastic programming model may be less sensitive to imperfect information than its deterministic counterparts.

3.4.2.2 Value of Stochastic Programming Solutions

The stochastic programming approach explicitly considers the entire range of uncertain scenarios, thus hedging better against uncertainty than its deterministic counterparts. However, it also increases computational complexity dramatically. The concept of Value of Stochastic programming Solution (VSS) (Birge and Louveaux 1997) can be used to justify whether the extra effort for modeling and solving stochastic programming is worthwhile.

Let us denote u* as the optimal solution suggested by a commonly used engineering approach. If the solution u* is implemented, the expected system cost across all possible damage scenarios is $EEV := E_{\xi}(Q(u^*,\xi))$. Similarly, let us denote the expected loss calculated in the stochastic programming model as $SP := \min_{u} E_{\xi} Q(u,\xi)$. Then the VSS is defined as $VSS = EEV - SP$. In general, a bigger VSS indicates a higher benefit of using the stochastic programming approach. We calculate VSS based on the comparison of the stochastic programming solution and the one from the most likely scenario (scenario 2 in the case study) given a 4M$ budget. The *EEV* is 8.79M and *SP* 7.69M, and *VSS* turns out to be 1.1 M$. The relative gain of using stochastic programming is 1.1/8.79=12.6%. The relatively large value of the stochastic programming solution justifies the use of more sophisticated modeling techniques and the extra computational efforts.

3.4.2.3 Evaluating the Reliability and Robustness of the Stochastic Programming Solution

As we mentioned in the previous section, the solutions from the risk-neutral ($\eta = 0$) and risk-averse ($\eta \in (0,1]$) approaches for case study II are the same. This may suggest that the solution from stochastic programming with risk-neutral objective is reliable. We conduct more analysis in this section to evaluate the stochastic programming solution in term of reliability and robustness.

The cumulative probabilities of not exceeding a certain cost threshold are plotted in Figure 3-6 for stochastic programming, most likely scenario, engineering ranking, and wait-and-see solutions. The Stochastic programming solution is more reliable than its deterministic counterparts in this case study. For example, at 80% reliability level, the SP solution produces a cost threshold of 10M$, but the most-likely-scenario solution produces a threshold of 17M$. If the goal is not to exceed 18M$ total cost, the SP solution has a 98% chance of achieving such a goal; while the wait-and-see policy (from scenarios 1 and 3) only has less than a 90% chance of achieving such a goal.

Planning decisions are usually made before the actual realization of random variables occurs. However, the public usually judges a decision in the aftermath of an incident when all the uncertain information is already revealed. If one knows perfectly which scenario will actually happen, one could make the optimal retrofit plan to achieve the minimum cost accordingly. The *regret* of a solution is the difference between the scenario cost from this solution and the least possible cost (scenario cost with perfect information), which is often used to measure the robustness of a solution. Mathematically, regret is defined as $Q(u,\xi)-Q(u(\xi),\xi)$, where $u(\xi)$ is the wait-and-see policy for scenario ξ, and u is the policy being evaluated. As shown in Table 3-5, the relative regrets ($\frac{Q(u,\xi)-Q(u(\xi),\xi)}{Q(u(\xi),\xi)}$) of the stochastic programming solution in all possible scenarios are small.

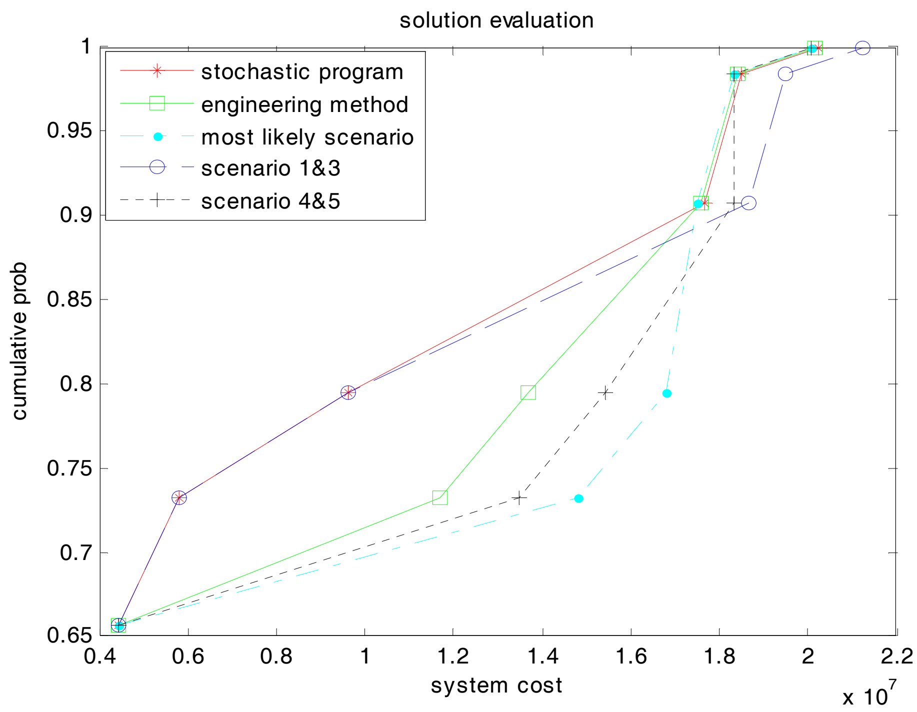

Figure 3-6 Reliability Evaluations of Different Retrofit Solutions

Results in this case study show that the stochastic programming solution performs well in terms of expectation, reliability, and robustness. However, we need to emphasize that this observation is case specific. Given different distributions of uncertain parameters or different problem settings, tradeoffs among expectation and risk may appear.

In order to evaluate the sensitivity of the stochastic programming solution to imperfect risk assessment, we test the solution in seven datasets. In the first two datasets, we include the original six scenarios plus four randomly generated scenarios. In the other five datasets, we include scenarios that are slightly perturbed from the original ones. The performance of the stochastic programming, most likely scenario, and engineering ranking solutions in all datasets are reported in Table 3-6.

Table 3-6 Expected system cost (million $) evaluated using different data sets

	Different Disaster Data Sets							
	Original	1	2	3	4	5	6	7
SP Solution	7.7	9.1	8.8	7.7	7.8	7.7	7.7	7.6
Most Likely	8.8	9.0	8.8	8.7	9.2	9.0	8.8	9.0
Engineering Ranking	8.4	9.0	9.1	8.3	8.7	8.0	8.4	8.5

The numerical results suggest the importance of having quality estimation of uncertain parameters. The Stochastic programming solution may still perform well if the estimation of uncertain parameters is slightly off. However, if the information about uncertain parameters is too unreliable, as in data sets 1 and 2 in the table, the choice of modeling methods will not matter - no matter what modeling approach is implemented, the results may be equally bad.

