Research Report – UCD-ITS-RR-15-13

Incorporating Behavioral Effects from Vehicle Choice Models into Bottom-Up Energy Sector Models

July 2015

David S. Bunch
Kalai Ramea
Sonia Yeh
Christopher Yang

Institute of Transportation Studies ◦ University of California, Davis
1605 Tilia Street ◦ Davis, California 95616
PHONE (530) 752-6548 ◦ FAX (530) 752-6572
www.its.ucdavis.edu

Incorporating Behavioral Effects from Vehicle Choice Models into Bottom-Up Energy Sector Models

David S Bunch, Kalai Ramea, Sonia Yeh, and Chris Yang*

**Graduate School of Management, University of California, Davis*
Institute of Transportation Studies, University of California, Davis

Suggested Citation: *Bunch, David S., Kalai Ramea, Sonia Yeh, Christopher Yang (2015) Incorporating Behavioral Effects from Vehicle Choice Models into Bottom-Up Energy Sector Models. Institute of Transportation Studies, University of California, Davis, Research Report UCD-ITS-RR-15-13.*

Abstract

Many different types of models are used for evaluating climate-change-related programs and policies, because analysis requirements can vary widely depending on the specific nature of the problem being investigated. Limitations on data and methodology typically ensure that models have various strengths and weaknesses, requiring researchers to make tradeoffs when choosing models. In the case of energy systems, a frequent distinction is between "top down" models (e.g., computable general equilibrium, or CGE models) that address energy systems within the context of the larger economy, versus "bottom up" models (e.g., so-called E4, or "energy/economy/environment/engineering" models), that model the energy system at a much higher level of detail, but simplify the relationship to the rest of the economy. Most attention has been on integrating these two types of models. However, researchers have also been concerned that E4 models, despite their vaunted high level of detail, produce results that are an unrealistic representation of consumer market behavior, calling into question their value for making policy decisions. This is particularly true for household vehicle technology choice, an important sub-sector of the energy system.

At the same time, there is a large and well-established literature on modeling household vehicle choice and usage decisions (using discrete and discrete-continuous models). But, the methods and approaches used in this literature differ dramatically from those used in E4 models, and so it has been unclear how to bridge the gap. This paper demonstrates a practical approach for incorporating behavioral effects from vehicle choice models into E4 models. It is based on principles of economic theory that form a common basis for all three types of models (CGE, E4, and vehicle choice/usage models). Derivations are provided that yield a theory-based approach for modifying E4 models that can be used without altering the basic software and modeling infrastructure widely used by many researchers. The approach is illustrated using an empirical application in which the behavioral assumptions from a nested multinomial choice model in an existing modeling system (MA^3T) are incorporated into a TIMES/MARKAL model.

Table of Contents

1. Introduction

There is a wide diversity of modeling approaches for analysis of policies for addressing climate change, where the goal is long-term reduction of greenhouse gases from the energy sector. The issues are challenging, because almost all aspects of economic activity require some form of energy consumption, using technologies that convert inputs (electricity, gasoline, etc.) into energy-related services (heating, lighting, transportation). Meaningful reductions in greenhouse gases will require major changes to complex energy systems with far-reaching implications for the broader economic system and society at large, across a wide variety of stakeholders. Models are used to understand and evaluate how alternative policies might affect the behavior of these systems, and modeling requirements can vary depending on the specific issues being addressed.

A number of reviews have appeared that summarize and classify models along multiple dimensions, such as scope (top down, bottom up, hybrid, general equilibrium, partial equilibrium), mathematical method (optimization, simulation), etc. For a recent high-level review focused on energy systems models, see Herbst et al. (2012). A typical practical consideration is the tradeoff between scope and level of detail. Focusing on a narrower scope by adding more detail helps improve realism and (hopefully) "accuracy," but this is frequently obtained at the risk of omitting important interactions with the larger system environment, as we discuss next.

In this paper, we focus on a specific family of bottom up energy sector models (MARKAL/TIMES)—also denoted energy/economy/environment/engineering (E4) models—that are considered to have a high level of technological and economic detail. One concern in the literature is that the price of this additional detail is the limitation of the scope to the energy sector: important interactions with the larger economy are not taken into account. However, there have been other concerns as well, namely, that despite their characterization as being "highly detailed," they still do not adequately capture important features of how consumers behave in real markets, calling into question the validity of their conclusions when using them to evaluate alternative policy scenarios. This latter concern is the subject of this paper.

However, as part of this discussion, it will be instructive to begin by reviewing the "scope versus detail" issue for top-down versus bottom-up models as well. Specifically, TIMES/MARKAL models can (at best) be viewed as partial equilibrium models designed to support more detailed analyses of a specific economic sector (energy). In contrast, top-down models strive to capture general equilibrium effects that take into account interactions across the entire economy, e.g., effects related to household income, disposition of capital and labor, and other important macroeconomic factors. However, top-down models do so at the risk of other types of simplification, applying a high level of aggregation to economic measures for the purpose of representing the outcomes of various components of the economic system. The Computable General Equilibrium (CGE) model is one particular type that has been widely used. Efforts have been made to integrate the two approaches, as discussed in section 2.

This paper moves in the other direction, seeking to add details that improve the representation of consumer behavior and market response in a *subsector* of the energy sector, specifically: *transportation services from personal vehicles*. In this case, the model with the larger scope is TIMES, which has limited detail on consumer choice and usage behavior for vehicles. The source of "behavioral content" we use for this purpose comes from the MA^{3}T (Market Acceptance of Advanced Automotive Technologies) model, developed by Zhenhong Lin, David Greene and co-workers at Oak Ridge National Laboratories (Lin, Greene, and Ward; Lin, Li, and Dong 2014). MA^{3}T has a narrower scope than E4 models, which is limited to projecting the behavior of the personal vehicle market in the U.S. under alternative policy scenarios. However, it also operates at a higher level of detail, capturing important aspects of consumer market behavior. At the same time, because its definition of "vehicle choice" focuses on future market shares of competing alternative fuel technologies, its vehicle-type definitions are similar to those used in TIMES energy sector models developed by researchers at ITS Davis—see Figure 1. The goal is to integrate the behavioral content of MA^{3}T into a TIMES modeling framework, so that the TIMES model produces vehicle choices as though they were determined by MA^{3}T.

Based on an initial examination of the two models, they would appear to rely on very different approaches. MARKAL/TIMES results are obtained by solving a deterministic linear program (LP), which is generated by a software product (VEDA) that acts as a front end (as well as a back end). In this regard, a MARKAL/TIMES model is frequently used as something of a "black box" by many researchers (even though the details of the LP model are visible), but in any case the fundamental structural approach cannot be easily modified. In most cases the LP objective is to minimize the total discounted monetary cost of the energy system over a relatively long time horizon, which includes investment costs from choosing future energy technologies, as well other fixed and variable costs (including fuel costs) for using those technologies to meet required end-use energy service demands (e.g., passenger vehicle miles traveled). Additional details are reviewed in section 3.

In contrast, MA^{3}T is essentially simulates total vehicle market behavior over time, where the core behavioral model is a nested multinomial logit discrete choice model that yields market shares of competing technologies for a large number of consumer segments that comprise the new vehicle market. Choices are a function of vehicle attributes (including purchase price, fuel operating costs, etc.), the refueling infrastructure, and characteristics/preferences of individual consumer segments. It simulates vehicle fleet turnover as vehicles age and disappear, and includes dynamic effects related to introduction of new technologies.

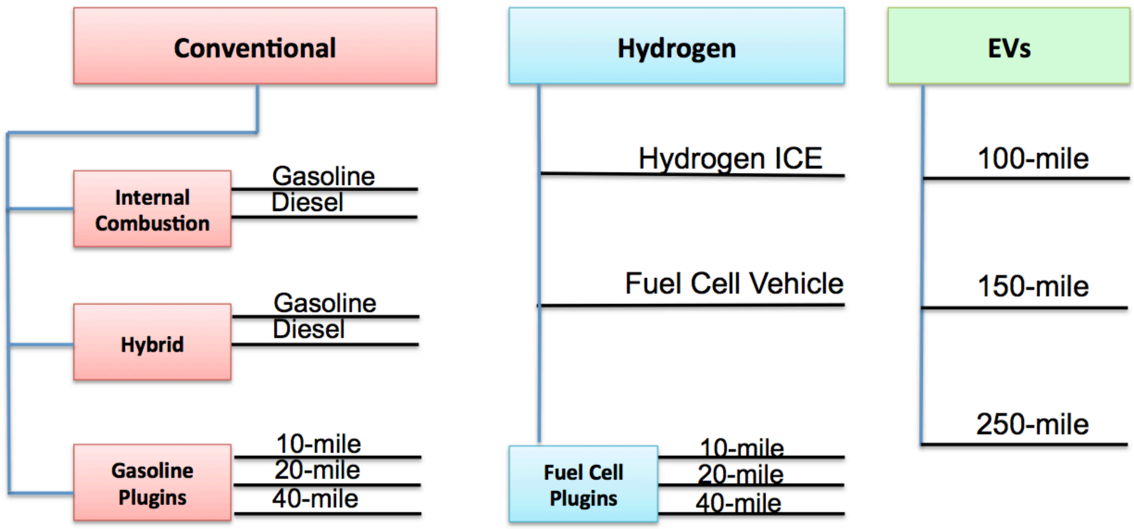

Figure 1. Vehicle Technology Options

The differences in behavior between the two models are illustrated in Figures 2 and 3. Figure 2 shows the "all or nothing" behavior that can occur in a TIMES model if it is relies only on basic inputs and assumptions (e.g., without any *ad hoc* intervention by researchers, such as arbitrary constraints on market growth for new technologies, manipulation of technology-specific hurdle rates, etc.). Figure 3 shows corresponding output from the MA³T model, which includes many additional behavioral factors.

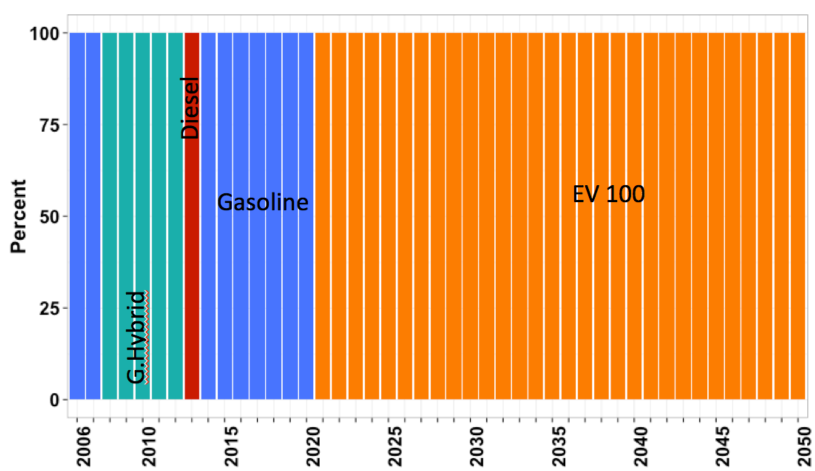

Figure 2. 'All or Nothing' New Vehicle Sales Shares in TIMES Model

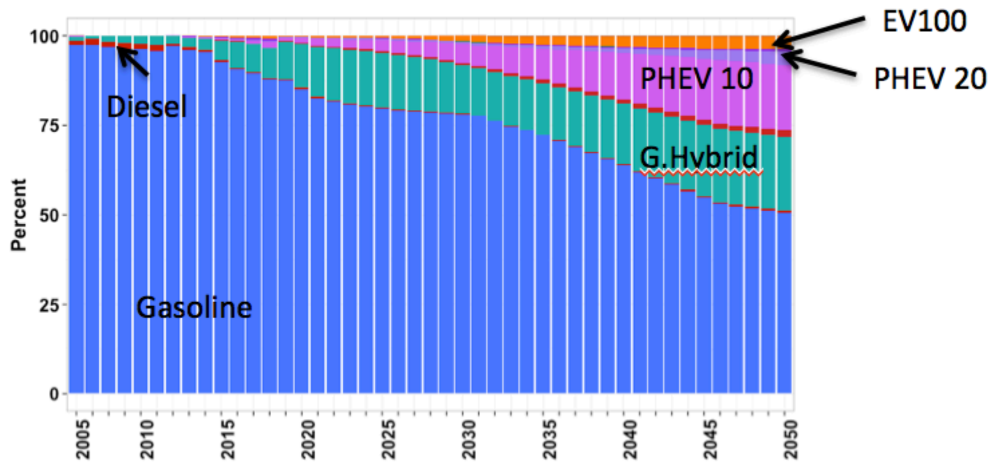

Figure 3. New Vehicle Sales Shares from MA^{3}T Model

The goal is to make modifications to TIMES so that it produces output that looks more like Figure 3, and does so on the basis of a valid theory of consumer choice behavior rather than through *ad hoc* user-imposed interventions. However, there is no obvious, straightforward way to integrate these two methodologies because the their implementation approaches are incompatible: TIMES is formulated as a standard (deterministic) LP model based on a single representative decision maker, whereas MA^{3}T performs a "simulation" that has, at its core, a nonlinear nested logit model of consumer purchase behavior typical of those used in econometrics.

One goal of this paper is to support the view that, when approaching modeling challenges of this type, it is helpful to develop a clear understanding of the relationship between specific modeling approaches and the underlying economic theory and assumptions on which they are based. All three of the model types considered here (top-down, bottom-up, and discrete/continuous) are based on the same theoretical economic framework. Frequently, researchers develop specific methodologies by adding many simplifying assumptions that are frequently motivated by practical considerations related to data availability and computing requirements. Passage of time and the relative complexity of the subject matter ensure that awareness and knowledge of the connection to underlying theory are often lost.

With this in mind, the structure of the paper is as follows. Section 2 provides additional context by briefly reviewing elements of the high-level economic framework shared by these models. Section 3 provides additional background on the MARKAL/TIMES models we seek to modify. Section 4 focuses on derivation of discrete and discrete-continuous choice models for vehicle choice and usage, providing results that support a theory-based approach for integrating behavioral content from these models with

MARKAL/TIMES, and reviews additional features specific to MA^{3}T and MARKAL/TIMES. Section 5 presents an empirical example in which behavioral factors from MA^{3}T are sequentially added to MARKAL/TIMES. Section 6 concludes with final comments.

2. Preliminaries

The decisions being modeled are assumed to occur in the context of some economic system. At the highest level of abstraction, a complete economy can be viewed as consisting of *H households* (sometimes referred to as *consumers*), *P producers*, and *C commodities*. Commodities represent all things that can be exchanged in the economy (goods and services). Households are endowed with (or otherwise control) a subset of these commodities, and these are frequently given a special designation as *factors of production* that can be "rented" by producers (providing *income* to the Households) and used as *inputs* to production processes (or *technologies*) to create other (output) commodities. These commodities can, in turn, be purchased and consumed by Households (using the aforementioned income) for the purpose of generating "utility." Households are assumed to choose quantities for all commodities so as to maximize utility, and Producers are assumed to make production plans so as to maximize profits.[1] These decisions clearly depend on the *prices* of the commodities, which must be determined in some way by the system. In addition to the previous assumptions, Households and Producers are assumed to be *price takers*, and there is a *circular flow* of resources in the economy that should obey various conservation properties. Under these and other assumptions, prices are determined by a market clearing process that yields a (Walrasian) equilibrium.

The above summary is the typical starting point for discussing *general* equilibrium models, and the basics of this theory appear in various textbooks (e.g., (Varian 2000), (Intriligator 2002)). Takayama and Judge (1971) is a comprehensive early reference that begins with this general framework and systematically adds assumptions that lead to various options for modeling general and partial (competitive) equilibrium solutions for economic systems, addressing both spatial and temporal dimensions. These simplifications usually involve replacing individual actors (Households and Producers) by representative agents, so that decisions are made in the aggregate rather than at the individual level. Sue Wing (2004) discusses the circular flow economy described above (see his Figure 1), and shows how CGE models can be implemented by assuming that the utility and profit maximization behavioral models for Households and Producers can be simplified using Cobb-Douglas utility and production functions, respectively.

The major simplifying assumption yielding a partial rather than a general equilibrium relates to how household income generation is modeled. The general equilibrium model assumes that choices governing both income and consumption are made

[1] Note that, for households, these choices affect income levels, which in turn determine the budget for purchasing other commodities, so decisions are interdependent in a complex way.

as part of the same process. If some other process (exogenously) determines household income, this yields a partial equilibrium. More generally, any analysis limited to a *subset* (e.g., sector) of the economy that takes incomes and prices of other (outside) goods as given yields a partial equilibrium solution. Although the theory starts with utility functions and profit equations for households and producers, respectively, practical approaches typically rely on being able to assume the existence and knowledge of (inverse) demand and supply functions. This is the approach taken by MARKAL/TIMES, which also makes Cobb-Douglas assumptions, and Sue Wing (2006) shows how these features support integration of a CGE model with a more detailed MARKAL model. Also, see Schaefer and Jacoby (2005).

Generally speaking, equilibrium conditions that must be satisfied are defined by mathematical equations (frequently inequalities), which may include expressions involving the aforementioned demand and supply functions. Rather than solving *systems of equations*, a frequent alternative approach is to formulate a mathematically equivalent *optimization problem* by defining an objective function that can be viewed as a measure of "quasi-social welfare" (or "social surplus"). In this case, demand and supply functions are used to produce expressions for consumer and producer surplus that appear as terms in the objective function. In this type of model, prices and quantities for all commodities in the system are endogenously determined by *maximizing social surplus*, subject to constraints (defined by the other required conditions).

