Departure Time Choices in Traffic Congestion

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Introducing myself

• Takamasa Iryo, Dr. Eng. Assistant Professor, Kobe University, working with Prof. Asakura.

• Studied in the University of Tokyo till 2002 and got a degree under the supervision of Prof. Kuwahara.

• Main topic: Transport Network Analysis, esp., DTA / Pedestrians
Kitamura sensei in my study

- Kobe is near to Kyoto.
- Easy to visit Kyoto.
- I had many chances to have discussions with “Sensei” and his staffs and students.
Today’s talk

- Departure Time Choices: in the context of the dynamic traffic assignment

- Equilibrium Approach: A joint work with Prof. Yoshii at Kyoto University.

- Non-equilibrium Approach
Behaviour vs. Network

• Modelling traveller’s behaviour of departure time choices: Behavioural Study (e.g. Small(1982))

• Modelling traveller’s interactions through departure time choices under congestion. Network Assignment Study (e.g. Vickrey(1969))
Time choice under congestion

- Define a schedule cost function. (bi-linear or non-linear)

- Define travel cost function, which is the sum of schedule cost and delay cost at a bottleneck.

- Assume travellers choose the departure time with the lowest cost in a day.
Equilibrium or Non-equilibrium?

- [Equilibrium] Find traveller’s departure time choice pattern satisfying Wardrop’s first principle within a day: Looking for a temporal equilibrium point

- [Non-equilibrium] Construct a day-to-day model and see change of traveller’s behaviour over days: Simulating a day-to-day dynamics of travellers
Equilibrium


• My contribution with Prof. Yoshii
  - There is an optimization problem that is equivalent to the equilibrium condition.
  - Total schedule cost of travellers is minimized in equilibrium.
  - The model is applicable to the more general situation; “single-bottleneck-per-route”.

Traveller’s choice (route and arrival time)  

Schedule cost (incl. detour cost)

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<th>Minimize</th>
<th>$\sum_{i,j} x_{(i,j)} \lambda_{(i,j)}$</th>
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<td>$10$</td>
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Subject to:

- $\sum_{j} x_{(i,j)} A_{i(j)} \leq 39.834$  
  Capacity Constraint

- $\sum_{i} x_{(i,j)} D_{j} \leq 39.0729$  
  Demand Constraint

- $x_{(i,j)} \geq 0$  
  Non-Negative Constraint

The KKT condition of the optimization problem is equivalent to the Wardrop’s first principle.
Maximize \[ \sum_{i=1}^{n} D(j) \theta_i(\phi) - \sum_{i=1}^{n} a(i) \phi_i(\phi) \times \frac{\phi_1}{\phi_{10}} \times 44.46 \]

subject to \[ w(i) \geq \frac{\theta_0}{\phi_{10}} 44.46 \Theta \geq 1 \]

\[ w(i) + T_{\phi(i,j)}T_{\phi(0,j)} \Theta \forall i \text{ and } j \]

Delay at the bottleneck Travel cost

The dual problem can solve delays at bottlenecks; delays are the dual variable of the primal problem.
Day-to-day dynamics

- A simple day-to-day dynamics is constructed after Smith(1984) and Mounce(2006).

- Travellers move towards a choice with smaller cost gradually.

- A system converges to equilibrium if travel cost function is monotone increase against number of travellers; it is normal in traffic assignments which is not dynamic.
Day-to-day dynamics

- Monotonicity of travel cost is not guaranteed where travellers are faced to a schedule constraint, especially when they try to arrive on time.

- Behaviour of travellers do not converge to equilibrium in some cases.

From system to behaviour

• Equilibrium: Simple.
  Good characteristic (equivalent to optimization)

• Equilibrium is not stable, may not be achieved??

• What is the meaning of equilibrium?
  How travellers behave under non-equilibrium situation?

• Behavioural viewpoint might be demanded to think of the meaning of equilibrium.