3.5 Summary

Assuming system optimal network flows, we have shown in this chapter that the STNP problem fits well into the framework of two-stage stochastic programming (SP). A risk-neutral SP model is first formulated, and then extended to including both expected system cost and risk-averse measures in the objective. The resultant mean-risk SP model better suits the needs of disaster mitigation and infrastructure protection planning, which seeks a balance between minimizing expected cost and reducing cost variation. We specifically adopt the mean-semideviation objective function which is proved in the literature to be convexity preserving. The convexity of objective function and special structure of the model facilitate the development of a numerical algorithm based on the techniques of decomposition, outer linear approximation, and successive approximation, similar to the L-shaped method and generalized Benders decomposition.

Numerical experiments demonstrate the efficiency of the BD-based algorithm compared with solving DEP directly. The case study on the Alameda County road network using realistic seismic risk and cost data illustrates the potential real world application of developed methodologies. Various analyses are also conducted to compare SP with other approaches (e.g. the wait-and-see approach or scenario analysis approach). SP solutions perform well in terms of being efficient, reliable and robust.

Some methodological questions remain. First, the SO flow assumption adopted in this chapter is justified by its applicability to various centralized networks, benefits to formulation and solution procedure, and provision of a valid lower bound to the system cost in reality. Nevertheless, it is a simplified assumption for transportation networks. In chapter 4, we will study the STNP problem with the UE flow assumption, which is a widely accepted assumption in transportation research.

The models developed here are also based on the assumption that once a link is retrofitted, its probability of being damaged is zero. A more realistic assumption is that the retrofitted links may still be damaged but with reduced probability. However, adopting this more general assumption will make system uncertainty depend on decisions. Stochastic programs with decision dependent uncertainty are very difficult to solve and currently available solution methods are only limited to convex problems having special structures. Moreover, the quantitative relationships among link damage probability, structure seismic performance and retrofit decisions are not well studied. Therefore our simplified assumption is a result of lacking knowledge and advanced modeling techniques.

The BD-based algorithm is demonstrated to be more efficient than solving DEP directly. However, we observed that the algorithm converges slowly, which is consistent with the literature (e.g. Magnanti and Wong 1981). In the future, we shall enhance this algorithm by adopting some acceleration techniques such as pareto-optimal cuts (see e.g. Magnanti and Wong 1981; Magnanti and Wong 1984; Wentges 1996; Rei, Cordeau et al. 2009).

It turns out that stochastic programming solutions are not very sensitive to the change of weighting coefficient η in the mean-semideviation objective for these two particular numerical examples. However this finding cannot be generalized. More numerical experiments are needed to investigate this issue. We may also explore other risk-averse models, such as optimizing Conditional Value at Risk (CVAR, see e.g. Rockafellar and Uryasev 2000; Rockafellar and Uryasev 2002) , chance constrained model (e.g. Prékopa 1995), and robust optimization (e.g. Kouvelis and Yu 1997). It would be interesting to compare results from different modeling approaches. Understanding how decision makers' risk preferences might affect their choices and eventually impact the effectiveness of the entire society will have significant policy implications.

4 Transportation Network Protection with UE Flows

This chapter tackles the Stochastic Transportation Network Protection (STNP) Problem with User Equilibrium (UE) flows. The problem setting and other model assumptions are the same as in chapter 3. The emphasis is on studying the effects of incorporating UE flows on system modeling and solution methods. We shall first restate the UE assumption and then analyze the complexity caused by such an assumption.

UE is the most common routing behavior assumption in transportation research and practice. It states that every user chooses the least cost path and as a result a stable traffic flow pattern called user equilibrium will be attained, where no one can reduce his/her cost by the unilateral action of changing routing decisions (Wardrop 1952). An UE flow pattern is the solution to a mathematic program or nonlinear complementarity problem (NCP) or Variational Inequality (VI).

A UE assumption in network optimization problems leads to a bi-level structure. In the upper level, network planning agencies (known collectively as the leader) make planning decisions (e.g. network expansion) which change network configurations. In the lower level, network users (known collectively as the follower) make routing decisions based on the new network configuration. The objective of the leader is to maximize social welfare while simultaneously taking account of the reactions of the follower. This class of problems is usually formulated as bi-level programs or mathematical programs with equilibrium constraints (MPEC) including mathematical programs with complementarity constraints (MPCC) as a special case

The STNP problem adds one more layer of complexity to the above bi-level structure. Network configuration is jointly determined by the leader and the realization of disasters. At the time of decision making, the leader only has limited information about disasters and at best gets probabilistic estimations of the uncertain post-disaster network configurations, for example, in the form of damage scenarios. Now the leader's problem is to maximize social welfare while considering multiple possible damage scenarios and reactions of the follower in each scenario. Such a structure could be captured by a stochastic bi-level program or a stochastic mathematical program with equilibrium constraints (SMPEC).

The nonconvexity of the stochastic bi-level program and SMPEC prohibits the application of the algorithm proposed in Chapter 3, which, like Benders decomposition and the L-shaped method, strongly relies on convexity of the problem. The Progressive Hedging (PH) method of Rockafellar and Wets (1991) might be a good choice in this context, which is not limited to convex problems. The PH method may be regarded as a scientific version of the practical scenario analysis approach. The basic idea is to iterate the process of solving perturbed scenario subproblems and aggregating scenario dependent solutions to an implementable policy. The presentation for the remainder of this chapter will also follow this flow of ideas. We first introduce the formulation of scenario subproblems as MPCCs, and consolidate them together with non-anticipativity constraints to build a stochastic programming model. The PH method is then used to detail how to iteratively aggregate scenario dependent solutions to an optimal solution. Scenario MPCC subproblem is solved by a relaxation approach. Finally results and observations gained from numerical experiments are reported.

4.1 Mathematical Models

This chapter uses same notations as chapter 3. Here we restate them for the readers' convenience. Consider a transportation network $G(N, A)$, where N is the set of nodes of size n and A is the set of network links of size m. Denote $\overline{A}$ ($\overline{A} \subset A$) as the set of candidate links that are subject to modification decisions. The size of $\overline{A}$ is $\overline{m}$. A link can be labeled by its link index as link a, or by its starting and ending node as link ij. The decision variable u_a represents the protection action on link a ($a \in \overline{A}$), which could be continuous or discrete. We focus on the discrete cases in this chapter. Consider the flow on the same link but destined to different nodes as distinguished commodities. For each commodity $k \in \kappa$, $x^k \in R_+^m$ is the link flow vector, and $q^k \in R^n$ is the vector of travel demands destined to node k. Denote f_a as the total flow on link a, i.e., $f_a = \sum_{k \in \kappa} x_a^k, \forall a \in A$. Let ξ_a represent the random hazard event on link a, and ξ be the vector of elements ξ_a ($\forall a \in A$). We may consider two possible outcomes of ξ_a (i.e., ξ_a=1 states that link a, if not protected, will be damaged in a disaster; and 0 otherwise). Apparently, the post-disaster condition of link a depends on both the protection decision u_a and the actual realization of ξ_a. We introduce the function $h_a(u_a, \xi_a)$ to represent the post-disaster capacity of link a. Assuming that a link, once retrofitted, will remain intact under any disaster scenario, $h_a(u_a, \xi_a)$ is represented as

$$h_a(u,\xi) = \begin{cases} (1-\xi_a(\xi_a - u_a))\, c_a, \forall a \in \bar{A} \\ 0, \quad \forall a \in A \setminus \bar{A} \end{cases}, \tag{4.1}$$

where c_a is the pre-disaster capacity of link a.