Because household choice/demand behavior is the main concern of this paper, it is instructive to review in more detail the constrained utility maximization model from economics:

$$\begin{aligned} &\max_{\mathbf{q}} U(\mathbf{q}) \\ &s.t. \sum_{j=1}^{C} p_j q_j = y \qquad (1) \\ &\mathbf{q} \geq 0 \end{aligned}$$

where q_j is the quantity of commodity j that is purchased/consumed, p_j is the per-unit price, y is the available household expenditure budget ("income") appropriate to the "planning horizon" (an aspect that is frequently not discussed, but which will be an important consideration in this work), and $U(\boldsymbol{q})$ is the utility from consuming the C-vector of commodities.

In the context of the first paragraph of this section (1) includes *all* commodities in the system. As previously noted, the key break between general and partial equilibrium models (as well as many others) is how y is determined. For our purposes, y is assumed to be exogenous. The general solution of (1) yields Marshallian demand functions, where, without additional structural assumptions, demand for each commodity is a function of prices for all commodities as well as income, i.e., $\boldsymbol{q} = w(y, \boldsymbol{p})$.[2]

As has been noted, MARKAL/TIMES is designed to produce partial equilibrium solutions for energy sector models. It does so using an approach consistent with the previous description, relying on specific assumptions about the form of demand functions for end-use energy services (discussed in the next section). Specifically, demand functions are assumed to take the form

$$DM_j = K_j p_j^{E_j} \tag{2}$$

where p_j is the price, E_j is an elasticity, and K_j is a constant. This is a considerable simplification versus the general result from (1); nevertheless, the details are still relatively complex.

Using MARKAL/TIMES requires initial projections of end-use energy service demands for a Reference Energy System, which are assumed to hold by definition. The initial solution for the system is obtained by minimizing the NPV of total system costs, subject to constraints (see the next section). Producing partial equilibrium solutions for alternative scenarios requires that at least some demands be treated as "elastic" using (2). The user must provide assumed values for elasticities, and K_j is calibrated using the initial results. Then, under alternative scenarios these demands are no longer treated as exogenous, and the objective function is augmented by terms that measure the change in consumer surplus associated with changes in demand for the relevant services.

However, using the model in this way has a number of challenges, so analyses are frequently performed that continue to assume exogenous (inelastic) demands. To provide additional context, a more detailed overview of MARKAL/TIMES is provided in the next section.

3. Overview of MARKAL/TIMES

As discussed in the introduction, MARKAL/TIMES models operate at a high level of detail, and implementing the approach described in the previous section is rather complex, so a complete treatment is well beyond the scope of this paper. The usual references are to detailed model documentation (e.g., Loulou et al. (2004) for MARKAL, and Loulou et al. (2005a, 2005b), for TIMES). However, even these references provide limited insight regarding the relationship between the models and the underlying theory. This section provides selected additional background to support the needs of this paper.

[2] The results in this paper rely heavily on microeconomic theory related to (1): for a useful reference, see Deaton and Muellbauer (1980).

From Loulou et al. (2005b),TIMES "is an economic model generator for local, national or multi-regional energy systems, which provides a technology-rich basis for estimating energy dynamics over a long-term, multi-period time horizon." Researchers at University of California, Davis have developed such a model for the state of California's energy system (McCollum et al. 2012), as represented in Figure 4. Scenarios require a database of energy technologies, including projected characteristics (e.g., efficiencies and costs) for the entire planning horizon. As shown in Figure 4, conversion technologies are used to process primary energy sources (e.g., crude oil or hydropower) to produce *energy carrier* commodities (e.g., gasoline or electricity) that are then used by *end-use technologies* (e.g., gasoline or electric cars) to produce *end-use energy services* (e.g., driving a specified number of miles). A major feature of TIMES models is that they make investment decisions, choosing from among competing technologies to meet overall system requirements. Examples of competing technology options for three types of end-use energy services are shown in Figure 5.

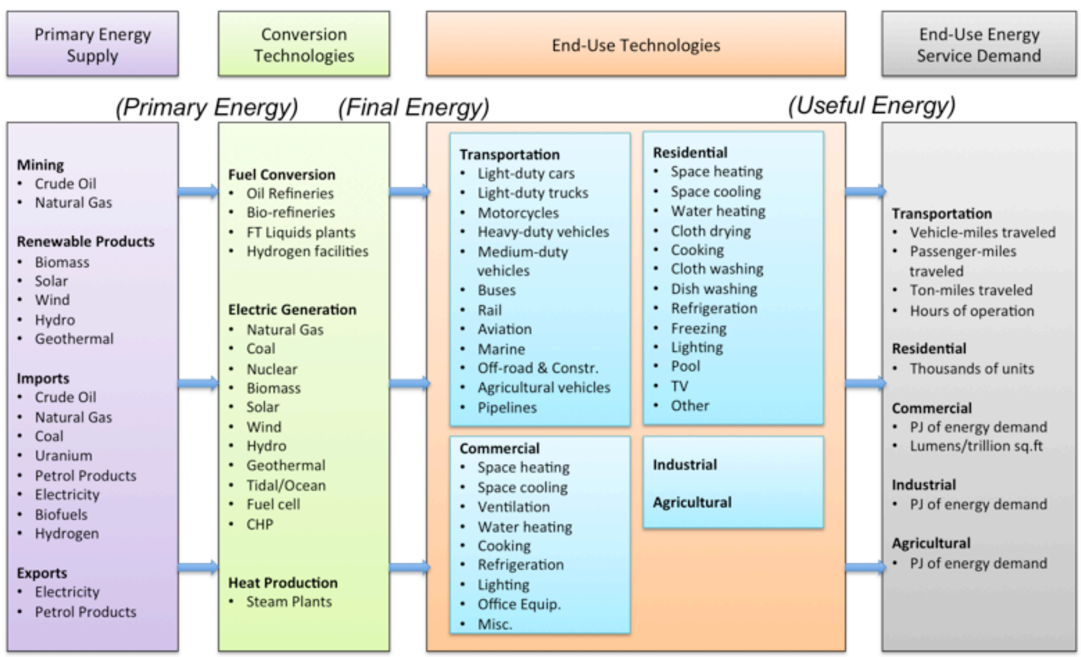

Figure 4. CA-TIMES: Bottom-Up Model of California's Energy Sector

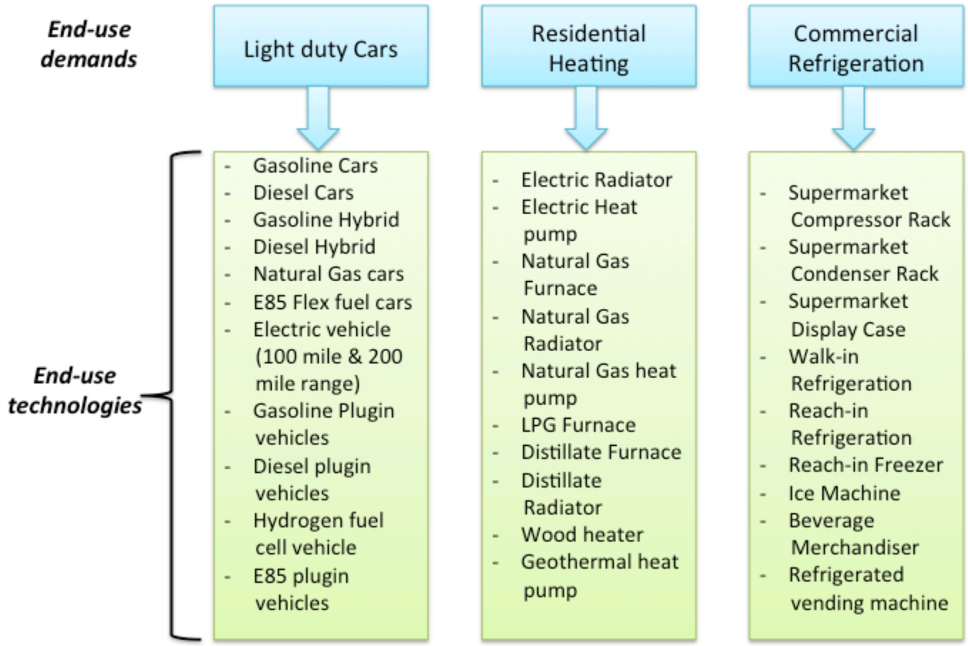

Figure 5. Technology Options for Three Types of End-Use Energy Services

The purpose of the energy system is to meet the demand for end-use energy services. As discussed in the previous section, using the model requires demand projections for an assumed Reference Energy System. An initial LP model is formulated and solved in inelastic demand mode, i.e., projections are treated as exogenous inputs that must be satisfied. In Loulou et al. (2005b) page 58, this model is depicted in the following simplified form:

$$\begin{aligned} &\min c'X \\ &s.t. \sum_k CAP_{k,i}(t) \geq DM_i(t) \quad i = 1,\dots,I;\ t = 1,\dots,T \\ &and \quad BX \geq b \end{aligned} \tag{3}$$

where X is the vector of decision variables, I is the number of demand categories, T is the number of periods (e.g., years), and $DM_i(t)$ is demand for demand category i in period t. The *installed* capacity of end-use technology k capable of satisfying $DM_i(t)$ is denoted $CAP_{k,i}(t)$, and is determined as a function of the X's[3]. In this mode, decisions for the energy system are interpreted as being made by a unitary "cost minimizing social decision maker." In inelastic mode, the model is modified as discussed in the previous section, so that the objective function is interpreted as "net social surplus," which is maximized to obtain a partial equilibrium solution.

[3] In this type of application, VMT is measured in millions of vehicle miles traveled (MVMT), and vehicles are measured in thousands of units. In this equation, it is assumed that $CAP_{k,i}(t)$ has been computed so as to have the same units as $DM_i(t)$. In fact, the implementation details for (3) differ somewhat from the literal depiction.

For this paper we developed a TIMES model (called COCHIN[4]) that limits the scope of decision making to household purchase and usage of vehicles, i.e., purchase of end-use technologies (vehicles), as well as fuel, to satisfy exogenously specified VMT demand (i.e., the inelastic case). We now provide a highly stylized example for purposes of illustration. At the highest level, the model seeks to minimize the net present value (NPV) of total system costs over a specified time horizon, for all regions:

$$NPV = \sum_{r=1}^{R} \sum_{y \in YEARS} \left(1 + d_{r,y}\right)^{REFYR-y} \times ANNCOST(r,y) \qquad (4)$$

where R is the number of regions (indexed by r), $YEARS$ is the set of years (indexed by y) that have costs that should be included, $d_{r,y}$ is a general discount rate, $REFYR$ is the reference year for discounting, and $ANNCOST(r, y)$ is the cost for region r in year y. In what follows, we drop the r index for simplicity, and assume a constant discount rate. Generally speaking, annual costs can be subdivided into: investment costs, fixed costs, variable costs, and salvage costs.[5] For details, see Loulou et al .(2005a) (page 145).

As noted, investment costs are incurred from making capital expenditures on technologies to create the necessary capacity for converting energy commodities into outputs. We now provide a specific example using variable names intended to reflect the style used in TIMES documentation. A decision variable *VAR_NCAP(v, k)* would be the number of units of technology type *k* purchased in year *v,* with associated capital cost denoted *INVC(v,k)*. For example, *VAR_NCAP(2006, gas-car)* and *VAR_NCAP(2006, elec-car)* would be the total number of new gasoline and electric cars purchased in 2006, respectively. These would comprise additional (new) capacity for satisfying VMT in 2006, and also in future years (determined by the respective vehicle lifetimes). The index *v* is used because vehicles are “vintaged,” i.e., model year 2006 vehicles have characteristics (e.g., fuel efficiency) that stay the same over their lifetimes.

The model also includes variables of the form *VAR_ACT(v,t,k)* to represent activity levels associated with technologies/processes. For example, *VAR_ACT(2006, 2007, gas-car)* could denote the activity level for a 2006 gasoline car in calendar year 2007 (in MVMT). This activity is limited by the capacity that was purchased in 2006, e.g.,

$$VAR\_ACT(2006, 2007, gas-car) \leq 16.8 \cdot VAR\_NCAP(2006, gas-car) \qquad (5)$$

[4] COnsumer CHoice INtegration

[5] In elastic mode, additional terms are added to represent (negative) changes in consumer surplus associated with shifts in end-use service demands that could occur under alternative scenarios, as discussed in section 2.

where the number of gasoline cars (000) is multiplied by the constant 16.8 (MVMT/000_cars) to convert capacity units into activity units. In theory, there could be a variable cost associated with *VAR_ACT(v,t,k)*, e.g., for operations and maintenance; however, this is assumed to be zero for simplicity.

The primary variable cost of interest is from fuel usage, e.g., *gas-car* requires an energy input commodity (*gasoline*) to generate this activity (measured in petajoules PJ), for the given year. This input is produced by another technology (*gas-station*), which for this example we treat as a production process requiring no additional capital investment. From the system perspective, there is a variable cost *VC(y,k)* from using a fuel technology *k* to produce, transport, and deliver fuel, which for *gas-stations* is measured in \$M/PJ. Extending the previous example for calendar year 2007, the model must include an equation that computes the total amount of gasoline produced and used by all vehicles in 2007 (in PJ), i.e., the activity level *VAR_ACT(2007, 2007, gas-station).*[6] One specific term included in this equation would be $\left(1 / EFF(2006, gas-car) \right) \bullet VAR\_ACT(2006, 2007, gas-car)$, where *EFF(2006, gas-car)* is the efficiency of a model year 2006 gasoline car, in MVMT/PJ. (These types of equations would appear in the matrix *B* shown above).

In this simplified model, the annual cost can be represented by:

$$
\begin{aligned}
ANNCOST(y) = & \sum_{k \in VEH\_TECH} INVC(y,k) VAR\_NCAP(y,y,k) \\
& + \sum_{k \in FUEL\_PROD} VC(y,k) VAR\_ACT(y,y,k)
\end{aligned}
\tag{6}
$$

where *VEH_TECH* and *FUEL_PROD* denotes sets of vehicle technologies and fuel production technologies, respectively.[7] In this stylized example we have included only investment costs and variable costs from fuel usage, ignoring many other types of costs (e.g., salvage costs, fixed operations/maintenance, etc.). Moreover, TIMES has many other features that we have omitted, e.g., technology-specific discount/hurdle rates, and complex features that subdivide periods into multiple years and time slices (e.g., day versus night). Main features to note are that the decision maker is assumed to have complete knowledge of all costs over the entire planning horizon, i.e., perfect foresight, and as noted, we are considering the inelastic case where the decision maker is minimizing total monetary costs.

---

[6] Note that this could include gasoline required by multiple types of vehicle technologies, not just gasoline vehicles, e.g., gasoline and plug-in hybrids. Also, *v* is set to *t* in this example because we are not treating gasoline stations as vintaged technologies.

[7] However, TIMES actually implements *ANNCOST(y)* using a more complex approach, so that it can always be viewed as the "annualized cost" associated with all decisions. This will be addressed in more detail in section 4.7.

To illustrate a point, note that *NPV* can be decomposed in a variety of ways, and alternative expressions are possible. For example, it is to isolate *NPV* of all costs associated with 2006 model year gasoline cars. Assume that the reference year is also 2006, and that the vehicle lifetime is *L* years, yielding:

$$
\begin{aligned}
&NPV(2006, gas-car) = \\
&\quad INVC(2006, gas-car) VAR\_NCAP(2006, 2006, gas-car) \\
&\quad + \sum_{y=2006}^{2006+L-1} d^{(y-2006)} \frac{VC(y, gas-station)}{EFF(2006, gas-car)} VAR\_ACT(2006, y, gas-car) \qquad (7)
\end{aligned}
$$

This is equivalent to how MA^3T expresses vehicle purchase cost from the perspective of a consumer making a decision in 2006. However, in MA^3T this is only one factor that affects the consumer's purchase decision. Additional factors can be included if they can be interpreted as "generalized costs" or "disutilities" that are measured in the appropriate units. Doing this correctly requires an understanding of the relationship between such costs and the original utility maximization problem defined in (1), and also provides the means for understanding the relationship between TIMES models and discrete choice models. This is the subject of the next section.

4. Theory-based Models of Consumer Decision Making

The sub-sector of the energy system considered here is household vehicle choice and usage, with the goal of improving the realism of behavioral response to market changes in TIMES models. Because analysis of personal vehicle demand has long been an important topic, there is a large literature on development of behavioral models and methodologies to address this problem that goes back decades, spanning multiple academic fields including economics, transportation, and marketing science. Vehicles are durable goods, infrequently purchased by households in very small quantities. Because the fundamental choice at the individual level is discrete (choose one option from a competing set), traditional approaches of modeling demand based on a single, representative consumer lack important structural features for adequately modeling aggregate-level demand (as illustrated by the comparison of Figures 1 and 2). Discrete and discrete-continuous models have been developed to address these issues, but the methodologies are not immediately compatible with TIMES models.

However, all of these models can be derived from the same underlying model in (1) by adding various structural assumptions. This section reviews aspects of the theory that illustrate how the two types of models can be viewed in a theoretically consistent way. Useful references are Deaton and Muellbauer (1980) and Pollak and Wales (1978).

An initial step is to partition commodities into groups (and perhaps partitioned further into sub-groups) based on their similarity in meeting consumer needs/benefits, yielding a tree structure. For example, the top level might consist of groups such as food, housing, clothing, etc. Two commodities within the same group would tend to be much closer substitutes (e.g., butter versus margarine) than would two commodities in different groups (butter versus golf balls). The utility function in (1) associated with such a structure may be assumed to have properties (separability) that impose restrictions on how preferences for goods in different groups may interact.

These and other assumptions allow household spending decisions to be viewed as a multi-stage budgeting process, where expenditures are first allocated across groups, and then group-level budgets are allocated across commodities within the same group. At higher levels of the "utility tree," total expenditure allocated to an entire group of commodities can be viewed as a *composite good* with an appropriate *price index*. At lower levels, modeling choices among competing alternatives within the *same* group can take into account more detailed preference tradeoffs, without the need to take into account similarly detailed tradeoffs between individual commodities in *different* groups (e.g., butter versus golf balls). These are the same notions used in developing CGEs to address the energy sector in relation to the larger economy: see, e.g., the utility tree in Figure 2 of Schaefer and Jacoby (2005) for the demand side, and Figure 1 of Sue Wing (2006) for the supply side, respectively.

For our purposes we specifically consider how this theory is applied to develop discrete and discrete-continuous models for analyzing demand for products within a specified group (or product category).

4.1 Demand in a Product Category

The specific problem being considered is household demand for personal vehicles, but other end-use demand categories could be addressed in a similar way. (Because the treatment presented here can be applied more generally, we may alternate between using terms specific to vehicles versus more general terminology.) Starting with (1), goods are partitioned into two groups: vehicles, and all other goods. Commodities within the product category being analyzed are referred to as *inside goods*. All other goods (*outside goods*) are collapsed into a *composite, Hicksian, numeraire good*, i.e., a single good with a normalized price of 1, so that consumption of all other goods is denoted by the expenditure z.

Assume there are J possible vehicle types (indexed by $j = 1, \ldots, J$). In the context of this paper, these correspond to competing vehicle technologies that vary based on the type of fuel(s) used, efficiencies, costs, and perhaps other characteristics (as in Figure 1). In addition, consideration will be limited to those households purchasing new vehicles in a given year, along with various other restrictions. However, the level of detail and other features could vary widely in other applications, e.g., vehicles could be defined using vehicle classes (e.g., subcompact car, minivan, pickup, large SUV, etc.), or more detailed vehicle definitions (e.g., make, model, engine type, etc.). Another possibility is considering both used and new vehicle purchase behavior.

Next, we provide a multi-step development of a framework starting with the following assumption: we consider an individual household making vehicle-related decisions using a planning horizon of one year, where the unit price for a vehicle of type *j* is p_j, and the household has an expenditure budget of *y* (e.g., annual income), so that the decision problem is defined as:[8]

$$\begin{aligned} &\max_{\mathbf{q},z} U(\mathbf{q},z) \\ &s.t. \sum_{j=1}^{J} p_j q_j + z = y \\ &\mathbf{q}, z \geq 0 \end{aligned} \tag{8}$$

The decision problem in (8) is only a starting point, requiring additional assumptions to produce specific behavioral models, e.g., the direct utility function in (8) is completely general with no special structure to address behavior-related factors. These are developed in the next sections.

4.2 The Discrete-Choice-Only Case

This section reviews assumptions and key features from the extensive literature on developing *discrete choice models* consistent with (8) at the individual household level, where, e.g., a household chooses one alternative from among *J* available options in a product category. The first requirement is to clarify the interpretation of q_j (whose units have thus far been left unspecified). If p_j is assumed to be the annual (fixed) cost of *renting* vehicle *j*, then q_j would be a measure of the *number* of vehicles of type *j* rented for the year. Without additional restrictions (and with an appropriately defined set of *J* vehicles), (8) could be used to represent a household's annual decision of what vehicle *fleet* to hold (addressing both the number and types of vehicles).

However, in this section we consider the case where a household chooses one alternative from among *J* available options so as to maximize its utility. In this notation, $q_c^* = 1$ for the (optimally) chosen option *c* (and $q_j = 0$ for $j \neq c$), and $U(\mathbf{q}^*, z^*) = U(0, \ldots, 1_c, \ldots, 0, z^*) = U(0, \ldots, 1_c, \ldots, 0, y - p_c)$ denotes the household's (maximum) utility. The behavioral interpretation is that the household derives a certain amount of "utility" from "consuming" vehicle *c* during the one-year period, and also from consumption of $z^* = y - p_c$ dollars worth of all other goods. Note that this completely ignores the variable cost associated with driving, an important issue to be discussed later. Next, we briefly summarize standard results for this case to provide context (and perhaps a

---

[8] Additional simplifying assumptions to address other behavioral questions that might arise are: the household repeats this decision process every year, all vehicle types are freely available to the household for a given year, and that vehicle choices can be made independently of choices in previous years. In other words, this is a "vehicle holdings choice model" with no transactions cost. (Later we will change some of these assumptions.)

review for those having prior familiarity with the topic). For a definitive reference with a rigorous treatment, see (Manski and McFadden 1981).

The notation for the direct utility function can be extended to include multiple types of behaviorally related factors. For example, if direct utility is assumed to be a function of observable vectors of product attributes ($\mathbf{x}_j$, *j=1,...,J*), an observable vector of household characteristics (**d**), and a vector of preference parameters (β), it can be written as $U(\mathbf{q},z;\mathbf{x},\mathbf{d},\beta)$. Note that including **d** implies that the solution to (8) could be used to represent market behavior that, e.g., differs across household demographic segments, and results would need to be combined to represent total market behavior.

Assumptions adopted thus far ensure that (8) can be solved by first determining the level of utility obtained under the *J* possible purchase options, and then seeing which one is *largest*. Additional behavior-related assumptions on *U()* play a role in this process. First, theory suggests that utility would depend *only on the attributes of the vehicle being consumed*. Second, utility is typically assumed to be non-decreasing in *z*, allowing the constraint in (8) to be expressed as an equality rather than an inequality (≤). Under these assumptions the (conditional, indirect) utility can be written as:

$$V(y-p_j;\mathbf{x}_j,\mathbf{d},\beta)=U(0,...,1_j,...,0,y-p_j;\mathbf{x},\mathbf{d},\beta),\ j=1,...,J, \tag{9}$$

and the household chooses the vehicle *c* for which this value is largest.

In this example the outcome is deterministic, but a major goal is to produce models appropriate for econometric/statistical analysis of data on consumer behavior. This is achieved by assuming the direct utility is also a function of additional factors unobservable by the analyst. It is typical to assume that, e.g., unobservable attributes yield a disturbance term for each of the *J* vehicles, i.e., $\varepsilon_j, j=1,...,J$. Additional unobserved factors (ξ) related to household characteristics and/or preferences may also be included, yielding $U(\mathbf{q},z;\mathbf{x},\mathbf{d},\beta,\varepsilon,\xi)$. For now, consider the case of $U(\mathbf{q},z;\mathbf{x},\mathbf{d},\beta,\varepsilon)$: repeating and extending the previous analysis yields *choice probabilities* given by:

$$\pi_c=\mathrm{Prob}\left\{V(y-p_c;\mathbf{x}_c,\mathbf{d},\beta,\varepsilon_c)\geq V(y-p_j;\mathbf{x}_j,\mathbf{d},\beta,\varepsilon_j),j=1,...,J\right\} \tag{10}$$

Because it is theoretically valid (under appropriate assumptions) to define the decision problem in terms of indirect rather than direct utility, publications frequently rely on (10) to provide the behavioral rationale for discrete choice models. Individuals are described as making choices consistent with *Random Utility Maximization* (RUM), where "random utility" is depicted as

$$V_j = \bar{V}_j + \varepsilon_j,\ j = 1,\ldots,J \tag{11}$$

but, with little reference to the underlying connection to (8), or (frequently) with minimal consideration of underlying theory-based requirements. The next section returns to a discussion of theoretical considerations: the remainder of this section reviews additional details that support empirical application of discrete choice models.

The most well-known discrete choice model (multinomial logit) assumes that the ε_j 's in equation (11) are iid Gumbel (with scale parameter μ), yielding the following closed form for choice probabilities[9]:

$$\pi_c = \frac{e^{\mu \bar{V}_c}}{\sum_{j=1}^{J} e^{\mu \bar{V}_j}},\ c = 1,\ldots,J \tag{12}$$

For practical empirical work, $\bar{V}$ is frequently specified as a linear-in-parameters form that may use any set of scalar functions of the form $f_k(p_j, \mathbf{x}_j, \mathbf{d})$ as explanatory variables, i.e.,

$$\bar{V}(p_j, y, \mathbf{x}_j, \mathbf{d}, \beta) = \sum_{k=1}^{K} \beta_k f_k(p_j, y, \mathbf{x}_j, \mathbf{d}) \tag{13}$$

Other functional forms might assume a correlation structure among the disturbance terms, yielding nested multinomial logit models (discussed later) or multinomial probit models. Random factors that capture unobserved (and perhaps correlated) variation in preference parameters can be modeled using mixed logit models.

4.3 Extension to Discrete-Continuous Choice

The pure discrete choice framework omits a potentially important behavioral consideration: a widely accepted tenet in the travel behavior literature is that household utility is not derived directly from vehicle ownership per se' (households do not literally "consume" vehicles), but is more properly viewed in association with the flow of *mobility services* that a vehicle provides. In the context of the previous section, a straightforward modification is to adopt an alternate interpretation of q_j: let q_j represent (annual) vehicle miles traveled (VMT), where p_j then represents the variable cost of driving (measured in \$/mile).

[9] For details on the definition of the Gumbel distribution used here, and a derivation of (6), see section 5.2 of Ben-Akiva and Lerman (1985).

Theory-based considerations aside, addressing VMT is important for our purposes because E4 models measure end-use demand for vehicles in these units, and VMT is the direct antecedent to emissions. At the same time, the (discrete) choice of vehicle type continues to be a critical concern, placing us squarely in the domain of *discrete-continuous* models.

The solution approach from the previous section is immediately extends to this case, but with an important difference. The *J* conditional optimization problems (i.e., *J* problems defined by assuming alternative *j* is chosen, for *j = 1,...,J*) are represented by

$$\begin{aligned}&\max_{q_j, z} U(0,..,q_j,...0,z;\mathbf{x},\mathbf{d},\beta,\varepsilon_j)\\ &s.t.\ p_j q_j + z = y \\ &q_j, z \geq 0\end{aligned} \tag{14}$$

but, unlike before, each problem must be solved to determine the optimal value q_j^* (conditional on *j* being chosen), and to determine *which* vehicle *c* is chosen, q_j^* must be substituted back into *U()* to obtain conditional indirect utility. More specifically, when developing behavioral models via this approach, the first step yields a *conditional ordinary demand function* $q_j^*()$, and the second step yields a *conditional indirect utility function V()*.

However, as implied in the previous section, researchers developing empirical models frequently take an alternate approach: instead of choosing *U()* and performing these steps, they choose a functional form for *V()*. In this approach it would in fact be possible to focus *only* on discrete choice without ever considering VMT, even though the formal framework is discrete-continuous. However, conditional demand (VMT) functions can always be derived from *V()* using Roy's identity (to be discussed later).

4.4 Aspects of MA^{3}T and TIMES to be addressed

The previous sections establish the initial theoretical results required to support the stated goal of this paper: integrating behavioral content from MA^{3}T into a TIMES framework. There are additional steps required, but before discussing them it will be helpful to first review some specific features of MA^{3}T and TIMES that have a bearing on how we proceed.

First, consider an important implication of what has been established thus far: *these results suggest that discrete and continuous choices are jointly determined, and, in general, the choice of VMT will differ depending on which vehicle is chosen*. This is a natural outcome of the assumption that households must allocate a limited monetary budget to cover both their vehicle-related travel needs and all other expenditures.

However, *this is inconsistent with the modeling assumptions adopted by both MA³T and TIMES* (as will be described). In particular, *both MA³T and TIMES (in inelastic mode) treat household VMT as exogenously determined and independent of vehicle type choice*. The details are a bit more complicated for the elastic case; however, the main goals of this paper are to (i) establish the relevant theoretical framework, and (ii) apply it for the inelastic case, which is much more widely used.

Another aspect that can raise complications: long-term vehicle choice and usage decisions require consideration of *both* fixed and variable costs, whereas the literature frequently considers only one or the other, for reasons that will be more apparent shortly. Addressing this feature requires extending (8), so that the household decision problem becomes:

$$\begin{aligned}&\max_{\mathbf{q},z} U(\mathbf{q},z)\\&s.t. \sum_{j=1}^{J} p_j q_j + \sum_{j=1}^{J} r_j I(q_j) + z = y\\&\mathbf{q},z \geq 0\end{aligned} \tag{15}$$

where (as before) p_j is the (variable) cost in \$/mile for vehicle type *j*, but in addition, r_j is a (fixed) annual rental cost (in \$), and $I(q)$ is an indicator function, i.e., $I(q) = 1$ if $q > 0$, and $I(q) = 0$ if $q = 0$. In the more standard formulation (8) the expenditure constraint is linear, which has many analytical advantages. In contrast, the constraint in (15) is nonlinear which is analytically more complex (yielding the aforementioned complications).

But, before addressing (15) more directly, the next section reviews material from the seminal paper by Hanemann (1984)(hereafter, "Hanemann"). Although based on (8) rather than (15), it provides useful insight useful for extending the discussion. For completeness, note that an important early reference addressing discrete-continuous choice with fixed and variable costs is Dubin and McFadden (1984), who develop models for household purchase and usage of energy-related appliances. In addition, Bento et al. (2009) estimate models for the specific case we are considering here (vehicle type and VMT choice).

A third feature shared by MA³T and TIMES is that both assume a household decision framework with a relatively long time horizon (as will be described) and perfect foresight. Interestingly, although both of these models happen to share this feature, this is, in fact, a rather uncommon assumption in the empirical choice modeling literature, which is generally more consistent (albeit perhaps implicitly) with the (one-year) framework discussed thus far. This will be addressed in section 4.7

To summarize, we identified three features shared by MA[3]T and TIMES that differ from more standard econometric behavioral models, and which also represent potential complications that need to be taken into account: (i) exogenous VMT, (ii) inclusion of both fixed and variable costs, and (iii) a long time horizon. Interestingly, the first two of these can interact in ways that can affect how these theoretical results might be applied (see section 4.6).

4.5 Hanemann (1984) Discrete-Continuous Framework

The framework developed thus has been extended by Hanemann to include features that are helpful for developing empirical models that are, simultaneously, practical, behaviorally meaningful, and consistent with theory. First, the utility function in (14) can be further refined to take the following form:

$$U(\mathbf{q},z;\mathbf{x},\mathbf{d},\beta,\varepsilon)=U(q_1,\ldots,q_J,z;\ \psi_1(b_1,\varepsilon_1),\ldots,\psi_J(b_J,\varepsilon_J)) \qquad (16)$$

where ψ_j is a *quality index* for alternative *j* with utility-related interpretations (to be discussed). It is a function of a vector b_j of explanatory variables, and a random disturbance term ε_j (both of which are associated with alternative *j*). The vector b_j can be an arbitrary function of vehicle attributes (x_j), demographics (**d**), and preference parameters. In addition, he identifies a general family of utility functions of the following form

$$U=U\left[\sum q_j,\sum\psi_j q_j,\sum\psi_j I(q_j),\ z\right] \qquad (17)$$

where $I(q)$ is an indicator function that equals *1* if $q > 0$ and *0* if $q = 0$. This family and its subclasses have a very important theoretical property: it incorporates a particular type of preference structure that ensures the purchase of *exactly one product* from the category. Although subtle, early development of discrete choice models (e.g., as described in section The Discrete-Choice-Only Case) essentially *imposed* this assumption exogenously, i.e., discrete choice was not necessarily an outcome directly attributable to the preference structure embodied in *U()*.

Moreover, in moving from (16) to (17), the role of the quality index ψ_j is made explicit, appearing in expressions of the form $\psi_j q_j$ or $\psi_j I(q_j)$, which have clear behavior interpretations. In the first case, ψ_j represents a (constant) *rate* of *sub-utility flow* from consuming *j*, on a per-unit basis.[10] In the second case, ψ_j represents a *fixed* amount of sub-utility that is generated only if alternative *j* is chosen.

[10] This is interpreted as a sub-utility, because the quantity appears in an argument of *U()*, which must be combined with other such quantities to obtain total utility.

For empirical work, Hanemann suggests two alternative forms for ψ_j:

$$\psi_j(b_j,\varepsilon_j)=\bar{\psi}_j(b_j)e^{\varepsilon_j}=\exp\left(\alpha_j+\sum_k\beta_k b_{jk}+\varepsilon_j\right), \tag{18}$$

and

$$\psi_j(b_j,\varepsilon_j)=\bar{\psi}_j(b_j)+\varepsilon_j=\alpha_j+\sum_k\beta_k b_{jk}+\varepsilon_j, \tag{19}$$

i.e., $\ln\left[\bar{\psi}_j(b_j)\right]$ and $\bar{\psi}_j(b_j)$ take linear-in-parameters forms in (18) and (19), respectively, where β_k 's are preference parameters, and α_j 's are alternative-specific constants. Hanemann provides many examples suitable for empirical work, focusing primarily on two special cases of the general family in (17):

$$U(\mathbf{q},z;\psi)=U^*\left(\sum\psi_j q_j,z\right) \tag{20}$$

and

$$U(\mathbf{q},z;\psi)=U^*\left(\sum q_j,\ z+\sum\psi_j q_j\right). \tag{21}$$

In both cases, *U*()* is a bivariate utility function, where the first argument is associated with the inside goods and the second argument is associated with the outside good. The choice of a mathematical form for *U*()* specifies the preference tradeoff between "units" of consumption in the two groups[11].