4.1.1 Formulation of Scenario Subproblem as a MPCC

If the future disaster scenario is known, that is if the exact value of ξ is known, the STNP problem with user equilibrium flow can be formulated as a MPCC:

$$\min_u Q(u, f) \tag{4.2}$$

$$s.t.\ \ u \in U \tag{4.3}$$

$$f = \sum_{k \in \kappa} x^k, x^k \in R_+^m \tag{4.4}$$

$$Wx^k = q^k - d^k, \forall k \in \kappa \tag{4.5}$$

$$0 \le x_{ij}^k \perp (t_{ij} + \tau_j^k - \tau_i^k) \ge 0, \forall k \in \kappa, \forall (i,j) \in A \tag{4.6}$$

with

$$t_{ij} = t_{ij}^0[1 + \alpha(\frac{f_{ij}}{h_{ij}(u_{ij}, \xi_{ij}) + \varepsilon})^\beta] \tag{4.7}$$

$$h_a(u,\xi) = \begin{cases} (1-\xi_a(\xi_a - u_a))\, c_a, \forall a \in \bar{A} \\ 0, \quad \forall a \in A \setminus \bar{A} \end{cases}, \tag{4.8}$$

Equation (1) states that the objective is to find the optimal retrofit strategy that will minimize system loss (i.e. repair cost, delay, and other penalty costs), quantified by function Q, at the given disaster scenario, where *u* is a $\bar{m}$ by 1 vector of elements u_a ($a \in \bar{A}$), and *f* is an *m* by 1 vector of elements f_a ($a \in A$). A suitable choice of function *Q* can be

$$Q(u, f) = < \rho, (c - h(u, \xi) > + \gamma < f, t(f, h) > + M \sum_{k=1}^{K} \left\| d^k \right\|, \tag{4.9}$$

which is the sum of total repair cost, the monetary value of the total travel delay on the network, and the penalty cost for unsatisfied travel demand. Explanation of the penalty term will be provided shortly. Parameter ρ represents the repair cost for each link, the parameter γ converts travel time to monetary value, and the operator < > represents inner product. Link time t_{ij} is a function of link flow f_{ij} and h_{ij} its remaining capacity (post-disaster capacity). When $h_{ij} = 0$, t_{ij} becomes very large prohibiting users from routing via this damaged link and when $h_{ij} = c_{ij}$, t_{ij} is in the form of the standard BPR function. Equation (4.3) specifies that the retrofit strategy must belong to the feasible set *U*, which may depend on budgetary and technological restrictions. Expression (4.5) and (4.6) define user equilibrium network flows. The quantity τ_i^k is the minimum time from node *i* to destination *D(k)*. The complementary condition (4.6) indicates that if a positive amount of flow travels on link *ij* toward destination *D(k)* (i.e., $x_{ij}^k > 0$), then link *ij* must be on the shortest path from *i* to *D(k) (i.e.,.* $t_{ij} + \tau_j^k = \tau_i^k$). Equation (4.5) defines the conservation of network flows, in which W represents the node-link adjacency matrix. Ideally, we wish to assign all travel demand to the network, i.e., $Wx^k = q^k - d^k, \forall k \in \kappa$ are satisfied.

However, the network may lose some capacity or even be disconnected under a severe earthquake, thus may not be able to accommodate all travel demand. In order to guarantee the feasibility of the model, we introduce the vector d^k to capture the amount of unsatisfied demand of each commodity *k*, and impose high penalty cost for any positive amount of d^k, i.e., $M\sum_{k=1}^{K}\left\|d^k\right\|$. The parameter M should be sufficiently large so that only the trips that cannot be accommodated by the network are captured by d^k and penalized.

The numerical difficulties of solving the problem defined by (4.2)-(4.8) come from the complementarity constraint (4.6), which makes the program nonconvex and the Mangasarian Fromovitz Constraint Qualification (MFCQ) not satisfied (Luo, Pang et al. 1996). Solving a MPCC directly by off-shelf solvers is difficult. In this work, we adopt a relaxation approach to convert a MPCC to a series of mixed integer nonlinear programs (MINLP). Details of this approach are given in the section on solution methods.

4.1.2 Formulation of the STNP problem as a SMPCC

Now, let us consider the real-world situation where decisions must be made without an exact foresight of the future. Let *S* be the set of possible scenarios for ξ, and *s* ($s \in S$) denote an individual scenario.

Solving the scenario sub-problems defined in (4.2)-(4.8) for all s ($s \in S$) will give us different s-dependent policies, denoted as u^s for each *s*. Note that u^s is a vector containing elements of u_a^s. However, these policies cannot be directly implemented, because at the time when the retrofit policy is implemented, one does not know yet which scenario is going to happen.

In order to consolidate the scenario dependent solutions to an *implementable* solution, we must impose the following condition:

$$u^s = u^{s'}, \forall s \in S, s' \in S, s \neq s' \tag{4.10}$$

or equivalently

$$u^s - z = 0, \ \forall s \in S \tag{4.11}$$

where z is a vector of free variables. This condition is called a *nonanticipativity constraint*, which states that an implementable policy should not require different actions relative to different scenarios if the scenarios are not distinguishable at the time when the actions are taken (Rockafellar and Wets 1991).