The class (20), where brands are considered "perfect substitutes," is a frequently studied case. The class (21) has been characterized as *cross-product repackaging*, and will play a role in subsequent discussions. For both classes it is worth considering the effect on utility of making a marginal change in one of the behavioral factors b_{jk}:

$$\frac{\partial U}{\partial b_{jk}}=\frac{\partial U^*}{\partial u_A}\frac{\partial u_A}{\partial b_{jk}}=\frac{\partial U^*}{\partial u_A}q_j\beta_k, \tag{22}$$

where u_A denotes the argument of *U* containing b_{jk}. The derivative in (22) is zero (i.e., there is no change in utility) unless $q_j>0$, i.e., there is no change in utility from a change in b_{jk} unless alternative *j* is being consumed. This property (*weak complementarity*) is a logical behavioral requirement, and consistent with these two classes. Moreover, the overall rate of utility change is a product of two terms: one associated with a change in *total*

[11] A potential question is what role units of measurement might play in each argument, but as implied previously, we would generally interpret the quantity in each argument to represent the total sub-utility obtained (directly) from each group.

sub-utility u_A, and one due to a conditional change in utility associated with the specific inside good being consumed.

For (20), it can be shown the alternative with the smallest value of p_j/ψ_j will be chosen, i.e., the alternative with the smallest "price per marginal utility". Similarly, for (21) it can be shown that the alternative with the largest value of $\psi_j - p_j$ will be chosen, i.e., the alternative with the largest "net utility per unit" (implying that ψ_j in this class is measured in dollars per unit). The main implication is that (for these classes) *discrete choice is independent of the functional form of U*()*, and depends only on prices and the index functions. However, the (conditional) *continuous* choice q_j *does* depend on the functional form *of U*()*.

In particular, if (18) is used in association with class (20), and ε_j's are iid Gumbel (with scale parameter μ), then all discrete choice models in the class are given by the same multinomial logit model:

$$\pi_j = \frac{e^{\mu v_j}}{\sum_{l=1}^{J} e^{\mu v_l}} \tag{23}$$

where $v_j = \ln[\bar{\psi}_j(b_j)] - \ln(p_j)$. Similarly, combining (19) with (21) yields (23) with $v_j = \bar{\psi}_j(b_j) - p_j$.[12] Note that, when estimating (23), the parameter μ (which appears in the terms $-\mu\ln(p_j)$ or $-\mu p_j$ for class (20) or (21), respectively) can be interpreted as a price coefficient. However, in statistical procedures it cannot be separately identified from the parameters (α,β) in (18) or (19). This means that, at the very least, a decision on whether to use $-\mu\ln(p_j)$ or $-\mu p_j$ when specifying a discrete choice model for applications with a potential continuous-choice aspect can be viewed as making an implicit assumption about the form of the underlying direct utility function.

We next summarize additional results that address relationships among direct utility, indirect utility, the continuous choice, index functions, etc. When solving (14) using classes of the form (20) and (21), the conditional utility maximization problem can be characterized as:

[12] Hanemann uses a different definition for the Gumbel distribution than the one we use here, so that his version of (23) uses v_l/μ in the exponents.

$$\begin{aligned}&\max_{w_I, w_{II}} \; U^*(w_I, w_{II})\\&s.t.\; p_I w_I + w_{II} = y\\&w_I, w_{II} \geq 0\end{aligned} \tag{24}$$

Applying the usual derivation from microeconomics (and standard assumptions), solving (24) yields ordinary demand functions $w_I^*(p_I, y)$ and $w_{II}^*(p_I, y)$, and the indirect utility function:

$$V^*(p_I, y) \equiv U^*\left[w_I^*(p_I, y), w_{II}^*(p_I, y)\right]. \tag{25}$$

As noted previously, one instead decides to directly specify rather than solve (24), the ordinary demand function $w_I^*(p_I, y)$ can be obtained using Roy's identity[13]:

$$w_I^*(p_I, y) = -\frac{\partial V^*(p_I, y) \big/ \partial p_I}{\partial V^*(p_I, y) \big/ \partial y} \tag{26}$$

Hanemann shows that, if $w_I^*(p_I, y)$, $w_{II}^*(p_I, y)$, and $V^*(p_I, y)$ are valid solutions to (24), then the following are valid solutions to (20):

$$\begin{aligned}\overline{q}_j(p_j, y; \psi_j) &= \frac{1}{\psi_j} w_I^*\left(\frac{p_j}{\psi_j}, y\right)\\\overline{z}(p_j, y; \psi_j) &= w_{II}^*\left(\frac{p_j}{\psi_j}, y\right)\\\overline{V}_j(p_j, y; \psi_j) &= V^*\left(\frac{p_j}{\psi_j}, y\right)\end{aligned} \tag{27}$$

Similarly, the following are valid solutions to (21):

$$\begin{aligned}\overline{q}_j(p_j, y; \psi_j) &= w_I^*\left(p_j - \psi_j, y\right)\\\overline{z}(p_j, y; \psi_j) &= w_{II}^*\left(p_j - \psi_j, y\right) - \psi_j w_I^*\left(p_j - \psi_j, y\right)\\\overline{V}_j(p_j, y; \psi_j) &= V^*\left(p_j - \psi_j, y\right)\end{aligned} \tag{28}$$

[13] Alternatively, one could start by assuming a form for w_I^* and derive V^* by solving the partial differential equation embodied in (17), as in, e.g., Mannering and Winston (1985).

The usual properties of indirect utility functions are that they are decreasing in the first argument, and increasing in the second argument. This is the reason for the earlier statements regarding discrete choice: Alternatives with the lowest values of $\frac{p_j}{\psi_j}$ and $p_j - \psi_j$ are chosen for cases (20) and (21) respectively.

However, note that this result depends heavily on the fact that *fixed costs have been omitted*. If the preceding results were re-derived based on (15) rather than (14), the solutions for classes (20) and (21) would be, respectively,

$$\begin{aligned}
\bar{q}_j(p_j, y; \psi_j) &= \frac{1}{\psi_j} w_I^* \left(\frac{p_j}{\psi_j}, y - r_j \right) \\
\bar{z}(p_j, y; \psi_j) &= w_{II}^* \left(\frac{p_j}{\psi_j}, y - r_j \right) \\
\bar{V}_j(p_j, y; \psi_j) &= V^* \left(\frac{p_j}{\psi_j}, y - r_j \right)
\end{aligned} \tag{29}$$

and

$$\begin{aligned}
\bar{q}_j(p_j, y; \psi_j) &= w_I^* \left(p_j - \psi_j, y - r_j \right) \\
\bar{z}(p_j, y; \psi_j) &= w_{II}^* \left(p_j - \psi_j, y - r_j \right) - \psi_j w_I^* \left(p_j - \psi_j, y - r_j \right) \\
\bar{V}_j(p_j, y; \psi_j) &= V^* \left(p_j - \psi_j, y - r_j \right)
\end{aligned} \tag{30}$$

.

Note that, because including both fixed and variable costs implies simultaneous changes to both arguments, the strong result that discrete choice depends only on (variable) prices and quality indexes no longer applies[14]. This will be addressed again below.

We next turn to an exploration of models at a higher level of detail. In addition to the previous general results, Hanemann provides many specific functional forms that yield practical empirical models. In our work we have found it helpful to work with direct utility functions if possible, because the behavioral interpretation of parameters can be much more straightforward. However, as implied above, much of the empirical work in the literature is based on indirect utility functions and/or demand functions, so not surprisingly, most of his examples are of this type. In fact, he specifically discusses only one

[14] For completeness, note also that, either way, these results exclude the notion of a fixed utility "bump" associated with the purchase of a specific vehicle (i.e., utility comes only from driving the vehicle).

form of direct utility, but it is one we find useful. This and other examples are discussed in the next sections.

4.6 The Blackburn Model

Hanemann's equation (3.29) is the following direct utility function, which he credits to Blackburn (1970)

$$u(q,\psi,z)=\sum_j q_j\left(1+\ln\theta-\ln\sum_j q_j\right)+hz+\sum_j \psi_j q_j,\ \ \theta>0,\ h>0, \tag{31}$$

where ψ_j takes the additive form in (19) to generate a MNL discrete choice model. He also notes that (31) does not explicitly appear, but is "implicit in his analysis."[15] Other researchers have also used this form for reasons that will become apparent, but make a variety of assumptions on how stochastic structure is introduced. For the current discussion, (31) will be treated as deterministic.

First, note that the two additional behavioral parameters introduced by (31) are θ and *h*. Second, Hanemann assigns (31) to a category of "other utility models"; however, when written in a slightly different but equivalent form, it belongs to the class (21) of cross-product repackaging models. Specifically, consider the bivariate utility function of the form

$$U^*(w_I,w_{II})=w_I(1+\ln\theta-\ln w_I)+hw_{II},\ \theta>0,h>0. \tag{32}$$

In this form, utility is additively separable between inside and outside goods. Utility units appear to be measured in units of consumption of the inside good, so *h* could be interpreted as a scale factor that converts units of the outside good ($) to units of the inside good (in our case, miles). Directly applying (32) to (21) yields the direct utility function

$$u(q,\psi,z)=\sum_j q_j\left(1+\ln\theta-\ln\sum_j q_j\right)+h\left(z+\sum_j \psi_j q_j\right), \tag{31'}$$

a slight variation of (31), and the conditional direct utility is

$$u(q_j,\psi_j,z)=U^*(q_j,z+\psi_j q_j)=q_j\left(1+\ln\theta-\ln q_j\right)+h\left(z+\psi_j q_j\right). \tag{33}$$

First consider the form of (33) and its interpretation in the context of cross-product repackaging. The first argument of *U*()* is a function of VMT only (i.e., there is no quality index), and yields the first term on the right-hand side of (33). For this term, the subscript *j*

[15] Interestingly, Blackburn developed this model in the context of analyzing aggregate consumer travel demand.

has no relevance: we interpret this term as yielding the contribution to household utility of VMT as a *commodity*, i.e., the utility of vehicle VMT as a "basic mobility service," which is the same *regardless of vehicle type.* This term has a clear behavioral interpretation worth mentioning. First, it approaches *0* as *q* approaches zero. It is increasing in *q*, but at a decreasing rate, reaching its maximum value at θ, with a slope of zero. In other words, θ is the maximum (sub-)utility that can be attained from VMT in its role as a commodity.

The second argument of *U*()* is associated with utility obtained from consumption of "all other goods," which is expressed as the second term on the right-hand side of (33). The interpretation of $z+\psi_j q_j$ is that vehicle *j* has certain qualities that generate additional services under the category "all other goods" (as a function of VMT) in addition to fulfilling its role as a provider of "basic transportation services."

As an example, suppose that vehicle *j* has special HOV or toll lane privileges so that the vehicle may be used in these lanes without paying congestion fees. This yields direct monetary savings that, in effect, relaxes the budget constraint to allow additional expenditures on outside goods, thus generating additional utility. In addition to direct monetary savings, these privileges also provide additional discretionary time, which can be devoted to activities that also increase utility.

To complete this discussion we make another modification, rewriting (31') as

$$u(q,\psi,z)=\frac{1}{h}\sum_j q_j\left(1+\ln\theta-\ln\sum_j q_j\right)+z+\sum_j \psi_j q_j \qquad (31'')$$

which is equivalent to (31'), except that this form clearly highlights that utility has the same units as *z* ($). The conditional ordinary demand function and conditional indirect utility function associated with (31'') are

$$q_j^*(p_j,\psi_j,y)=\theta e^{\psi_j-p_j} \qquad (34)$$

$$V_j^*(p_j,\psi_j,y)=y+\frac{\theta}{h}e^{\psi_j-p_j} \quad . \qquad (35)$$

These yield additional observations. First, both household VMT demand and vehicle choice are unaffected by income in this model. Second, consider the case where $\psi_j=0$ for all *j*, so that vehicle quality differences have no impact on decision making. It is clear that the vehicle with the lowest operating cost (p_j) would be chosen, and that it would be driven further than any of the other vehicles. As p_j gets smaller, VMT demand approaches θ and total household utility approaches $y+\theta/h$ (the maximum possible utility, measured in dollar units).

Now, reintroducing ψ_j, it is clear that (as in earlier discussions) the vehicle with the largest value of $\psi_j - p_j$ is chosen, and VMT demand increases with increasing ψ_j. If ψ_j is specified as in (19) along with the usual assumptions, the discrete choice model is given by (23).

Now, suppose that fixed costs are included and the above analysis is repeated for (15). Interestingly, for this case the conditional VMT demand *does not change*. However, the conditional indirect utility function in this case is:

$$V_j^*(p_j,\psi_j,y) = y - r_j + \frac{\theta}{h} e^{\psi_j - p_j} \tag{36}$$

which, as discussed previously, complicates the determination of which vehicle is chosen. Although this form can be readily addressed using today's computational methods for model estimation, this result is unhelpful for the purpose of this paper.

However, it will now be helpful to *consider the implications of imposing an assumption that VMT is exogenously determined*. Let us continue to assume that (31'') represents a household's true utility, but that a constraint is added to impose an exogenously determined level of VMT: $\sum_j q_j = q^*$. In this case, the household's conditional utility under each of the *J* vehicle options is given by:

$$\begin{aligned} u(q_j,\psi_j,z) &= U^*(q^*, y - r_j - p_j q^* + \psi_j q^*) \\ &= \frac{1}{h} q^* \left(1 + \ln\theta - \ln q^*\right) + y - r_j - p_j q^* + \psi_j q^* \end{aligned} \tag{37}$$

In this case, the utility from the first term (basic transportation service) is the same for all vehicles and plays no role in the discrete choice. As before, the level of income also has no effect, and the chosen vehicle is the one with the smallest value of

$$r_j + p_j q^* - \psi_j q^* . \tag{38}$$

The first two terms of (32) are the total monetary cost of using the vehicle, and $-\psi_j q^*$ can be interpreted as the (negative of) the cross-product repackaging benefits due to the unique quality characteristics of vehicle *j*. Because (37) is based on a direct utility model for which utility is measured in dollar units, (38) implies a vehicle choice decision criterion of "minimizing costs." The first two terms are monetary costs measured in the usual way, but the third term must be interpreted as a *generalized cost* arising from other factors. This interpretation of "disutility" as a generalized cost is common in the transport literature, and is used by Lin and Greene (2011) for the MA^{3}T model.

Alternative terminology arises from dividing (38) by q^* and incorporating (19), yielding

$$lgc_j = \frac{1}{q^*} r_j + p_j - \psi_j = \frac{1}{q^*} r_j + p_j - \alpha_j - \sum_k \beta_k b_{jk} - \varepsilon_j \qquad (39)$$

where lgc_j denotes the *levelized generalized cost* for vehicle *j* (or, "generalized cost per mile"), extending the notion of levelized cost commonly used in the energy systems literature. An MNL model can be defined by using $v_j = -lgc_j$ in (23). However, if specific discrete choice model specifications are developed using (39), it would be important to recall this behavioral interpretation as a levelized cost.

As a technical matter, it is important to understand that the main factor leading to these results was the assumption/interpretation of cross-product repackaging, i.e., that the utility model is a member of class (21), and that these conclusions are true for any $U^*(w_I, w_{II})$ as long as there is some type of separability between the two categories of goods (inside versus outside):

$$u(q_j, \psi_j, z) = U^*(q_j, z + \psi_j q_j) = u_I(q_j) + u_{II}\left(z + \psi_j q_j\right) \text{ or } u_I(q_j) \bullet u_{II}\left(z + \psi_j q_j\right). \qquad (40)$$

Of course, such assumptions are typical for this type of analysis[16]. The Blackburn model assumes that $u_{II}()$ in the additive version of (40) is linear, but this linearity assumption is not what drives the result.

To review, equation (38) provides the essentials for a theory-based approach to solving the specific problem addressed in this paper: Incorporating behavioral content from MA^{3}T into the TIMES model (when used in inelastic mode). In both approaches, the assumption that VMT is determined exogenously implies that discrete choice is determined on the basis of minimizing "generalized cost." This interpretation is aided by the cross-product repackaging framework, which is consistent with the notion VMT affects household utility in two different ways: (i) a direct impact from VMT as a "commodity" (which is the same regardless of vehicle type), and (ii) an indirect effect due to those features that differentiate vehicles from one another, through their impact on how utility can be generated via "all other goods." However, the tractable forms of equations (38) and (39) only arise under the additional assumption of exogenous VMT.

As has been shown, this assumption that vehicles with different characteristics would all be driven the same distance is inconsistent with the underlying economic theory (as well as empirical observation), and one cost of making this assumption is the loss of the conditional ordinary demand functions for VMT produced by the theory. This may be one reason for the apparent gaps between how discrete choice models are typically applied in

[16] Note that the last term in (40) can be transformed into an equivalent additive form by taking logs.

the econometrically oriented literature versus applied energy modeling, adding to the challenge of cross-fertilization of results. Although the exogenous VMT assumption works in our favor here, this issue re-emerges for the case of elastic demand and partial equilibrium modeling. We hope to extend these results to this case in future work.

4.7 *L*-Period Household Decisions in TIMES and MA^{3}T

The theory and results provided thus far were developed using a 1-year time horizon, as is typical in the discrete choice modeling literature. However, as shown in section 3, TIMES assumes a multi-period planning horizon, incorporates a discount rate to evaluate future costs in current dollars, and assumes that purchasing a vehicle can be viewed as an investment with a specified lifetime. MA^{3}T also simulates multi-period vehicle market behavior, and adopts a similar view of household decision-making. Both models share the following assumptions:

1. Households buy one vehicle at a time, with the expectation of holding the purchased vehicle for *L* years (the lifetime of the vehicle).

2. Vehicles are purchased in the new vehicle market, and are driven by households that purchase them for their entire lifetime. When vehicles are retired, households repeat the process.

Under these assumptions there is no such thing as a *used* (or *secondary*) *market* in either model. In addition, all households are essentially treated as one-vehicle households (i.e., there is no notion of multi-vehicle households where VMT can be allocated across vehicles), and all vehicles are identical except for body type (limited to two levels: car versus truck) and fuel technology (see, e.g., Figures 1 and 5). As already discussed, both models assume that all vehicles are driven the same distance (regardless of type), and that VMT is exogenously determined. Because any of these simplifying assumptions have behavioral implications household choice of vehicle fuel technology, they could the subject of future investigation.

In the remainder of this section, we address features related to the *L*-period planning horizon. As a preview (and focusing for the moment on monetary costs only), what we will show is that (at the individual household level) equation (7) in section 3 is the extension of equation (38) (the one-period result), to the *L*-period case.

A frequent assumption in the economics literature is that household utility is additively separable over periods (years) with a discount rate d (which represents time preference for utility), and an associated *discount factor* defined as $\rho = \frac{1}{1+d}$. Under this assumption, consider the (conditional) direct utility function for vehicle *j*:

$$U^L(\mathbf{q},\mathbf{z}) = \sum_{t=1}^{L} \rho^{t-1} U(q_t, z_t) \tag{41}$$

where the subscript *j* has been suppressed (for notational convenience, to allow introduction of subscript *t*), and *U()* is an appropriate one-period (bivariate) utility function of the type used previously. The bold notation here denotes *L*-vectors of consumption quantities, e.g., q_t is VMT for period *t*, and $\mathbf{q} = (q_1, \ldots, q_L)$.

The household is also assumed to have an expenditure constraint in each period with known income (y_t), and a known variable cost ($p_t q_t$) of using the vehicle. However, a complication is that the household can now be viewed as making a capital investment (*C*) in period *1* when the vehicle is first purchased (corresponding to the vehicle purchase price), which would have a differential impact on the period 1 budget constraint. This can be mitigated by an additional assumption: households are allowed borrow and lend money in the form of bonds, at a constant interest rate (*R*).

It can be shown that the initial formulation of the household's decision problem can be expressed as

$$\begin{aligned} &\max_{\mathbf{q},\mathbf{z}} U(\mathbf{q},\mathbf{z}) = \sum_{t=1}^{L} \rho^{t-1} U(q_t, z_t) \\ &\text{subject to} \\ &\quad \mathrm{PV}_c = PV_y \\ &\quad \mathbf{q},\mathbf{z} \geq 0 \end{aligned} \tag{42}$$

where PV_c is the present value of consumption expenditure, and PV_y is the present value of income, so that the constraint in (42) can be written as

$$\begin{aligned} PV_c \equiv C + p_1 q_1 + z_1 + \frac{p_2 q_2 + z_2}{1+R} + \frac{p_3 q_3 + z_3}{(1+R)^2} + \cdots + \frac{p_L q_L + z_L}{(1+R)^{L-1}} = \\ y_1 + \frac{y_2}{1+R} + \frac{y_3}{(1+R)^2} + \cdots + \frac{y_L}{(1+R)^{L-1}} \equiv PV_y \end{aligned} . \tag{43}$$

The Lagrangian for solving this problem is

$$L(\mathbf{q},\mathbf{z},\lambda) = \sum_{t=1}^{L} \rho^{t-1} U(q_t, z_t) + \lambda \left(PV_y - PV_c \right) \tag{44}$$

and from considering the first-order conditions it can be seen that additional simplification is possible by making a number of assumptions. First, assume that household's discount rate is the same as the borrowing interest rate, i.e., $d = R$, or equivalently, $(1+R)\rho = 1$. Second, assume that income and variable cost is the same for all periods: behaviorally, this could be interpreted as an assumption on the household's expectations for future income and fuel prices.

Under these conditions, it is possible to show that the constraint in (43) can be replaced by

$$PV_c \equiv \sum_{t=1}^{L} \frac{r+pq+z}{(1+R)^{t-1}} = (r+pq+z)\sum_{t=1}^{L} \frac{1}{(1+R)^{t-1}} = y\sum_{t=1}^{L} \frac{1}{(1+R)^{t-1}} \equiv PV_y \tag{45}$$

where the capital cost in period 1 has been redistributed (using the borrowing mechanism) to create an equivalent expression that uses an annual rental cost *r*, and annual VMT (*q*), which under these assumptions will be the same in all periods. For (44) and (45) to be equivalent, the rental cost *r* must be chosen so that

$$\begin{aligned} C &= r\sum_{t=1}^{L} \frac{1}{(1+R)^{t-1}} = r \bullet \frac{1-\left(\frac{1}{1+R}\right)^{L}}{1-\left(\frac{1}{1+R}\right)} \\ \Rightarrow r &= C \bullet \frac{1-\left(\frac{1}{1+R}\right)}{1-\left(\frac{1}{1+R}\right)^{L}} = C \bullet CRF \end{aligned} \tag{46}$$

where CRF is called a capital recovery factor. So, the lifetime cost of vehicle ownership (for vehicle *j*) has been redistributed so that it has the same annual cost in all periods: *r + pq*. In fact, TIMES actually implements cost calculations this way, using the CRF in (46) to create the same annual cost for any particular vehicle investment (see footnote 7). At the same time, the NPV calculations are mathematically equivalent to what would be obtained using the equations in section 3. Most importantly, under these conditions *the solution to this multi-period problem is equivalent to the one-period problem discussed in section 4*, so that the results and insights from the previous sections can be immediately applied.

Consider the specific case of developing a choice model based on equation (38), under the assumption that there are *S* consumer segments (indexed *s = 1,…, S*). Assume also that explanatory variables in equation (19) can vary by segment (but not over time), but that preference parameters and other assumptions are otherwise the same. To be consistent with TIMES, assume that future fuel prices might vary, but are known to consumers (who have perfect foresight). Finally, assume that annual VMT is exogenously specified (but can vary by segment).

The model determining vehicle choice for any particular year (which can be arbitrarily labeled '1') is given by

$$
\begin{aligned}
NPVGC_{js1i} &= \sum_{t=1}^{L} \frac{r_j + p_{jt} q_s^* - \psi_{jsi} q_s^*}{(1+d)^{t-1}} = \sum_{t=1}^{L} \frac{r_j + p_{jt} q_s^* - \left[\bar{\psi}_j(b_{js}) + \varepsilon_{ji}\right] q_s^*}{(1+d)^{t-1}} \\
&= C_j + \sum_{t=1}^{L} \frac{p_{jt} q_s^*}{(1+d)^{t-1}} - \gamma q_s^* \left[\alpha_j + \sum_k \beta_k b_{jsk} + \varepsilon_{ji}\right] \qquad (47) \\
&= \overline{NPVGC}_{js1} - \varepsilon_{ji}^s
\end{aligned}
$$

where $NPVGC_{js1i}$ is the NPV of vehicle *j*'s *generalized cost* for a consumer *i* randomly chosen from segment *s* (in period 1), q_s^* is VMT for *s* (assumed to be the same for all members), ε_{ji} is a random term capturing the difference between consumer *i*'s quality index for vehicle *j* and the average for segment *s*, γ is a constant obtained by summing the discount factors, $\overline{NPVGC}_{js1}$ is the average *NPVGC* of vehicle *j* for segment *s*, and $\varepsilon_{ji}^s \equiv \gamma q_s^* \varepsilon_{ji}$. Some features of this example will be used in later discussions.

Although highly specific, (47) illustrates how theory-based assumptions and their implications can be understood and maintained. Earlier assumptions now yield a model where "units" are in NPV-dollars, and the first two terms in the middle equation of (47) coincide with standard financial calculations. The additional generalized cost terms based on equation (19) apply assumptions so that $\bar{\psi}_j(b_{js})$ and ε_{ji} are interpreted as "generalized variable costs" measured in units of dollars-per-mile. This is certainly appealing based on theoretical considerations.

However, at the same time, these expressions rely on the assumption of fixed and known VMT, so the "variable cost" interpretation is in a sense arbitrary. Combined with discussion in this section, which shows costs can be expressed in multiple equivalent ways, it would be possible to develop alternative measures based on fixed annual costs. In fact, once everything is expressed in NPV dollars for the current period, the quantity $\overline{NPVGC}_{js1}$ could be reinterpreted as being equivalent to a single pseudo-capital expenditure in period 1. This is, in effect, what will be done in the next section. This is convenient, because it provides a mechanism to include generalized costs that, while based on VMT, could be nonlinear rather than linear, as in equation (19).

One final point: although (47) might be considered "complex", it still includes many strong assumptions. For example, preference parameters are assumed to be the same for all segments, which is an assumption that is frequently relaxed in discrete choice modeling applications.

To summarize, individual-level consumer vehicle is assumed use a criterion of minimizing NPV of total lifetime (generalized) costs. To start, these costs are treated as coming from two sources: a fixed investment cost in the first period (*C*), and variable costs

(p_t) that occur over all periods during the lifetime of the vehicle.[17] Computing the full NPV requires the annual VMT (q_t), and a discount rate (d). This calculation can be performed as in equation (7) for monetary costs. This can be extended from monetary costs to generalized costs by augmenting C and/or p_t with appropriate generalized cost measures, which depend on the nature of the cost (investment versus variable), or through other equivalent approaches. Extending this to include features from discrete/continuous choice modeling theory, these measures could include uncertain, unobservable random factors from some specified statistical distribution.

5. Incorporating MA3T Consumer Behavior into TIMES

This section applies material from section 4 to produce empirical results. There are two main aspects of the MA³T choice model that will yield more realistic market response when integrated into TIMES: (i) introducing additional product dimensions that differentiate among competing vehicle fuel technologies, and (ii) introducing consumer heterogeneity to yield vehicle purchase shares rather than "all or nothing" behavior.

The first aspect (i) recognizes that vehicle choice is affected by factors other that simple monetary costs, which requires development additional generalized cost measures that can be justified by theory. These include, for example, lost time due to differences in refueling characteristics and/or station availability. As discussed below, these can often interact with the second aspect (consumer heterogeneity). One specific feature of MA³T is its inclusion of effects that can change over time due to the dynamics of market penetration—see, e.g., section 5.6.

The second aspect (consumer heterogeneity) can be classified into two types: observable and unobservable. The first (observable) is associated with consumer segmentation. For example, some households might live in housing types that can accommodate at-home recharging of plug-in vehicles, whereas others may not. This affects the practicality and convenience of owning plug-ins (which can be represented as a generalized cost), which will differ across segments. Other segmentation dimensions considered here include: annual VMT (three levels), availability of workplace recharging, and attitude toward risk for new technologies.

The second type (unobservable) is associated with the random disturbance term assumed for implementing discrete/continuous models, as in, e.g., (18) and (19). We propose introducing these effects into energy systems models by generating random draws from appropriately specified distributions to create multiple "clones" from each consumer segment.

[17] Annual fixed costs could also be included, but in an NPV framework an equivalent calculation could be performed that would absorb these into the period 1 cost.

To begin, it is obvious that the two models should be essentially equivalent if the same basic input assumptions are adopted (vehicle capital costs, efficiencies and lifetimes; fuel costs; average annual VMT; household population projections; etc.), and decisions were based only on monetary costs, with no introduction of random disturbance terms.[18] Choices in both models are based on cost minimization using NPV of lifetime vehicle costs. From this starting point, in what follows we sequentially introduce additional behavioral factors from MA³T into a TIMES model. Section 5.1 reviews results based on monetary costs only. Sections 5.2 through 5.6 add generalized cost factors that are based on vehicle characteristics and consumer segmentation (observable heterogeneity). In each of these sections the basis for computing relevant generalized costs is described. In terms of implementation, these costs are computed to represent the total lifetime cost associated with a vehicle purchase, which are added to the investment cost input data in TIMES. Section 5.7 is focused on introducing unobservable heterogeneity.

To be clear, the results produced here are from a TIMES model intended to address the same domain as MA³T (i.e., behavior of the household vehicle market in the United States for the period 2005 to 2050) for purposes of demonstrating the approach. We are also in the process of introducing these modifications into a model of the California energy sector (CA-TIMES).

A common set of input assumptions was developed to support comparison and validation. As noted previously, focus is on choice among vehicle fuel technologies shown in Figure 1, which are available in two versions: passenger cars and light trucks. All vehicles compete with one another in the household market, in contrast to many TIMES models where demand for passenger cars and light trucks are treated separately.

5.1 Capital investment and fuel operating costs

As noted previously, the starting point for this process is to base choice only on monetary costs, assuming no consumer heterogeneity. The relevant input data (new vehicle purchase prices, vehicle fuel efficiencies, fuel costs, the year in which a new vehicle technology is introduced, etc.) are adopted from MA³T, which uses vehicle price and efficiency attributes from the Autonomie Model (2013). One question that can arise in this type of modeling is how to address the possibility of price reductions from technological learning that might occur during the period immediately after introduction. To keep things simple, and because this issue is not central to the purpose of this paper, we use "learned out costs" to represent vehicle capital investment.

There are many other details required for setting up TIMES models that are beyond the scope of this paper but could be of interest to some researchers. For example, these

[18] In fact, there would be one slight difference: TIMES assumes perfect foresight and knowledge of future fuel costs, whereas MA³T assumes that consumers will expect future fuel costs to remain the same as current fuel costs.

include details on household population and VMT projections (which grow over time), the requirement to create 15 initial "cohorts" for the base year vehicle fleet (corresponding to the vehicle age distribution, assuming a common vehicle lifetime of 15 years), etc. In addition, as we add consumer segmentation dimensions in later sections, this requires additional sub-division of these initial cohorts. Another issue is how to properly address the case of vehicles that use multiple fuel types.

The initial results using monetary costs only are shown in Figure 6. These show new vehicle sales shares by fuel technology type, i.e., results have been aggregated over cars and light-duty trucks. These reveal a number of important features. First, these are the same results as in Figure 2, used to illustrate the well known "all or nothing" (or "knife edge") behavior associated with models of this type. Second, the complete switch to battery-powered EVs starting in 2020 is based on a major oversimplification, specifically, all households were assumed to have the capability of recharging a plug-in vehicle at home, and all households are assumed to have the same daily VMT (with no variation). Under the technology input assumptions, all VMT can be satisfied by an EV that starts out each day with a full charge, and the household chooses EVs in later years because they have the smallest levelized cost. However, all of these assumptions are unrealistic and will be relaxed in later sections. A related observation is that diesel is preferred (briefly) in 2012. However, this basic model omits the effect of limited diesel refueling stations (see section 5.3). Finally, we note that all of the purchased vehicles are cars (no light-duty trucks).

5.2 Consumer Segmentation by VMT Group

The first added factor recognizes that households can vary in their VMT patterns. This heterogeneity is introduced by dividing the household population into three VMT groups based on their average annual driving profile: low annual VMT (8,656 annual miles), average annual VMT (16,068 annual miles), and high annual VMT (28,288 annual miles)—see Lin and Greene (2011).The effect of adding this dimension is that total lifetime vehicle costs vary by group for all vehicle technologies, yielding the results in Figure 7.

These show the effect that VMT heterogeneity can have in a cost-minimization context. In 2012, each of the VMT groups chooses a different vehicle technology (low VMT = gasoline, average VMT = diesel, and high VMT = gasoline hybrid). High VMT households purchase the most fuel-efficient vehicle (gasoline hybrid) despite its higher capital cost, and when EVs are introduced in 2013, these households are the first to switch to EVs (which have higher capital cost, but also lower fuel-operating cost). Under the adopted input assumptions, future vehicle and fuel costs change over time in a way that cause all segments to eventually shift to EVs.