For simplicity, we set the objective of the stochastic program to be expected system cost, although the mean-risk objective could be incorporated without adding complexity. The formulation of the stochastic program is as follows.

$$\min_{u^s} \sum_{s \in S} p^s Q^s(u^s, f^s) \tag{4.12}$$

$$s.t. \quad u^s \in U^s, \forall s \in S, \tag{4.13}$$

$$f^s = \sum_{k \in \kappa} x^{k,s}, x^{k,s} \in R_+^m, \forall s \in S \tag{4.14}$$

$$Wx^{k,s} = q^k - d^{k,s}, \forall k \in \kappa, \forall s \in S \tag{4.15}$$

$$0 \leq f_{ij}^{k,s} \perp (t_{ij}^s + \tau_j^{k,s} - \tau_i^{k,s}) \geq 0, \forall k \in \kappa, \forall (i,j) \in A, \forall s \in S \tag{4.16}$$

$$u^s - z = 0, \ \forall s \in S \tag{4.17}$$

with

$$Q^s(u^s, f^s) = < \rho, (c - h^s > + \gamma < f^s, t^s(f, h^s) > + M \sum_{k=1}^{K} \left\| d^{k,s} \right\|, \tag{4.18}$$

$$t_{ij}^s = t_{ij}^0 [1 + \alpha (\frac{f_{ij}^s}{h_{ij}^s + \varepsilon})^\beta] \tag{4.19}$$

$$h_a(u, \xi) = \begin{cases} (1 - \xi_a(\xi_a - u_a))\, c_a, \forall a \in \bar{A} \\ 0, \qquad \forall a \in A \setminus \bar{A} \end{cases}, \tag{4.20}$$

where a quantity with a superscript *s* indicates that the quantity is scenario dependent.

4.2 Solving the Stochastic Program with UE Flows

The key computational difficulties in solving the SMPCC model (4.12)-(4.20) stem from the large problem size and the complementary conditions. Complementary conditions cause nonconvexity and make cutting plane based procedures such as the L-shaped method and generalized Benders decomposition not suitable, which strongly rely on the convexity assumption. In this section, we propose a numerical procedure based on the PH method. It iterates the process of solving perturbed scenario subproblems and aggregating scenario dependent solutions to an implementable policy. Each subproblem (a MPCC model) is converted to a series of MINLPs via a relaxation approach, and solved by commercial solvers. We shall first sketch the algorithm framework based on the PH method and then detail the procedure of solving MPCC subproblems.

4.2.1 The Algorithm Framework Based on the PH method

Let us denote G_s as the feasible solution set defined by constraints (4.13)-(4.16) in each scenario *s*. Define

$$L_r(U,X,z,W)=\sum_{s\in S} p_s[Q_s(u^s,x^s)+(w^s)'\cdot(u^s-z)+\frac{1}{2}r\left\|u^s-z\right\|^2] \qquad (4.21)$$

as the augmented Lagrangian, where W is the vector of dual variables for the nonanticipativity constraints in (4.17) and $r>0$ is a penalty parameter associated with violation of the nonanticipativity constraints. Therefore, the augmented Lagrangian integrates the nonanticipativity constraints with the original objective function. The STNP problem becomes

$$\text{minimize } L_r(U,X,z,W) \text{ over all } (u,x)\in G_s \qquad (4.22)$$

Due to the nonseparable penalty term $\frac{1}{2}r\left\|u^s-z\right\|^2$ in (14), the problem cannot be decomposed directly. The PH method achieves decomposition by alternatingly fixing the scenario solutions (u,x) and the implementable solution z . The detailed procedure is described below.

The progressive hedging algorithm (PH)

Step 1.

Set the iteration index ν to 0. Solve for each scenario sub-problem defined in (4.2)-(4.8) and obtain $(u^s,x^s)\ \forall s\in S$. Initialize $z^\nu=\sum_{s\in S} p_s u^s$. If $(u^s)^\nu=z^\nu,\forall s\in S$, then the optimal solution is found, otherwise continue with step 2.

Step 2.

Repeat step 2 until the termination criterion

$$\varepsilon=[\|z^\upsilon-z^{\upsilon-1}\|^2+\sum_{s\in S} p_s\,\|(u^s)^\upsilon-z^\upsilon]^{1/2}\approx 0 \qquad (4.23)$$

is reached.

Solve for each scenario

$$((u^s)^{\nu+1},(x^s)^{\nu+1}) \in \arg\min_{(u^s,x^s)\in G_s} \left\{ Q_s(u^s,x^s) + ((w^s)^\nu)' \cdot u^s + \frac{r^\nu}{2}\left\|u^s - z^\nu\right\|^2 \right\}, s \in S \qquad (4.24)$$

Obtain a new implementable solution

$$z^{\nu+1} = \sum_{s\in S} p_s (u^s)^{\nu+1} \qquad (4.25)$$

Update the dual variable estimates

$$(w^s)^{\nu+1} = (w^s)^\nu + r^\nu((u^s)^{\nu+1} - z^{\nu+1}), s \in S \qquad (4.26)$$

Increase the iteration index ν by 1.

One may also adjust the penalty parameter r^ν as the iteration proceeds. We will have more discussion on the choices of parameter *r* in the subsequent sections.

4.2.2 Solving MPCC problem via relaxation

The computationally intense part of the PH-based solution procedure is in solving many MPCC scenario subproblems. Thus it is crucial to select an effective algorithm for solving MPCC problems. In this work, we adopt an approach of reformulating a MPCC into a mixed integer nonlinear program (MINLP) through relaxation of complementarity constraints. Two relaxation schemes are considered: regularization and penalization. The regularization scheme relaxes the right hand side constant of the complementarity constraints from zero to a positive number, in which case constraint (4.6) becomes

$$0 \le x_{ij}^k, (t_{ij} + \tau_j^k - \tau_i^k) \ge 0, x_{ij}^k(t_{ij} + \tau_j^k - \tau_i^k) \le \mu^k, \forall k \in \kappa, \forall (i,j) \in A \qquad (4.27)$$

In the penalization scheme, the complementarity constraints are added to the objective as penalty terms:

$$\min_{u} Q(u, f) + \sum_{ij,k} \frac{1}{\mu^k} x_{ij}^k (t_{ij} + \tau_j^k - \tau_i^k) \tag{4.28}$$

A series of MINLPs are generated when the relaxation parameter μ^k, initialized as a relatively large positive number, is gradually reduced to close to zero. These resultant MINLPs are solved directly by commercial solvers. The complete procedure reads as follows.

<u>The Relaxation Approach for Solving MPCC</u>

Step 1 Initialization

Choose an initial relaxation parameter $\mu^{k0} > 0$ for each commodity $k \in \kappa$. Set the update factor $0 < \lambda < 1$, iteration index υ=0, iteration limit L.

Step 2 Iteration

If $\upsilon < L$, repetitively solve the current relaxed single-level MINLP. Update $\mu^{k(\upsilon+1)} = \lambda \mu^{k\upsilon}$, $\upsilon = \upsilon + 1$.

Otherwise, go to step 3.

Step 3 Final Solve

Solve the exact SMPCC model (4.12)-(4.20) . If it is successful, the solution is a local optimum to the SMPCC; otherwise, an approximate solution is achieved from the last run of step 2.

Ralph and Wright (2004) showed that under certain conditions, the above relaxation schemes can solve MPCC to a local optimum. Ban et al. (2006) also reported a successful experience of applying the relaxation approach to solving deterministic network design problems. One can control solution accuracy by adjusting the "track" of parameter μ, i.e., the initial and final values of μ and the reducing factor λ. For example, a possible choice of the track of μ can be (10, 1, …, 10^{-6}) with $\lambda = 0.1$. A coarse track of μ results in speed-up but leads to a less accurate solution. Since the PH method does not require scenario subproblems to be solved accurately, we shall keep a balance between solution accuracy and speed so that a good approximate solution to the subproblem can be generated rather quickly.

Thus far, we have shown that in order to solve a large scale stochastic network optimization problem with equilibrium constraints, we can rely on the PH algorithm to decompose the large scale problem to subproblems of manageable sizes, and use relaxation approaches to convert a MPCC subproblem to a series of mixed integer nonlinear programs which can be directly solved by commercial solvers. In summary, the major advantages of this PH-based solution procedure applied to our work are

(1) Each sub-problem is in a class of network design problems for which many specialized solution algorithms are available.

(2) The core of PH algorithm is an augmented Lagrangian, which is not limited to problems of convexity.

(3) The PH algorithm only requires that the sub-problems be solved approximately. In our work, solving each MPCC subproblem for the exact solution is highly time consuming, but a

good approximation can be found much more easily through relaxing the complementarity constraints.

(4) The PH algorithm allows easy use of parallel processors and can greatly save computing time when the number of scenarios is large.

4.3 Numerical Example

Rigorous convergence proof of the PH algorithm in convex and continuous problems is given by Rockafellar and Wets (1991). For nonconvex and continuous problems, the best theoretic knowledge about the convergence of the PH algorithm is that if all scenario subproblems are solved to local optimal solutions in each iteration, and if the sequences of the primary and dual variables do converge, they converge to the optimal solutions (Rockafellar and Wets 1991). These convergence theorems are valid unconditional to any particular choice of the penalty parameter r. However, it was also pointed out by the authors (Rockafellar and Wets 1991) that parameter r plays an important role in convergence in practice. Mulvey and Vladimirou (1991; 1992) and Lokketangen and Woodruff (1996) reported some important factors that may influence the setting of penalty parameter r in convex problems, which provide valuable numerical results for our research. Since our problem is nonconvex and discrete, for which no previous numerical implementation of the PH algorithm is available, we designed the following numerical experiments to explore the applicability of the PH algorithm to an extended range of problem types.

Three well known networks in transportation network literature are used in our numerical experiments, including the Braess network (Hagstrom and Abram 2001), the Sioux Falls city network (Leblanc, Morlok et al. 1975), and the network used by Harker and Friesz (1984). The numerical results obtained from the three networks are consistent. Here we only provide detailed numerical results for the Sioux Falls network.

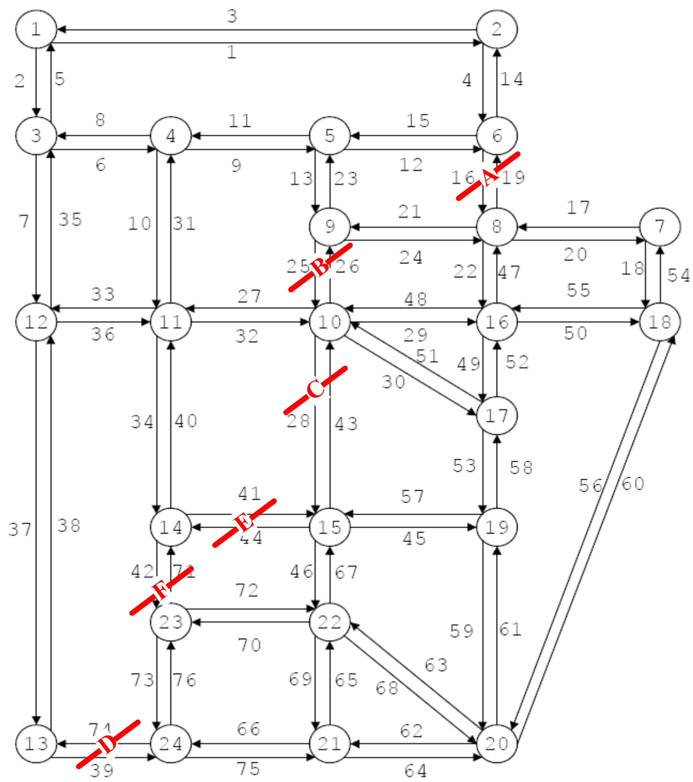

Figure 4-1 Sioux Falls city network

The figure of Sioux Falls city road network is repeated for the reader's convenience. The problem data used are the same as in chapter 3. The parameters are set as $\rho = 1.5, \gamma = 1, \alpha = 0.15, \beta = 1,$ and $M = 10^6$. It is assumed that six road segments (twelve links), labeled as A to F in Figure 4-1, are subjected to potential hazards. These twelve links are the candidate links for receiving retrofit action. However, due to insufficient resources, only four links can receive immediate retrofit. Two links of opposite directions on the same road segment or bridge should receive the same retrofit action. The question is: which set of four links should be retrofitted so that the total expected loss caused by future hazards is minimized?

For this particular model and data setting, $d^k = 0$, i.e., travel demand can be accommodated by the network in all post-disaster scenarios. The optimal solution obtained from the PH algorithm is to retrofit bi-directional links $13 \leftrightarrow 24$ and $14 \leftrightarrow 15$, which leads to an optimal objective value of 45.55. Through enumeration, we found that the worst strategy is to retrofit links $6 \leftrightarrow 8$ and $9 \leftrightarrow 10$, which leads to an objective value of 54.98. The gain of following an optimal retrofits strategy can be as high as 20% in this case study.

4.3.1.1 Effects of penalty parameter r on convergence

The PH algorithm was implemented with different values of *r* for solving the 64-scenario stochastic program. In Figure 2, the sequences of convergence resulting from different *r* values are plotted. The *x* axis corresponds to the number of iterations, and the *y* axis corresponds to the value of the error term ε defined in (4.23). When *r* is set between 0.2-0.7, the algorithm was able to converge to the optimal solution, even though the convergence speeds varied. When *r* is set to be 1, the sequence started to oscillate after a few iterations. This oscillation continues if the value of *r* remains the same. Usually reducing the *r* value can break the oscillations. As shown in Figure 4-2, reducing r from 1 to 0.5 terminated the oscillation and led to convergence.