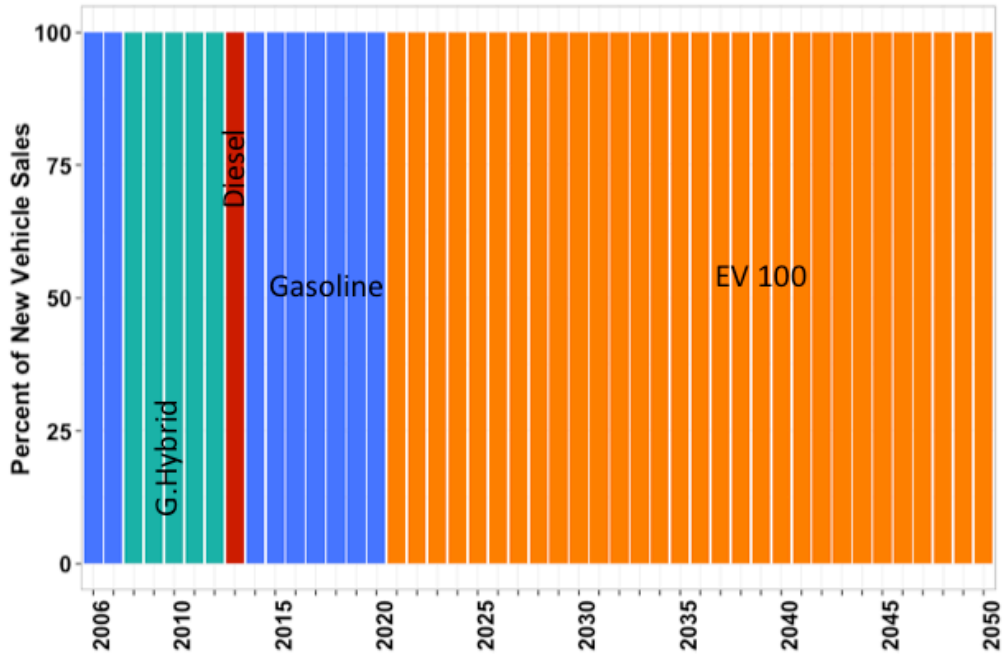

Figure 6. Model: New Vehicle Sales Shares--Monetary Costs Only

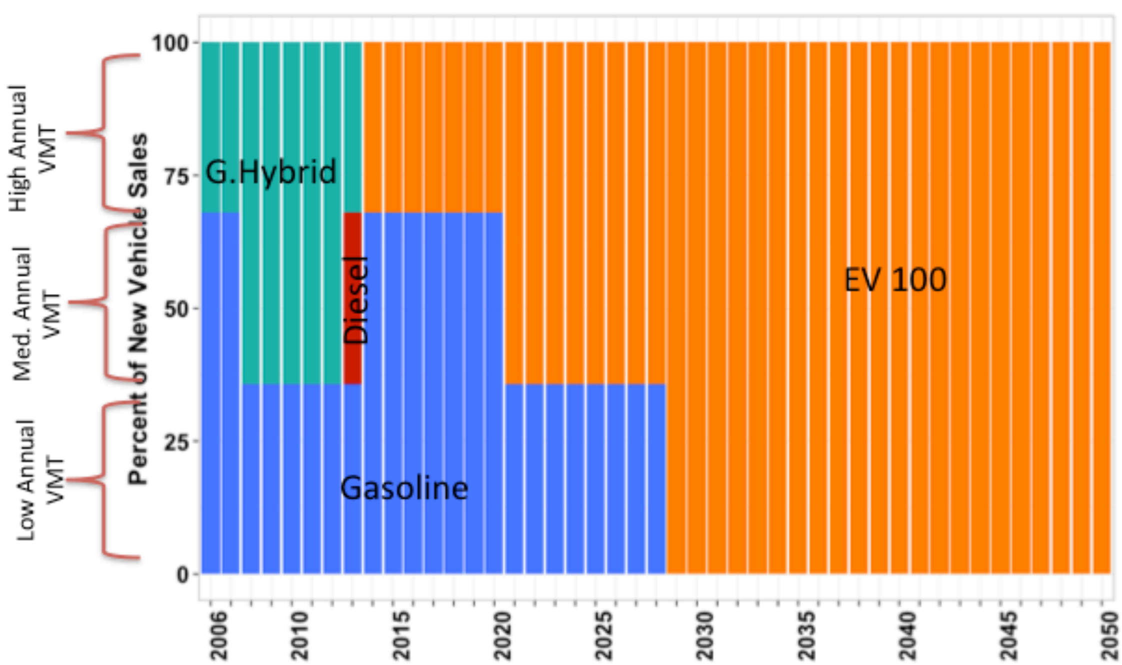

Figure 7. New Vehicle Sales Shares: Monetary Costs + Three VMT Groups

5.3 Inconvenience Cost for (Non-electric) Refueling Infrastructure

The next factor introduced relates to the spatial coverage of refueling station infrastructure. This specifically applies to liquid or gaseous fuels, e.g., diesel, ethanol, hydrogen, or natural gas, under the assumption that these fuels are only available at stations away from home. Under the input assumptions adopted here, this turns out to be relevant only for diesel vehicles. Although the term "inconvenience cost" may suggest some type of psychological distress or disutility associated with driving out of one's way to refuel a vehicle, the measure of inconvenience cost developed for MA^{3}T is consistent with the generalized cost discussion in section 4.6.

Specifically, while utility from "VMT as a commodity" is the same for all vehicles, a vehicle requiring fuel with limited refueling infrastructure would, on average, require a reallocation of time away from other, more desirable activities relative to a comparable vehicle with ubiquitous infrastructure (e.g., gasoline). This would create a loss of utility, as measured from a reduction in consumption of other goods. Recall that the outside good is viewed as a numeraire good (measured in dollars), and in such cases it is common practice to evaluate its reduction from lost time by using the household *wage rate*.

Greene (1994) develops an estimate for refueling inconvenience measured as the additional annual time required by a vehicle fuel technology relative to a comparable gasoline vehicle. First, required number of (annual) refueling events is estimated. This depends on the VMT usage pattern, fuel efficiency, and energy storage capacity of the vehicle. Fuel efficiency and storage capacity are used to compute vehicle range when fully refueled. Combining this with VMT yields the number of refueling trips required. Second, the amount of time required per trip depends on two factors: the actual time to refuel the vehicle, and any extra time required to travel to a refueling station. The first factor depends on the technology, and the second factor depends on the availability of stations. Station availability relative to gasoline is measured using an index, for which 100% would be "the same as gasoline." Using the estimated number of trips, and extra time required per trip, the total extra time (relative to gasoline) is computed. This is converted to dollars by multiplying the estimated average wage rate for households (Lin and Greene 2010; Melaina, Bremson, and Solo 2013)[19].

Under the input assumptions, the results when adding this cost are essentially identical to Figure 7 (so they are not shown). The only difference is that the diesel vehicles purchased in 2012 are shifted to gasoline hybrids. This factor becomes much more important when considering scenarios involving, e.g., vehicles using hydrogen or natural gas.

5.4 Range Limitation Costs for Vehicles Using Battery Technology

As already discussed, a narrow focus on capital and fuel-operating cost yields the results shown in Figure 6 where EVs achieve 100% new vehicle sales share starting in 2020. However, the range and recharging limitations of these vehicles have been a major subject

[19] The details are rather complex, and are not included here.

of concern and discussion for decades. Discrete choice models using stated choice experiments have been used to empirically estimate measures of "consumer disutility" associated with range limitations: see, e.g., Bunch et al. (1993) or Brownstone, et al. (2000). While these empirical measures could be used as a measure of generalized cost, Greene and co-authors have taken a different approach that relies on a more detailed "bottom up" exploration of the implications of limited range and its interaction with data on actual vehicle usage behavior.

For example, suppose that the household driver: (i) has a completely fixed driving routine that is exactly the same every day, (ii) the routine accounts for all of the household's VMT, (iii) the vehicle can be recharged at home overnight and start each day with a full charge, and (iv) the daily VMT requirement is less than the vehicle range. Then, the results in Figure 6 might be possible. However, the reality is much more complex.

First, daily VMT requirements are not constant and fixed, but are randomly distributed. So, there is at least some positive probability that it would be difficult to use a limited range vehicle on a variety of occasions. Moreover, because VMT needs can be uncertain even during a given day for an individual driver, as the remaining vehicle range approaches zero a driver would face the prospect of running out of charge, which would be extremely inconvenient. Psychologically, concern about this potential risk would lead to "range anxiety" which, as noted in Lin (2012) is difficult to measure. In the economics-based behavioral framework developed here, it is possible to perform a type of risk analysis leading to a measure of (expected) "range limitation cost." This involves estimating the probability of "insufficient range events", and also a way of assigning a cost measure to such outcomes. For example, if on a particular day a vehicle's range was insufficient to satisfy all mobility needs, this would require procurement of additional mobility services (e.g., a cab, rental car, or asking a favor of one's brother-in-law) to cover the shortfall. The goal is to develop an estimate of expected (generalized) cost that, as in earlier discussions, could be viewed as a measurement of lost utility associated with a reduction in consumption of the outside (numeraire) good.

However, there are a number of factors that would play a role in such an analysis. Until now, we have assumed that anyone owning a plug-in vehicle could obtain a full charge by plugging it in overnight. This is clearly incorrect, because there will be households that (for all practical purposes) lack home recharging capability. The initial scenario in the previous sections assumed that all households had access to home recharging. Based on an analysis of National Household Transportation Survey data (2009), the MA^3T model assumes that only 52% of households have access to home recharging infrastructure.

But, even if home recharging is unavailable, there are other options. Some percentage of households will have access to recharging at work (Lin and Greene 2011). Anecdotally, many EV owners with workplace recharging rarely use home recharging, even if they have a level 2 charger installed at home. In any case, the availability of both options would clearly affect the details of the "risk analysis" calculation described above. Finally, a complete analysis would include the impact of public recharging infrastructure. As in section 5.3, one concern is the availability of stations; however, the overall problem addressed here is very different. The major focus here is the effect that all of these factors have on the probability that the on-battery driving range of a plug-in vehicle will be exceeded, affecting the distribution of additional range-related costs.

To review, key factors affecting the probability that on-battery range will be exceeded are: vehicle range on a fully-charged battery, the distribution of daily VMT (not just the average), home recharging capability, workplace recharging availability, and public recharging station availability. Two of these factors (home and work recharging) expand the number of consumer segments by a factor of 4, and an assessment of how households are distributed across segments becomes another input assumption requirement. Two of these (workplace and public station availability) could vary substantially over time, and could be heavily influenced by policy makers. Also important to note: Although for now our focus will be on dedicated BEVs, these factors would affect calculation of vehicle fuel cost estimates for multiple-fuel vehicles (e.g., plug-in gasoline hybrids).

The type of calculations used to estimate range inconvenience costs are discussed in Lin and Greene (2011). First, they use NHTS data to estimate the distribution of daily VMT (assumed to be gamma) for each of the three groups discussed previously. Taking into account the various factors discussed above (which vary by consumer segment), they estimate the probability that range will be insufficient on any given day (P_I). Specifically, computing P_I requires evaluating an integral that depends on the daily VMT distribution for the segment, the vehicle range on a full charge, and the nature of the portfolio of recharging options available to the segment. Multiplying P_I by 365 gives the expected number of "insufficient range" days per year. Finally, this is multiplied by a per-day cost penalty to obtain an annual expected cost.

The daily inconvenience cost penalty (in $/day) is based on researcher judgment apply arguments discussed previously (i.e., need for substitute mobility services), and we adopt those used in MA^3T. In their approach, they link this figure to another consumer segmentation dimension: attitude toward risk. Households are assumed to fall into one of three categories (early adopter, early majority, and late majority), and the cost penalty is assumed to increase with increasing risk aversion (specifically, $10/day for early adopters, $20/day for early majority and $50/day for late majority)—see Lin and Greene (2011).

Note that these calculations only apply to dedicated BEVs, and not dual-fuel vehicles such as, plugin hybrid vehicles. In those cases, the approach used to compute the effect of available recharging options is used to determine the share of electricity used as a fuel. For example, in a scenario where there is no work or public recharging available, a household with no home refueling capability would operate a plugin hybrid vehicle the same as a pure gasoline hybrid vehicle.

The next sequence of results has been produced to illustrate the effects of the various factors that have been discussed. First, we add range inconvenience cost to the previous input assumptions, and assume that all households are "late majority."

Specifically, all households have access to home recharging (but not work recharging or public recharging), are “late majority” in their attitude toward risk, but there are three VMT groups. See Figure 8. Adding range inconvenience cost removes all EVs from the previously observed mix, and vehicle purchases are almost exclusively gasoline. In some periods the high VMT households prefer gasoline hybrids based on their lower lifetime costs.

The next results introduce public recharging, using the assumptions shown in Figure 10. The number of recharging locations starts small, and levels off at a maximum of about 15,000 in 2035, with an average of three charge-points per station, all Level II chargers. The results from adding public recharging are shown in Figure 11, and the effect is relatively small. In later years the high annual VMT drivers switch from gasoline, and gasoline hybrids, to PHEV 10s.

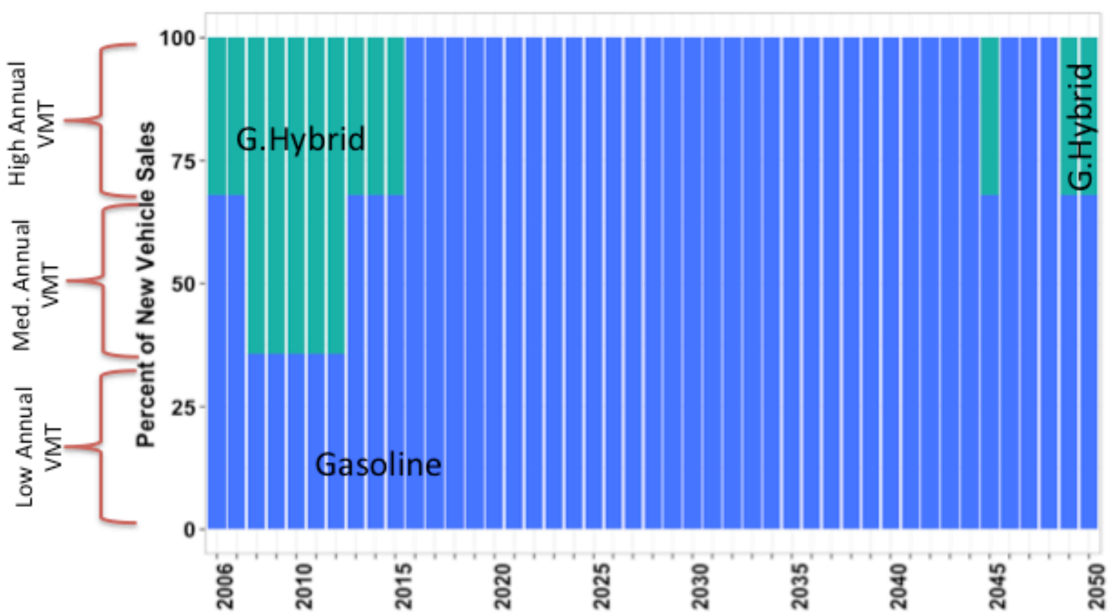

Figure 8. New Vehicle Sales Shares: Monetary Costs, Refueling Inconvenience Cost, Range Limitation Cost + Three VMT Groups

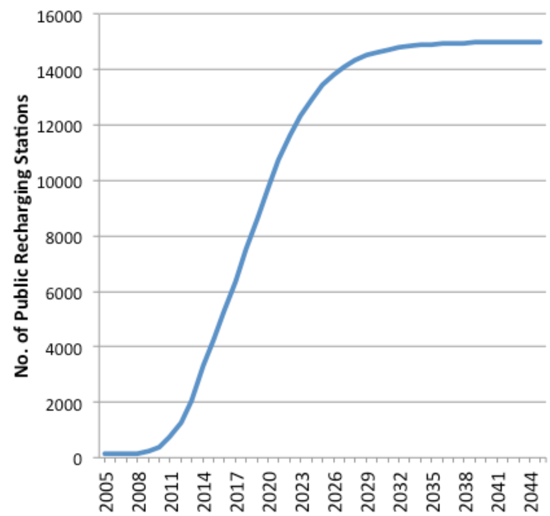

Figure 9. Nationwide Public Recharging Station Trajectory Assumption

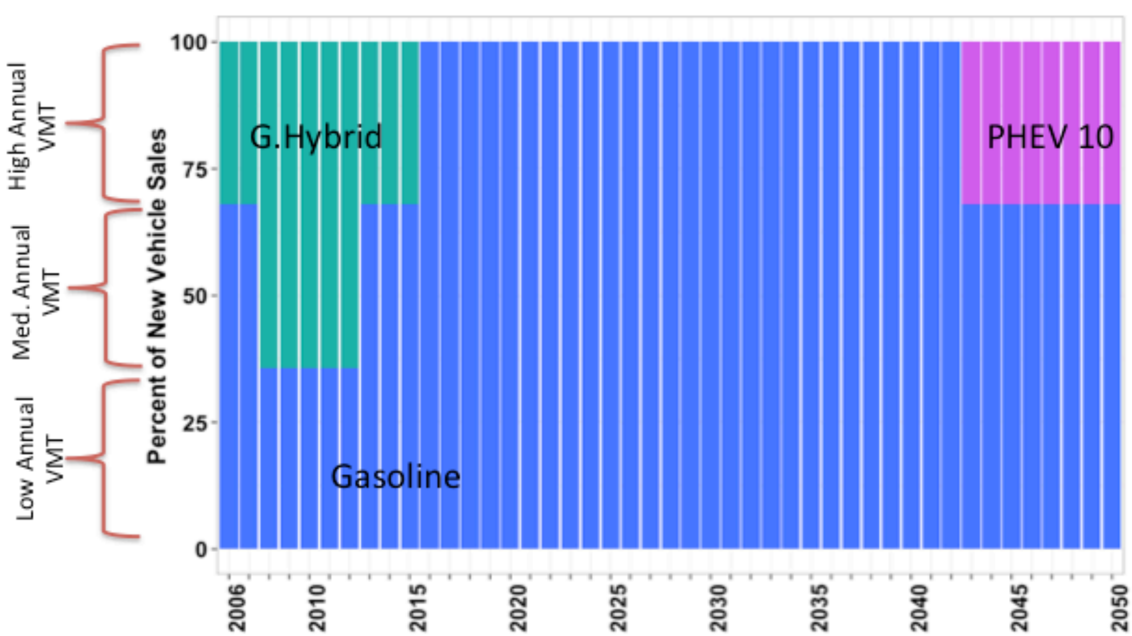

Figure 10. New Vehicle Sales Share: Monetary Costs, Refueling Inconvenience Cost, Range Limitation Cost, Three VMT Groups with Public Recharging Infrastructure

It is important to remember that the Figure 10 assumes that all households have home recharging. However, MA³T assumes only 52% of households have home recharging capability. Introducing this factor doubles the number of segments from three to six, yielding the results in Figure 11. PHEV 10 demand is cut in half, presumably because of switching by households without home recharging.