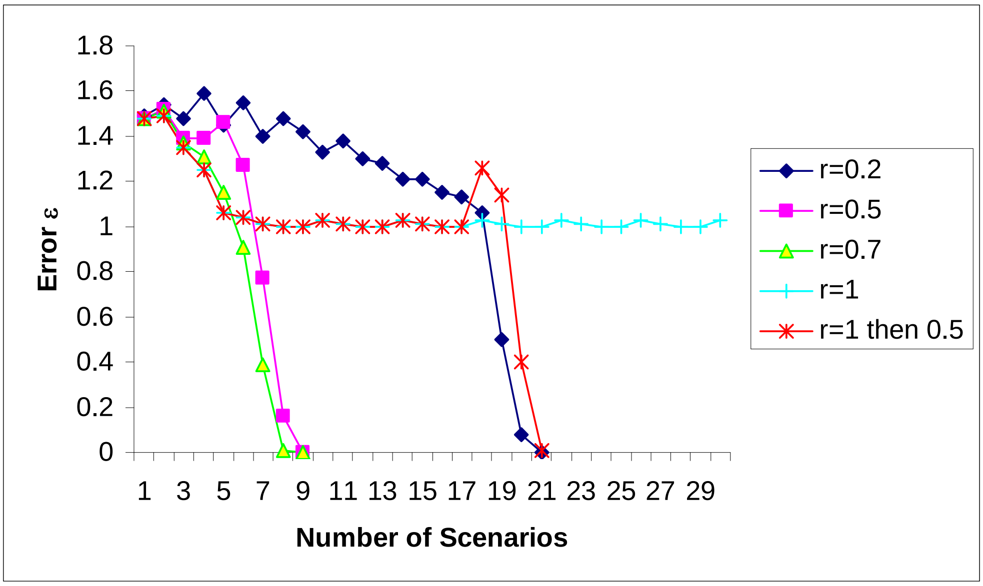

Figure 4-2 Sequences of convergence resulted from different values of the penalty Parameter *r*

In order to generalize the observation of the effects of *r* on the convergence performance of the PH algorithm, different datasets were generated using the same Sioux Falls network. Only the ten most likely scenarios out of the 64 scenarios are included to speed up the computation. Six cases correspond to six different probability distributions of the 10 random scenarios as shown in Figure 4-3. Consistent observations about the effects of *r* on convergence of the PH algorithm have been obtained in all cases. For demonstration purpose, Table 4-1 lists the convergence performance of the PH algorithm with different settings of *r* in six cases. Overall, when *r* is set between 0.5 and 1.5, the PH algorithm performed quite well. When *r* is set too low, it took longer for the sequence to converge. However, when *r* is set too high (*r*=2), the algorithm did not converge to the optimal solution in some of the cases.

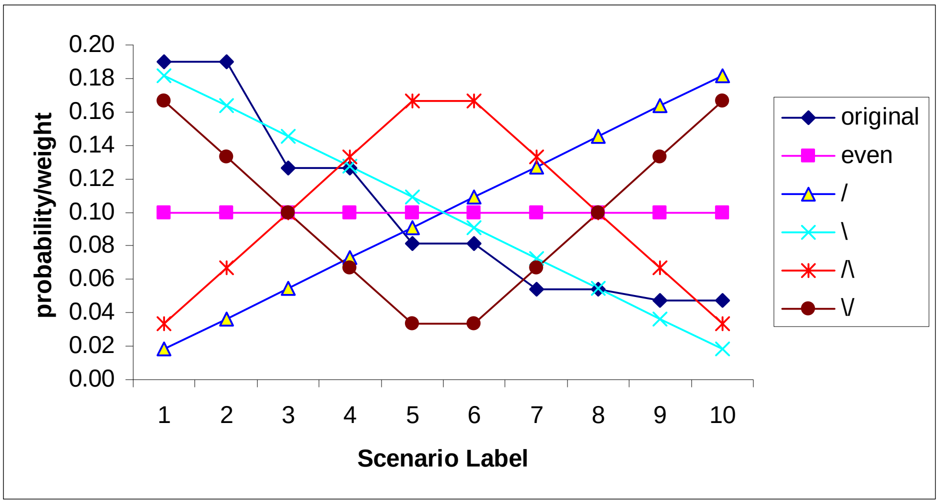

Figure 4-3 Six probability distributions of the ten scenarios

Table 4-1 Convergence of the PH algorithm (number of steps needed to converge) with different values of the penalty parameter *r* in six cases*

r	original	even	"/"	"\"	"/\"	"\/"
0.2	19	17	18	21	18	19
0.5	10	10	8	13	10	9
0.7	8	8	11	13	8	8
1.0	5	5	7	10	7	8
1.2	5	5	11	6	8	9
1.5	12	6	11	8	5	6
2	Oscillate	6	Oscillate	Oscillate	6	5

Note: * The six cases correspond to the six probability distributions plotted in Figure 3.

One can discern from Figure 4-2 and Table 4-1 that among all feasible values of *r*, some perform better than others. How should one select an appropriate *r* that could lead to fast convergence to an optimal solution? Previous research has demonstrated in convex cases that the setting of *r* is strongly influenced by the sensitivity of the objective function with regard to changes in the first-stage decision variables. In order to investigate this issue in the context of discrete and nonconvex problems, we implemented the PH algorithm using different *r* values for problems with different repair costs (i.e., different ρ values). To speed up the computation, again only the ten most likely scenarios are included. The convergence performances of the PH algorithm in different settings are summarized in Table 4-2. A cell with symbol * indicates convergence to an optimal solution. For example, in problems where ρ =1.5, the PH algorithm with *r* = 0.5 solved the problem optimally within 10 iterations, but when *r* = 50 the algorithm only converged to a suboptimal solution. The scale of ρ has a direct impact on the setting of *r*. As ρ increased from 1.5 to 1500, the effective range of *r* value also changed from around the neighborhood of 0.5 to the neighborhood of 500. In addition to the repair costs, other parameters that affect the sensitivity of the objective value with respect to a change in the first stage variables (u_a), such as β, also matter to the setting of *r*. For example, as β increased from 1 to 4, the good range of *r* values changed from the neighborhood of 1 to the neighborhood of 15.

Table 4-2 Penalty parameter *r* and the objective value: *r* vs. ρ ($\beta = 1$)

	ρ =1.5	ρ =1.5x10	ρ =1.5x100	ρ =1.5x1000
r = 0.15	*23 iterations	>40 iterations	>40 iterations	>40 iterations
r = 0.5	*10 iterations	*34 iterations	>40 iterations	>40 iterations
r = 5	oscillation after 2 iterations	*6 iterations	*31 iterations	>40 iterations
r = 50	Suboptimal, converged in 4 iterations	*7 iterations	*5 iterations	*31 iterations
r = 500	Suboptimal, converged in 4 iterations	Suboptimal, converged in 4 iterations	Suboptimal, converged in 4 iterations	*5 iterations

Based on our numerical experiments, we draw the following general rules for choosing an appropriate range for the parameter *r*. Most of these rules are consistent with those drawn for convex problems in Mulvey and Vladimirou (1991; 1992) and Lokketangen and Woodruff (1996):

(1) A small *r* usually results in gradual convergence to the optimal solution; while a big *r* generally produces faster initial convergence, but may arrive at a suboptimal solution. Thus an intermediate *r* is preferred for the best overall performance of the PH algorithm.

(2) When r is not carefully chosen, oscillation may appear. This phenomenon is unique in discrete problems.

(3) The choice of *r* depends on the sensitivity of the objective to changes in the first stage variables.

In addition to the above three remarks, Mulvey and Vladimirou (1991; 1992) also reported that in convex problems the scale of *r* is dependent on the structure of the problem. If the nonanticipativity constraints are highly restrictive, a bigger value of *r* should be used.

4.3.1.2 Effects of number of scenarios

Numerical tests with different numbers of scenarios (10, 20, and 64 scenarios) are carried out to study the effects of number of scenarios on the performance of the PH algorithm. It was found that the number of iterations required by the PH algorithm to converge to an optimal solution is not conditional on the number of scenarios. For example, in all testing problems, an optimal solution was reached in about eight or nine iterations (*r* was set to be 0.7 in all cases). The approximate computing time is about 10 min for a 10-scenario problem, about half hour for a

20-scenario problem, and about 2 hours for a 64-scenario problem[5]. The increasing computing time associated with larger problems is due to the increasing number of MPEC sub-problems that need to be solved at each iteration of the PH procedure. In fact, more than 99% of the computing time was devoted to repeatedly solving scenario subproblems. A significant amount of computing time can be saved via parallel computation of the subproblems. Because the subproblems in the PH procedure are of similar size and complexity, and they are independent of each other, it is quite straightforward to implement parallel processors in the PH procedure. This makes the PH algorithm particularly favorable for problems with large number of scenarios. More detailed discussion on parallel computation implemented for the PH procedure can be found in Mulvey and Vladimirou (1991; 1992).

4.3.1.3 Effect of initial solution on the convergence

In the PH procedure description provided in Section 4.2.1, we stated that an initial solution to the first-stage decision variables can be found by simply aggregating the scenario-dependent solutions. This is an easy choice, but may not be the most efficient. One may first solve the problem without the equilibrium constraints (i.e., to solve a stochastic programming problem with system optimal (SO) flows), and then use that solution as the initial solution to the corresponding problem with equilibrium conditions. The cost of this choice is the additional computing efforts spent on solving a stochastic programming problem that is large in size but convex. The benefit is the reduced number of iterations in the PH procedure. In most problems where the number of scenarios is not trivially small, such a tradeoff would be worthwhile. This is

[5] The tests are done using a desktop PC with Intel Xeon 3060 CPU@2.39 and 2.40 GHZ, 2 GB RAM.

because the computational complexity introduced by equilibrium constraints is tremendous. In our case, the time required for solving a MPCC subproblem is about 10 times that for the corresponding SO subproblem. Considering that each iteration of the PH procedure involves many MPCC problems, the time saved from reducing one iteration can be significant. In addition, since the proper settings of *r* do not differ by much between problems with or without equilibrium constraints, starting with an SO problem can help tune the *r* value in a much less expensive manner.

4.3.1.4 Notes on integrating MPEC solvers with the PH algorithm

As mentioned earlier, the effectiveness of the solution method for MPCC subproblems is critical to the performance of the entire solution procedure. We adopt the NLPEC (nonlinear program with equilibrium constraint) solver by Ferris et al. (2002) which can automatically reformulate an MPCC into a series of MINLPs, and call GAMS solvers to solve the MINLPs. Users can specify the choices of the MINLP solver, reformulation type, and relaxation setting through the NLPEC option files.

For solving scenario subproblems, we observe that the regularization scheme (also called multiplication in NLPEC) provides higher solution accuracy and the penalty scheme has faster convergence, which is consistent with the results reported by Ban et al (2006). One can also adjust the setting of relaxation parameter μ to control the balance between solution accuracy and speed. For example, one setting for the sequence of μ is $(10,1,0.1,\ldots,10^{-6})$ with a reducing factor 0.1, whereas a more approximate setting can be (10,1,0.1). Obviously the latter setting achieves a faster solution speed but a less accurate solution. In the experiments, we found that the PH algorithm performed well in terms of both solution accuracy and speed when the regularization (multiplication) reformulation type is adopted with the track of μ set as (10, 1, 0.1), and the resultant MINLPs are solved by GAMS/SBB solver.

4.4 Summary

Stochastic network optimization problems with equilibrium constraints are important in many areas of science and engineering. However, due to their computational complexity, numerical implementation for such problems has been lacking. In this chapter, we have demonstrated that such problems can be successfully solved via the progressive hedging method if some important parameters are carefully chosen and an efficient solution method for MPCC subproblems is integrated. Previous research has demonstrated the applicability of the PH algorithm to problems of convex and in most cases continuous nature. This work extends the PH method to a broader range of applications including discrete and nonconvex problems as well.

5 Conclusions and Discussion

Policy makers in the area of disaster management and mitigation often encounter an important class of decision making problems: how should limited mitigation resources be allocated to critical system components for protection, response, and recovery in order to minimize societal loss from disasters? A specific instance of this general resource allocation problem is to determine which network components should be protected (e.g. retrofitted or strengthened) before disasters, given limited resources. This problem is not well solved in the research and practice of disaster mitigation, primarily due to the challenges of capturing infrastructure interdependency and making decisions under disaster uncertainty. This dissertation is devoted to developing a rigorous approach to finding effective resource allocation strategies for pre-disaster network protection. The developed methodologies are general, but the context of discussion is on transportation network protection under seismic hazards.

Section 5.1 summarizes our major contributions for this effort. Section 5.2 discusses some future research directions.

5.1 Summary

The engineering method used in the current practice of disaster mitigation often prioritizes the retrofit of network facilities by their importance, which is estimated by some descriptive measures (e.g. network link flow). This prioritization approach fails to capture the interdependency of interconnected network components. On dealing with uncertainty, a popular approach is scenario analysis. System cost is evaluated under each disaster scenario, and scenario dependent policies may be generated. One then can aggregate these scenario dependent policies into an implementable policy or simply take the policy from the most likely scenario. However, this scenario analysis approach has little possibility to ensure an optimal policy in the sense of optimizing mathematically well defined system measures (e.g. expected system cost). This dissertation develops a mathematical modeling approach, based on stochastic programming and network optimization, with the capability of capturing system component interdependency and incorporating uncertainty.