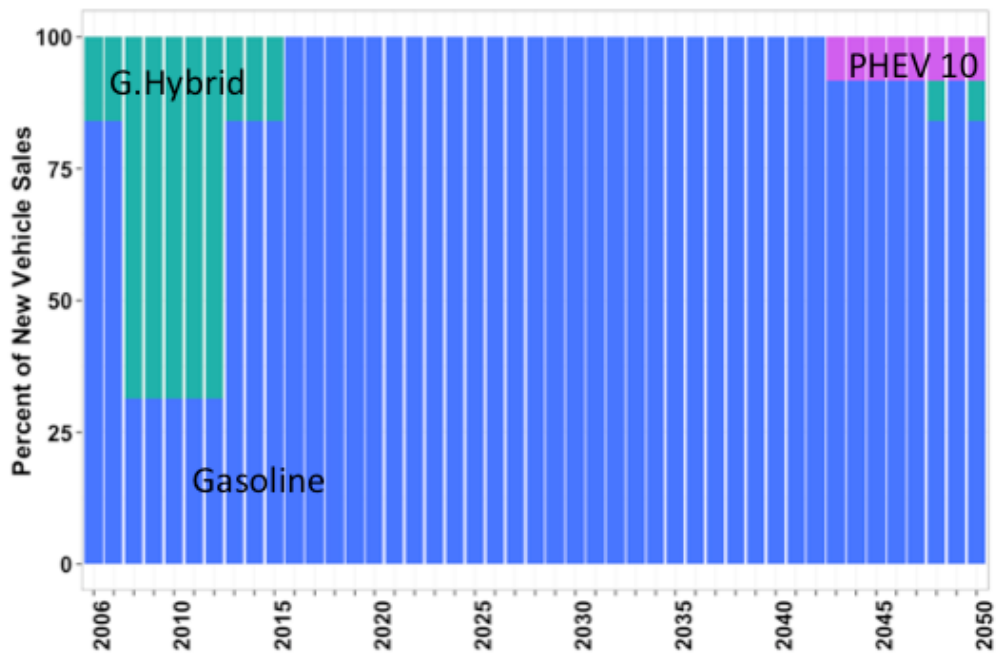

Figure 11. New Vehicle Sales Share: Monetary Costs, Refueling Inconvenience Cost, Range Limitation Cost, Six Consumer Groups, No Public Recharging Infrastructure

The next step is to add the dimension of workplace recharging. This doubles the number of consumer segments (again) to 12 groups. Similar to public recharging infrastructure, we assume a gradual increase in workplace recharging investment over time, so that in 2050 about 5% of the US population has access to workplace recharging—see Figure 12. The associated results are shown in Figure 13. The impact of adding this dimension is that now there are consumer segments for which BEVs are the best option. Households with both home and workplace recharging have small enough range limitation costs that these vehicles become viable.

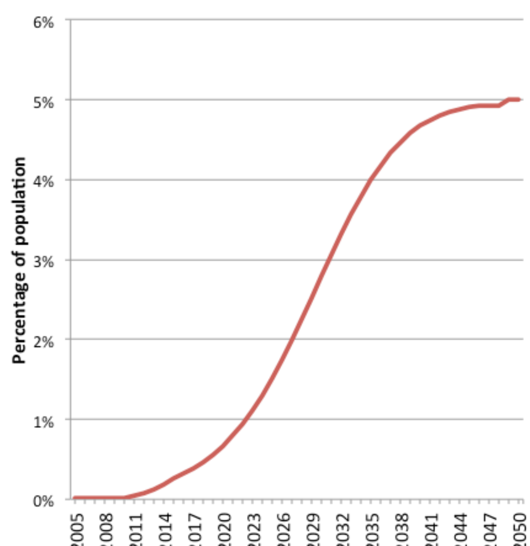

Figure 12. Nationwide Percentage of Population having access to Workplace Recharging Trajectory Assumption

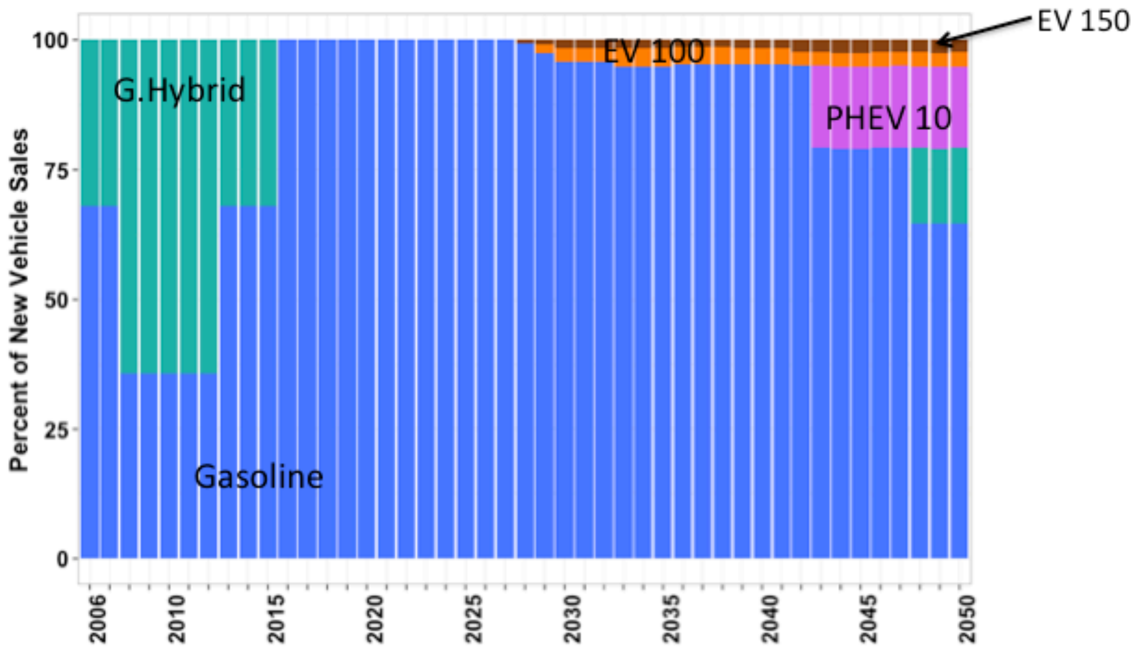

Figure 13. New Vehicle Sales Share: Monetary Costs, Refueling Inconvenience Cost, Range Limitation Cost + Twelve Consumer Groups

5.5 Effect of Risk Attitude Consumer Groups

In the previous section we focused on introducing segmentation dimensions related to recharging infrastructure, while assuming that all segments belonged to the same risk-attitude group (late majority, i.e. the most risk averse). We now formally add this dimension, tripling the number of consumer segments to 36. (Recall there are three categories: early adopters, early majority and late majority). This has two major implications. First, each group is assigned a different range limitation cost, as described in section 5.4. Early adopters and early majority have lower costs, and Figure 14 shows results from adding this factor. Compared to Figure 13, Figure 14 shows increased sales of EV 100 and PHEV 20 vehicles in the later years.

The other effect associated with this dimension is a more direct implementation how attitude toward risk affects preference for new technologies. For example, a newly introduced BEV would be perceived as risky by late majority households, which could be represented by an additional generalized cost, lowering overall preference. As cumulative sales for these vehicles increases over time, this cost would get smaller, eventually reaching zero. Similarly, early adopters (a.k.a. innovators) would actually have a higher preference for newly introduced technology (or a positive utility), which would also diminish as sales accumulate. MA^{3}T includes these factors: for a discussion, see Greene (2001). Adding these yielded results that were indistinguishable from Figure 14, so they are not shown.

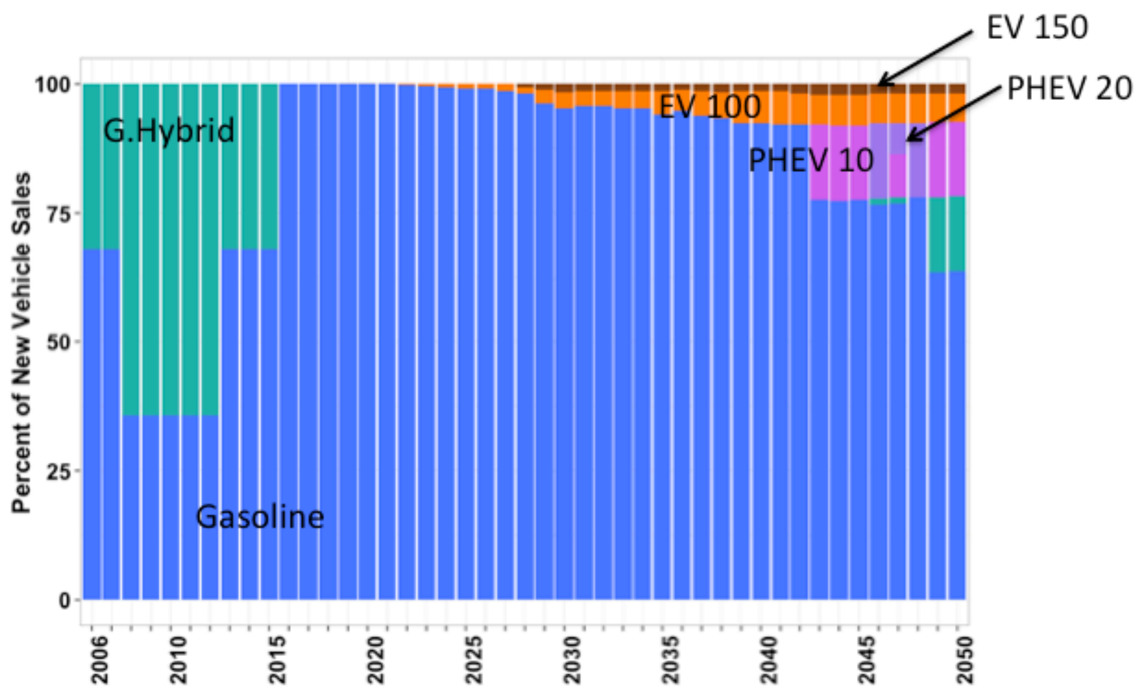

Figure 14. New Vehicle Sales Share: Monetary Costs, Refueling Inconvenience Cost, Range Limitation Cost, Thirty-six Consumer Groups

5.6 Vehicle Make/Model Diversity

The next factor we consider relates to the number of vehicle models that are available for a given vehicle technology. This is a phenomenon that has been studied in the discrete choice literature. When modeling vehicle choice using a system of "vehicle classes," it is important to recognize that the number of makes and models will vary across vehicle classes, and that the attractiveness of a class increases with the number of models, due to the increased diversity of the offerings. This additional attractiveness can be represented as a (negative) generalized cost.

In the context of this paper, note that the set of vehicle technologies in Figure 1 is, in effect, a system of vehicle classes. When a new vehicle technology is introduced to the market it will only be available in a limited number of makes and models. Relative to, e.g., the established class of gasoline cars, this new class should have an additional generalized cost because it has many fewer models. However, as the new technology penetrates the market, manufacturers will add more makes and models, and this cost would diminish.

This model availability cost is modeled in MA^{3}T as described in Greene (2001). A key feature is that its value changes dynamically as a function of previous sales. For this application we extract this measure from MA^{3}T and use it as input data. For more general use in TIMES models, we have developed iterative procedures to capture this effect.

Introducing this factor yields the results in Figure 15, and it has a significant effect. This model availability cost severely penalizes all non-conventional technologies, so that only a very small number of BEVs and PHEVs are purchased in the latest years.

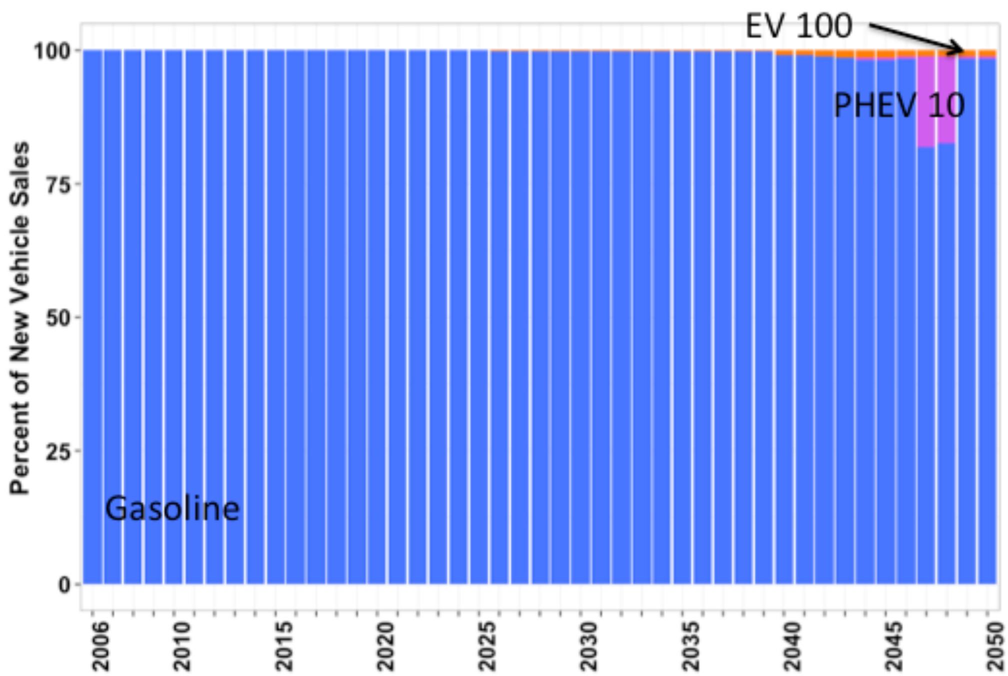

Figure 15. New Vehicle Sales Share: Monetary Costs, Refueling Inconvenience Cost, Range Limitation Cost, Model Availability Cost + Thirty-six Consumer Groups

5.7 Incorporation of Unobserved Consumer Heterogeneity

Previous sub-sections sequentially added behavioral factors related to generalized costs and consumer segmentation, and an obvious conclusion drawn from the results is that any combination of such factors could have a dramatic effect on the projected outcome of vehicle market behavior in a TIMES modeling framework. This is clearly related to the "all or nothing" phenomenon that occurs as a consequence of the underlying behavioral assumptions implicit in TIMES. However, it is worth noting that the varying qualitative nature of these results, while not unexpected, was not necessarily a foregone conclusion: an alternative possibility was that introducing these factors might have had little effect on the outcome, e.g., that the results could have been dominated by monetary costs alone.

In any case, because making this determination first required a systematic development of factors by drawing on insights from discrete choice theory and applications, it is clearly important to now consider implications from the other aspect of the theory, namely, unobservable effects that are randomly distributed from the perspective of the analyst.

Equation (47) provided a prototype example for this process. Although the specific assumptions differ, the previous sub-sections represent sequential development of appropriate segment-level estimates for $\overline{NPVGC}_{js1}$ based on the MA³T model. Recall that the choice model used by MA³T is a nested multinomial logit model, which is consistent with the RUM framework discussed in section 4.2, represented by equation (11). The multinomial logit (MNL) model in equation (12) is a simpler special case of this model.

For purposes of illustration, assume that MA³T is MNL rather than nested logit, and that a preference model has been formulated as in equation (13) to represent the negative of $\overline{NPVGC}_{js1}$. The information required to produce choice probabilities using equation (12) is knowledge of the parameter μ, which is a scale parameter for the Gumbel distribution. Specifically, if the $\bar{V}_j$ terms in (12) are replaced by $\overline{-NPVGC}_{js1}$ terms, a correctly determined value of μ will produce good estimates of purchase shares for segment *s* (assuming MNL is an appropriate model). Without going into details, μ can be determined using statistical estimation or calibration procedures. It would be typical to use the model to compute vehicle market shares for analysis.

However, the TIMES methodology is very different. Our approach is to create multiple "clones" for each segment, where, for each clone, random draws are generated for all the competing vehicles from the appropriate Gumbel distribution and then subtracted from their mean NPVGC values. This creates additional heterogeneity by replacing the single representative household for each segment (used previously) by the collection of clones. The random variation is consistent with the assumptions of MA³T, where parameters (e.g., μ for the MNL model) were calibrated based on empirical data.

Note again that the procedures required for generating random draws from the MA³T nested logit model are actually much more complex than for MNL, and to our knowledge this is the first time anyone has taken this approach in the literature. We are, in effect, turning TIMES into a simulation-based model to product market share projections that approximate the behavior of a nested logit model. The reason this has not been done before is that, in the discrete choice literature, one of the main appeals of nested logit is that it has a closed-form expression for computing choice probabilities, so there would seem to be no need for simulating nested logit model disturbance terms. The technical details of how this is done are provided in a separate report (Bunch 2015).