Specifically, in chapter 3, we demonstrate that the decision and physics process underlying the network protection problem fits the framework of two-stage stochastic programming very well. Assuming network flows are completely controllable to achieve system optimum (SO), we formulate the problem as a one-level two-stage stochastic mixed-integer nonlinear program with binary variables in the first stage. In considering the context of infrastructure protection, we include semi-deviation of scenario cost as a risk-averse measure into the objective. The resultant mean-risk two-stage stochastic program seeks a balance between minimizing expected system cost and reducing system cost variation and better suits the needs of disaster mitigation planning. Due to the nonlinearity of developed stochastic programs, the well-known L-shaped method cannot be directly applied to solve the model. But following the general idea of generalized Benders decomposition and the L-shaped method, we are still able to develop an efficient algorithm based on the techniques of decomposition, outer linearization and successive approximation. Using numerical examples, we demonstrate that the developed algorithm is much more efficient than directly solving the deterministic equivalent program (DEP). We also compare stochastic programming and engineering method solutions. Stochastic programming solutions perform better than engineering method solutions in terms of providing smaller expected system cost, higher probability of system cost within a predefined threshold, and better robustness (i.e., smaller regret).

Chapter 4 studies the same problem as chapter 3 except assuming network flows are in user equilibrium (UE) condition. The emphasis is on studying the effects of incorporating this more realistic assumption for transportation neworks on system modeling and solution methods. A bi-level modeling structure is necessary in order to accommodate the inconsistent objectives of network planners and users. Much effort has been spent in solving this non-convex stochastic bi-level mixed-integer nonlinear program. We have shown the feasibility of a numerical algorithm based on the Progressive Hedging (PH) method with scenario MPCC problems solved via a relaxation approach. Previous research has demonstrated the applicability of the PH algorithm to problems of a convex and in most cases continuous nature. This work extends the PH method to a broader range of applications including discrete and nonconvex problems as well.

In summary, the major contribution of this dissertation is twofold. Firstly it integrates multiple disciplines to build a modeling framework for the transportation network protection problem. Secondly in the computational aspect, it develops effective solution algorithms by extending and adapting existing decomposition methods, and enriches numerical experience for solving stochastic network optimization problems. Additionally, the methodologies developed herein can be directly applied to general network design problems under uncertainty.

5.2 Discussion

While we have successfully fulfilled our goal to formulate and solve the transportation network protection problem, we realize that this work is only an important first step in assisting effective decision making involved in infrastructure protection activities. Some methodological questions remain to be further investigated.

1. Optimization VS. Engineering method

We have pointed out the limitations of the practical engineering method (including the ranking and scenario analysis approach) for being unable to analyze the problem from a system view and deal with uncertainty in a scientific way. However, the engineering method is widely used in practice due to its simplicity and flexibility. The ranking approach is intuitive and operationally simple. The scenario analysis approach is a typical first response when people encounter uncertainty. Moreover, as we reviewed in chapter 2, the scenario analysis approach can conduct a relatively complex analysis, for example, to evaluate economic loss from an earthquake. On the other hand, the optimization method requires problems to be mathematically well-defined. The objective and constraints must be in functional form of variables and parameters, and the functions should have some good mathematical properties, otherwise the model could be difficult to solve. This requirement restricts the application of the optimization method. For example, it is hard to include broad social and economic impacts in the objective of our network protection problem since these impacts are difficult to analytically describe as functions of variables such as traffic flow.

A possible way to overcome the above difficulty in incorporating more realistic disaster loss estimates is to integrate simulation into ordinary optimization models. The performance of solutions is evaluated by a simulation module and hence objectives do not need to be analytical expressions of variables. Specifically, retrofit strategies could be input to some earthquake damage assessment software (e.g. REDAS[6]), and a set of measures of disaster loss are then generated including network damage and social & economic impacts. Combining this simulation module with our optimization model framework would result in a stochastic program with simulated system cost. This philosophy of enhancing optimization by simulation is often referred to as *simulation optimization* in the literature (e.g. Fu 2002; Olafsson and Kim 2002). Common techniques for solving such models include stochastic approximation (e.g. Nemirovski, Juditsky et al. 2009) and metaheuristics (e.g. simulated annealing, tabu search and genetic algorithms)

2. Modeling Assumptions

1). Network users' travel behavior in terms of flow and travel demand patterns need to be incorporated into network design models. In this work, we assume that travel demand is fixed, and flow is either system optimal or in user equilibrium condition, which works well in a normal

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[6] The Rapid Earthquake Damage Assessment System (REDAS) is a simulation software for seismic hazard estimation and risk assessment.

transportation system but may be arguable in a post-disaster environment. An immediate step of this research is to incorporate more realistic assumptions from studies on post-disaster travel behavior (see e.g. Cho, Fan et al. 2003; Walton and Lamb 2009). The effects of different behavior al assumptions on modeling framework, numerical methods, and solutions delivered are also of interest.

2). We assume that if a link is retrofitted, its probability of being damaged is zero. A more realistic way is to assume reduced but nonzero damage probabilities for retrofitted links. Such an extension leads to a stochastic programming problem with decision-dependent uncertainty. Computational experience with this class of problems is very sparse, and is only limited to convex problems of special structures. Moreover, quantitative relationships among link damage probability, structure seismic performance and retrofit decisions are not well studied. Finding a suitable solution method for such problems is an ongoing endeavor.

3. Retrofit Practice

Several issues arising from retrofit practice are not yet considered. For example, in construction practice bridges are often grouped during a retrofit project and thus the retrofit decisions would be made over clusters instead of individual bridges. If the clusters are predefined, then the proposed models are still suitable. However, if the clustering decisions needs to be made simultaneously with the retrofit decisions, this requirement would impose one more layer of complexity to the model.

4. Risk Preferences

It turns out that stochastic programming solutions are not very sensitive to the change of weighting coefficient η in the mean-semideviation objective for the particular numerical examples in this dissertation. However this finding cannot be generalized. More numerical experiments are needed to investigate this issue. We may also explore other risk-averse models, such as optimizing Conditional Value at Risk (CVAR, see e.g. Rockafellar and Uryasev 2000; Rockafellar and Uryasev 2002) , chance constrained model (Prékopa 1995), and robust optimization (e.g. Kouvelis and Yu 1997). It would be interesting to compare results from different modeling approaches. Understanding how decision makers' risk preferences might affect their choices and eventually impact the effectiveness of the entire society will have significant policy implications.

5. Computational Improvements

1) The BD-based algorithm is demonstrated to be more efficient than solving DEP directly. However, we observed that the algorithm converges slowly, which is consistent with the literature (e.g. Magnanti and Wong 1981). In the future, we shall enhance this algorithm by adopting some acceleration techniques such as pareto-optimal cuts (see e.g. Magnanti and Wong 1981; Wentges 1996; Rei, Cordeau et al. 2009).

2) The PH algorithm is able to solve the numerical example based on Sioux Falls network, but its efficiency needs to be tested on more examples. The PH method also seems promising in solving other types of stochastic bi-level programming models, e.g. network design models under travel demand uncertainty.

The discussion of this dissertation is focused on highway networks. However, the modeling and solution methods are general and can be tailored to other transportation modes and a broad range of critical infrastructure systems that can be analyzed as networks. It is our hope that this work will attract more research effort into this important subject of strategic resource allocation for critical infrastructure protection and hazard prevention.

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