The impact of this process is shown by first examining the effect of one clone per segment (recall there are 36). The no-clone results were shown in Figure 15. Figure 16 shows the results of 9 different sets of random draws (where each set generates 36 clones). Adding even this small number of random draws produces results that are very different from Figure 15, and as might be expected, there is a substantial amount of variability from panel to panel.

Theory would suggest that as the number of clones per segment increases, the variability should diminish and results will approach smoother patterns of the type produced by the MA³T model. However, there will be a practical tradeoff between number of clones used and computational resources required (in both time and memory). Figure 17 shows results for 5 clones per group, which yields a marked improvement in stability.

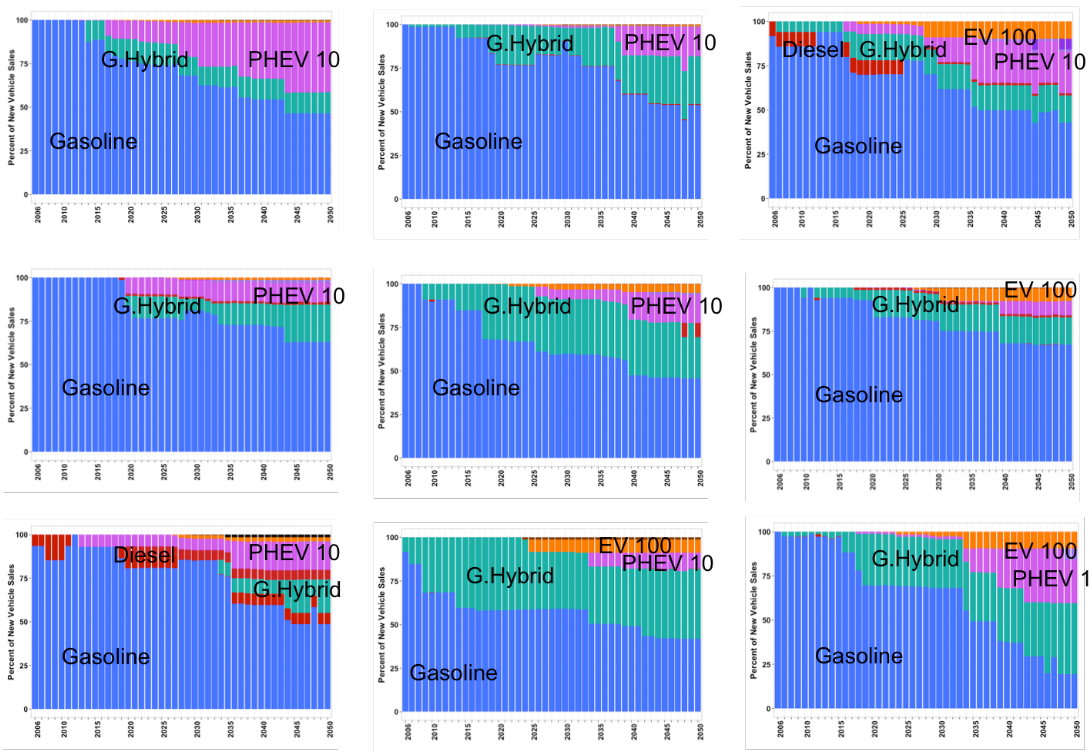

Figure 16. New Vehicle Sales Shares: One clone per Consumer Group

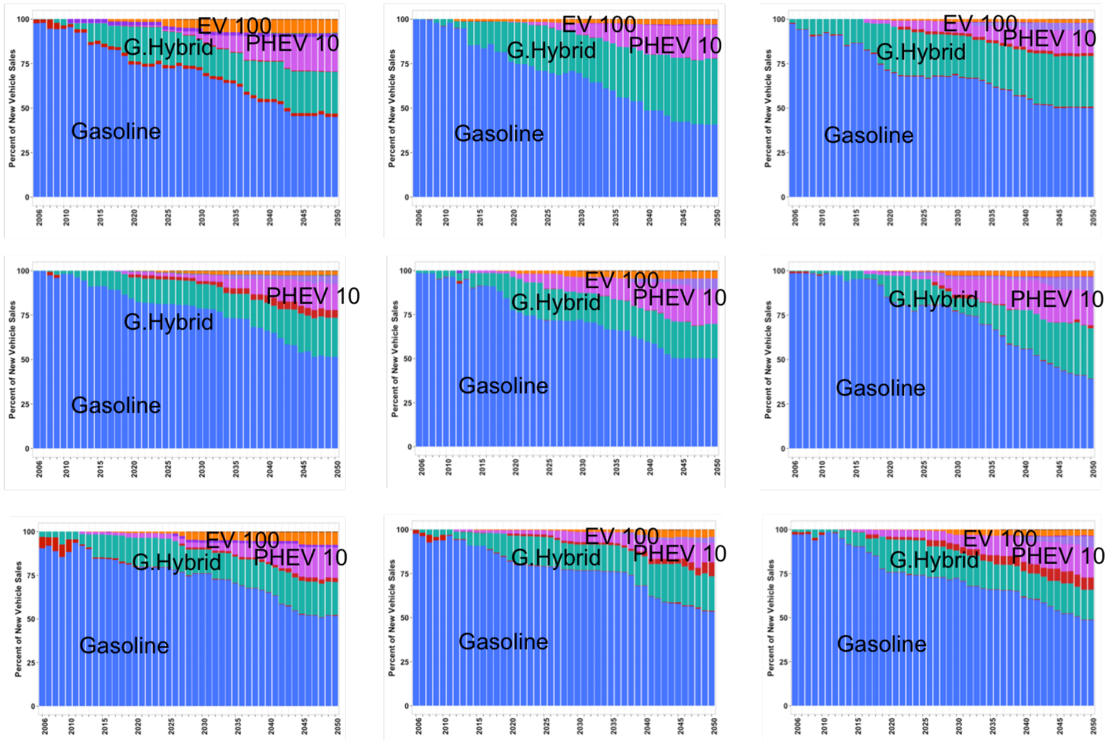

Figure 17. New Vehicle Sales Shares: Five clones per Consumer Group

Figure 18 shows the effect of increasing the number of clones to 20 per segment. At this level it is difficult to distinguish differences across the panels.

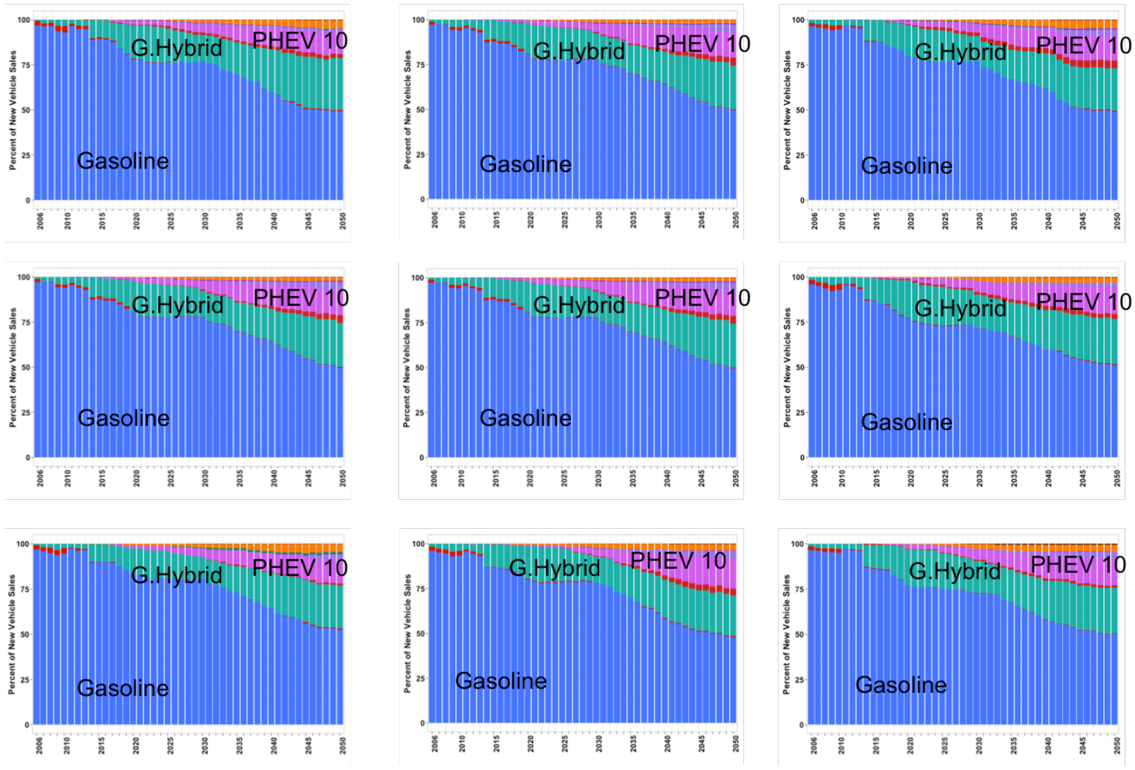

Figure 18. New Vehicle Sales Shares: Twenty clones per Consumer Group

Finally, Figure 19 makes a direct comparison between the 20-clone results and the results from MA³T. They are very similar, even though they rely on slightly different assumptions about consumer knowledge of future fuel prices. This difference could have a larger effect in some scenarios, if they were to have volatile changes in fuel prices over the planning horizon.

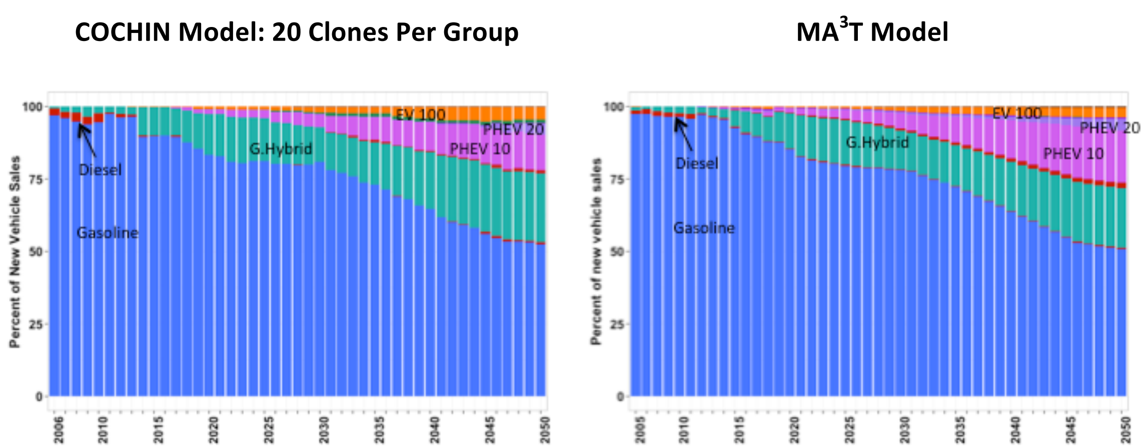

Figure 19. Comparison of New Vehicle Sales Shares between COCHIN and MA³T Models

6. Summary and Conclusions

This report addresses a problem that has been frequently mentioned in the energy modeling literature: the challenge of producing results consistent with realistic consumer market response in energy systems models. The treatment here is comprehensive, and emphasizes the importance of developing methods based on a solid theoretical framework, which yields insights that allow results and approaches from multiple literatures to be integrated into the same basic framework.

Our current plans are to apply these approaches in larger energy systems models, and in addition, to extend the theory to address the issue of elastic demand in a similar fashion. Finally, in performing this work a number of questions arose about some of the underlying assumptions that are routinely used in these models. For example, if households drive vehicles different distances, why would the vehicles all be treated as having the same lifetime? Similarly, in considering discrete choice models, would the scale of the random error term be different across segments? These are questions that have not typically arisen in either literature.

References

ANL, ed. 2013. *Autonomie Model*. Argonne National Laboratory. http://www.autonomie.net/.

Ben-Akiva, M E, and S R Lerman. 1985. "Discrete Choice Analysis: Theory and Application to Travel Demand."

Bento, A M, L H Goulder, and M R Jacobsen. 2009. "Distributional and Efficiency Impacts of Increased US Gasoline Taxes." *The American Economic Review*.

Blackburn, Anthony J. 1970. *A Non-Linear Model of the Demand for Travel*.

Brownstone, David, David S Bunch, and Kenneth Train. 2000. *Joint Mixed Logit Models of Stated and Revealed Preferences for Alternative-Fuel Vehicles*.

Bunch, David S. 2015. "Simulating Random Disturbance Terms for Nested Logit Models for Use in Large Scale Energy Systems Models." *Working Paper (in Preparation)*.

Bunch, David S, Mark Bradley, Thomas F Golob, Ryuichi Kitamura, and Gareth P Occhiuzzo. 1993. "Demand for Clean-Fuel Vehicles in California: a Discrete-Choice Stated Preference Pilot Project." *Transportation Research Part a: Policy and Practice* 27 (3): 237–53. doi:10.1016/0965-8564(93)90062-P.

Deaton, A, and J Muellbauer. 1980. "Economics and Consumer Behavior - Angus Deaton, John Muellbauer - Google Books."

Dubin, Jeffrey A, and Daniel L McFadden. 1984. "An Econometric Analysis of Residential Electric Appliance Holdings and Consumption." *Econometrica* 52 (2): 345. doi:10.2307/1911493.

Greene, D L. 1994. *Alternative Fuels and Vehicles Choice Model*. doi:10.2172/28301.

Greene, D L. 2001. "TAFV Alternative Fuels and Vehicles Choice Model Documentation."

Hanemann, W Michael. 1984. "Discrete/Continuous Models of Consumer Demand." *Econometrica* 52 (3): 541. doi:10.2307/1913464.

Herbst, A, F Toro, F Reitze, and E Jochem. 2012. "Introduction to Energy Systems Modelling." *Swiss Journal of Economics and Statistics*.

Intriligator, Michael D. 2002. *Mathematical Optimization and Economic Theory*. SIAM.

Lin, Z, and D L Greene. 2010. "Rethinking FCV/BEV Vehicle Range: a Consumer Value Trade-Off Perspective."

Lin, Z, D Greene, and J Ward. "User Guide of the ORNL MA 3 T Model (V20130729)." *Cta.Ornl.Gov.*

Lin, Z, J Dong, C Liu, and D L Greene. 2012. "PHEV Energy Use Estimation: Validating the Gamma Distribution for Representing the Random Daily Driving Distance." *TRB 91st Annual Meeting …*.

Lin, Z, J M Li, and J Dong. 2014. "Dynamic Wireless Power Transfer: Potential Impact on Plug-in Electric Vehicle Adoption."

Lin, Zhenhong, and David Greene. 2011. "Promoting the Market for Plug-in Hybrid and Battery Electric Vehicles." *Transportation Research Record: Journal of the Transportation Research Board* 2252 (December). Transportation Research Board of the National Academies : 49–56. doi:10.3141/2252-07.

Loulou, R, G Goldstein, and K Noble. 2004. "Documentation for the MARKAL Family of Models." *Energy Technology Systems Analysis Programme*.

Loulou, R, U Remme, A Kanudia, A Lehtila, and G Goldstein. 2005a. "Documentation for the TIMES Model Part II." *Energy Technology Systems Analysis Programme*.

Loulou, Richard, U Remme, A Kanudia, A Lehtila, and G Goldstein. 2005b. *Documentation for the TIMES Model: Part I*. Energy Technology Systems Analysis Programme.

Mannering, F, and C Winston. 1985. “A Dynamic Empirical Analysis of Household Vehicle Ownership and Utilization.” *The RAND Journal of Economics*.
Manski, Charles F, and Daniel McFadden. 1981. *Structural Analysis of Discrete Data with Econometric Applications*.
McCollum, D, C Yang, S Yeh, and J Ogden. 2012. “Deep Greenhouse Gas Reduction Scenarios for California–Strategic Implications From the CA-TIMES Energy-Economic Systems Model.” *Energy Strategy Reviews*.
Melaina, M, J Bremson, and K Solo. 2013. “Consumer Convenience and the Availability of Retail Stations as a Market Barrier for Alternative Fuel Vehicles.”
Pollak, Robert A, and Terence J Wales. 1978. *Demographic Variables in Demand Analysis*. Vancouver: Department of Economics, University of British Columbia.
Schaefer, A, and H D Jacoby. 2005. “Technology Detail in a Multisector CGE Model: Transport Under Climate Policy.” *Energy Economics*.
Takayama, Takashi, and George G Judge. 1971. *Spatial and Temporal Price and Allocation Models*.
Varian, Hal R. 2000. *Microeconomic Analysis*.
Wing, I S. 2004. “Computable General Equilibrium Models and Their Use in Economy-Wide Policy Analysis.” *Technical Note*.
Wing, Ian Sue. 2006. “The Synthesis of Bottom-Up and Top-Down Approaches to Climate Policy Modeling: Electric Power Technologies and the Cost of Limiting US CO2 Emissions.” *Energy Policy* 34 (18): 3847–69. doi:10.1016/j.enpol.2005.08.027.
2009. *National Household Travel Survey*. Oak Ridge National Laboratory. http://nhts.ornl.gov/.